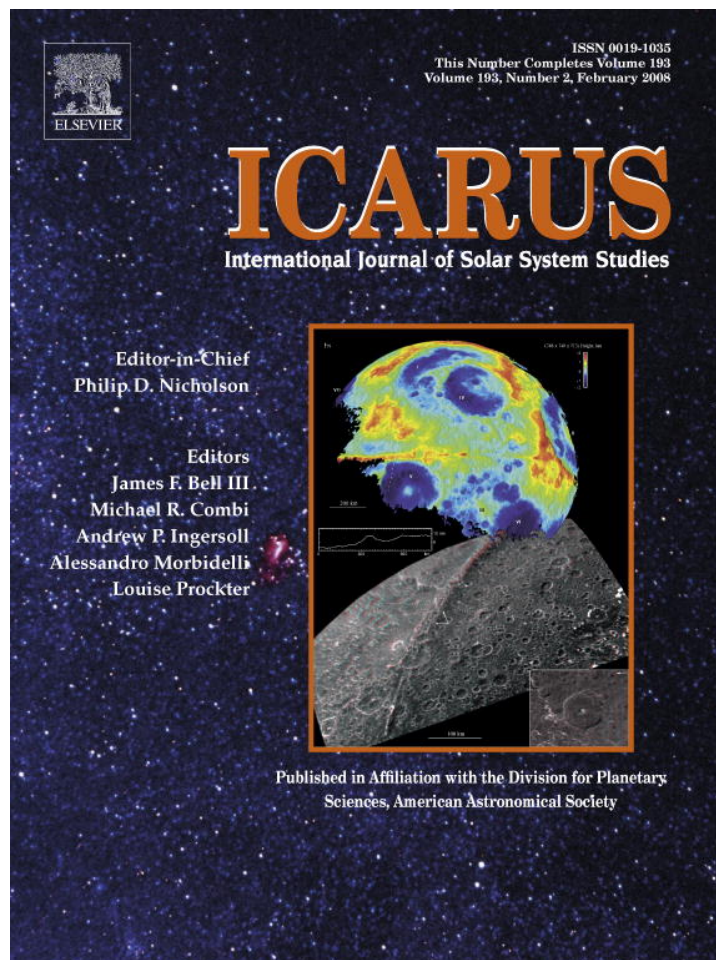


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# A steady-state model of NEA binaries formed by tidal disruption of gravitational aggregates

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## Abstract

We present results of a simulation of a steady-state binary near-Earth asteroid (NEA) population. This study combines previous work on tidal disruption of gravitational aggregates [Walsh, K.J., Richardson, D.C., 2006. *Icarus* 180, 201–216] with a Monte Carlo simulation of NEA planetary encounters. Evolutionary effects include tidal evolution and binary disruption from close planetary encounters. The results show that with the best known progenitor (small Main Belt asteroids) shape and spin distributions, and current estimates of NEA lifetime and encounter probabilities, that tidal disruption should account for approximately 1–2% of NEAs being binaries. Given the best observed estimate of a ~15% binary NEA fraction, we conclude that there are other formation mechanisms that contribute significantly to this population. We also present the expected distribution of binary orbital and physical properties for the steady-state binary NEAs formed by tidal disruption. We discuss the effects on binary fraction and properties due to changes in the least constrained parameters, and other possible effects on our model that could account for differences between the presented results and the observed binary population. Finally, we model possible effects of a significant population of binaries migrating to the near-Earth population from the Main Belt.

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*Keywords:* Asteroids, dynamics; Satellites of asteroids

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## 1. Introduction

### 1.1. Motivation

The number of known binary NEAs has been growing rapidly in recent years with estimates for the binary fraction consistently around ~15% (see Richardson and Walsh, 2006 for a review). Richardson et al. (1998) predicted that roughly 15% of NEAs could become binaries via tidal disruption of gravitational aggregates, or “rubble piles,” when an asteroid is disrupted during a planetary close encounter and some debris remains in orbit around the original body. This is also the value predicted as the percent of binaries needed to explain doublet craters on Earth and Venus (Bottke and Melosh, 1996a, 1996b). The work by Walsh and Richardson (2006), characterized the binaries formed from tidal disruption. However, this provided only a snapshot of the binaries’ properties after disruption, and

did not account for the dynamical, thermal or tidal evolution that would shape the entire population of binary NEAs during their lifetimes.

The NEA population is essentially transient, with the bodies having lifetimes on the order of 10 Myr, and constantly being replaced from the Main Belt (Bottke et al., 2002). In our model, we compare the properties of binaries formed in our simulations with the observed population, taking into account the dynamical lifetimes of the bodies and known evolutionary effects. The modeled population changes over time, as binaries are created and evolve, and are replaced with new single bodies at the end of their lifetime. The steady-state population is then compared with the observed population, revealing the importance of tidal disruption in forming binary NEAs.

### 1.2. Near-Earth asteroid population

The NEA population consists of those asteroids with perihelion distances  $q \leq 1.3$  AU and aphelion distances  $Q \geq 0.983$  AU (Rabinowitz, 1994; Bottke et al., 2002). Earth and

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Moon cratering records suggest an impact flux that has remained roughly constant over the past 3 Gyr, which implies an NEA population that has not varied drastically in numbers over that time (Grieve and Shoemaker, 1994). Therefore bodies being removed from the NEA population (via collision with a planet or the Sun, or by ejection from the inner Solar System) must be replaced to keep the size of the population roughly constant. Various resonances in the Main Belt have the ability to increase an asteroid's eccentricity quickly, possibly sending it into a Mars- or Earth-crossing orbit. However, these resonances must be supplied material at appropriate rates to maintain the steady-state NEA population. The thermal Yarkovsky effect has been shown to be a viable mechanism for moving small bodies in the Main Belt into resonances which help to replenish the supply of NEAs (Bottke et al., 2006).

Recent numerical simulations of large numbers of NEAs on well-determined orbits have estimated a median lifetime of 10 Myr (Gladman et al., 2000). The lifetimes were significantly shortened compared to previous works due to various resonances pushing the asteroids into high-eccentricity orbits eventually leading to collisions with the Sun. More complex simulations by Bottke et al. (2002) tracked objects from various source regions in the Main Belt as they became NEAs, and produced a bimodal set of lifetimes, where objects with  $a > 2$  AU have significantly shorter lifetimes than objects with  $a < 2$  AU.

### 1.3. Observed binary population

The observed population of binary NEAs and MBAs was detailed in the review chapter Merline et al. (2002) and more recently in Richardson and Walsh (2006). Nearly all binary NEAs discovered to date have primaries with a rotation period faster than 4 h, a size ratio between 0.2 and 0.6 secondary over primary radii ( $R_{\text{sec}}/R_{\text{pri}}$ ), and separations between 2.5 and 5  $R_{\text{pri}}$ . The binary MBAs, until recently, showed a diverse range of properties and generally appeared entirely different from the NEAs.

Recent discoveries among small MBAs using lightcurve observations have begun to remove the observational biases that have obscured any similarities in the two populations (Pravec and Harris, 2007). Previously binary MBAs and NEAs were discovered primarily using two distinct techniques: lightcurves for NEAs and high-resolution direct imaging for MBAs. These two techniques have different biases, with lightcurves only sensitive to binaries with small separation and moderate (1.0–0.2) size ratios, whereas direct imaging is primarily sensitive to binaries with a large separation and can cover a wider range of size ratios.

The application of lightcurve techniques to small MBAs may define the role that asteroid size plays in binary fractions. These recent discoveries suggest the previously perceived differences between NEA and MBA binaries may have been dominated by the relative difference in size of observed bodies as well as the fundamental observing biases affecting each. If the small MBA binaries have very similar properties to NEAs, then the formation mechanism for small MBAs and NEAs are likely closely related, and the potential transport of MBAs to the NEA

population might be important. The first substantial reports of lightcurve-discovered small MBA binaries suggest that there is a population of binaries in the Main Belt that share the distinctive traits previously observed only in NEA binaries (Pravec and Harris, 2006). These binaries with rapidly rotating primaries, small size ratios and small separations suggest that a formation mechanism for NEA binaries may also be creating binaries of a similar sort in the Main Belt.

Until the recent discoveries of small binary MBAs different formation mechanisms were typically invoked to explain binary NEAs and MBAs: rotational spin-up and collisional processes, respectively (Merline et al., 2002; Richardson and Walsh, 2006). Tidal disruption fit the binary NEAs, as encounters with large planetary bodies are unique to that region and frequent enough to affect a large percentage of the population. In the Main Belt collisions are more important as bodies have longer dynamical lifetimes, and there are no bodies large enough to tidally disrupt asteroids. The work on small binary MBAs calls into question these separate formation mechanisms and may demand a new formation mechanism which will affect both populations. The role that tidal disruption plays is something that can now be tested directly using previous numerical simulations.

### 1.4. Previous work

Substantial modeling work has been done on the tidal disruption of rubble pile asteroids in regards to the formation of binary NEAs. Richardson et al. (1998) investigated tidal disruption outcomes in terms of the body's close approach distance and speed, shape and spin, spin-axis and body-axis orientation. This work suggested that tidal disruption could account for 1–2 observed crater chains on the Moon, as well as the population of binary NEAs (Bottke et al., 1997). Using similar but improved methods Walsh and Richardson (2006) did an exhaustive set of simulations of tidal disruption to characterize the properties of binaries formed during a disruption.

Walsh and Richardson (2006) found that binary asteroids formed during a tidal disruption event share many characteristics with the observed population of binary NEAs, namely:

1. The semi-major axis distribution of the binaries is strongly peaked below 10  $R_{\text{pri}}$ , though the simulations also show a long tail out beyond the Hill sphere for 1 AU at 130  $R_{\text{pri}}$ . Large separations are neither expected nor observed in the NEA population because close planetary encounters will easily separate very wide binaries and all but one of the observed binary NEAs have semi-major axes smaller than 10  $R_{\text{pri}}$ , with all but 4 smaller than 5  $R_{\text{pri}}$ .
2. The size ratios in the simulations are peaked between  $R_{\text{sec}}/R_{\text{pri}} = 0.1$ –0.2, with a significant tail towards higher values, i.e., equal size components. The observed population, though biased against size ratios below  $R_{\text{sec}}/R_{\text{pri}} \sim 0.2$ , almost all have values between 0.2 and 0.6. There is only one observed binary with equal-sized components, also a rarity in simulations.

3. The rotation rate of the primary body is narrowly bracketed between 4 and 6 h in the simulations. Nearly all binary NEAs have rapidly rotating primaries though they typically have somewhat faster rotation rates between 2.2 and 3.5 h.

Some questions, either unanswered or subsequently raised in Walsh and Richardson (2006), are:

1. What is the overall steady-state binary fraction for NEAs caused by tidal disruption?
2. Why do simulations not match the rapid rotation rates of the primary bodies?
3. Will the binaries created by tidal disruption, which generally start with high eccentricity, survive long enough to have their eccentricity tidally damped to the observed low values (almost all observed below 0.1 for the few well-measured systems)?
4. Do binary NEAs with large semi-major axes exist unobserved as Walsh and Richardson (2006) suggest, or are they disrupted during subsequent close approaches with Earth?

The main focus of the current work is to apply the results of Walsh and Richardson (2006) to determine how many present-day NEA binaries may be tidal disruption outcomes, and what the population of this subset of binaries will look like in steady-state. We use these results in a Monte Carlo routine to simulate the transport of bodies from the Main Belt to the near-Earth population, their encounters with Earth, and the formation and subsequent evolution of binaries. We also determine the effect of pre-existing binary MBAs migrating into the near-Earth population.

## 2. Steady-state model

The steady-state model consists of a set number of asteroids (usually 2000), simulated over 1 Gyr to estimate the number and properties of the steady-state binary NEA population. During each timestep (typically 0.01 Myr), each asteroid may be removed and replaced, have a close encounter with a planet, evolve (if it is already a binary), or not change. The model uses recent estimates from the literature for NEA lifetimes, planetary encounter probabilities, binary asteroid formation via tidal disruption, Main Belt binary formation via catastrophic collision, and tidal evolution (details are given below). This model does not consider NEA orbits directly; rather it is a statistical approach that does not account for resonant encounters or the different dynamics of the NEA orbital classes (see Section 3 for the caveats of our method).

### 2.1. Initial shape and spins

One of the primary results of previous tidal disruption work is the strong dependence on the shape and spin of a progenitor on the outcome of a disruption (Richardson et al., 1998; Walsh and Richardson, 2006). Elongated and/or faster-rotating asteroids are more likely to disrupt and form binaries. However, due to NEAs' frequent interactions with terrestrial planets, the

observed distribution of shape and spin properties for NEAs is likely different from its source population. Scheeres (2002) quantified the changes to the rotational states of asteroids for a steady-state population of NEAs (assuming rigid bodies, and a distribution of spin rates for the source bodies from collisional experiments) and found that an overall spin-up of the population might be expected, with a maximum spin period for any given body close to the observed maximum  $\sim 2$  h (near the estimated critical spin rate).

Main Belt asteroids of a similar size as NEAs, for the sake of this model, qualify as NEA progenitors. However, obtaining lightcurves to estimate shape and spin of 1–3 km asteroids in the Main Belt is sufficiently challenging that these properties remain a point of some uncertainty. The archived data of asteroid lightcurves (Harris et al., 2005) and recent results (Walsh and Richardson, 2005) provide data for 78 MBAs with  $D < 5$  km. This data provided relative frequencies for each of the parameters contained in the tidal disruption database (shape and spin, see Fig. 1).

Different distributions were tested in the simulation, with the outcomes varying accordingly. The distributions used were: (a) one derived from the small MBA (SMBA) lightcurve data described above; (b) and one based on only NEA spin and shape data.

### 2.2. NEA lifetimes and planetary encounters

Recent numerical results place the median NEA lifetime around 10 Myr. The simulations by Gladman et al. (2000) show a rapid decay in surviving particles with a tail of long-lived particles surviving for the length of the 60 Myr integration. The lifetimes of asteroids in each of the present work's simulations are assigned when they are created, with a distribution designed to match Gladman et al. (2000) exactly up to 60 Myr. Beyond 60 Myr the number of asteroids surviving is made to tail off to zero at 100 Myr, which is the longest lifetime used (see Fig. 2, Section 3). When an asteroid or binary exceeds its lifetime it is removed from the simulation and replaced by an asteroid/binary with properties designated for the SMBA population.

The encounter probabilities used were a combination of those for Earth and Venus. The probabilities for an NEA encounter with the two planets differs, as do the encounter parameter probability distributions. However, the gravitational properties of Venus are quite similar to those of Earth, with a density of  $5.2 \text{ g cm}^{-3}$  and comparable radius  $\sim 6000$  km. For these reasons the tidal effects of close encounters are quite similar in terms of close approach distance in units of planetary radii. Due to these similarities the same tidal disruption database is used, and planetary encounters are not distinguished as being with Earth or Venus, as differences were found to be small for this work. We generally refer to encounter distances in terms of Earth radii ( $R_{\oplus}$ ), as the tidal disruption simulations were done in a geocentric system.

At each timestep the probability for each asteroid to encounter a planet was calculated. Encounters within 3 planetary radii were the maximum for a binary forming by tidal disruption.

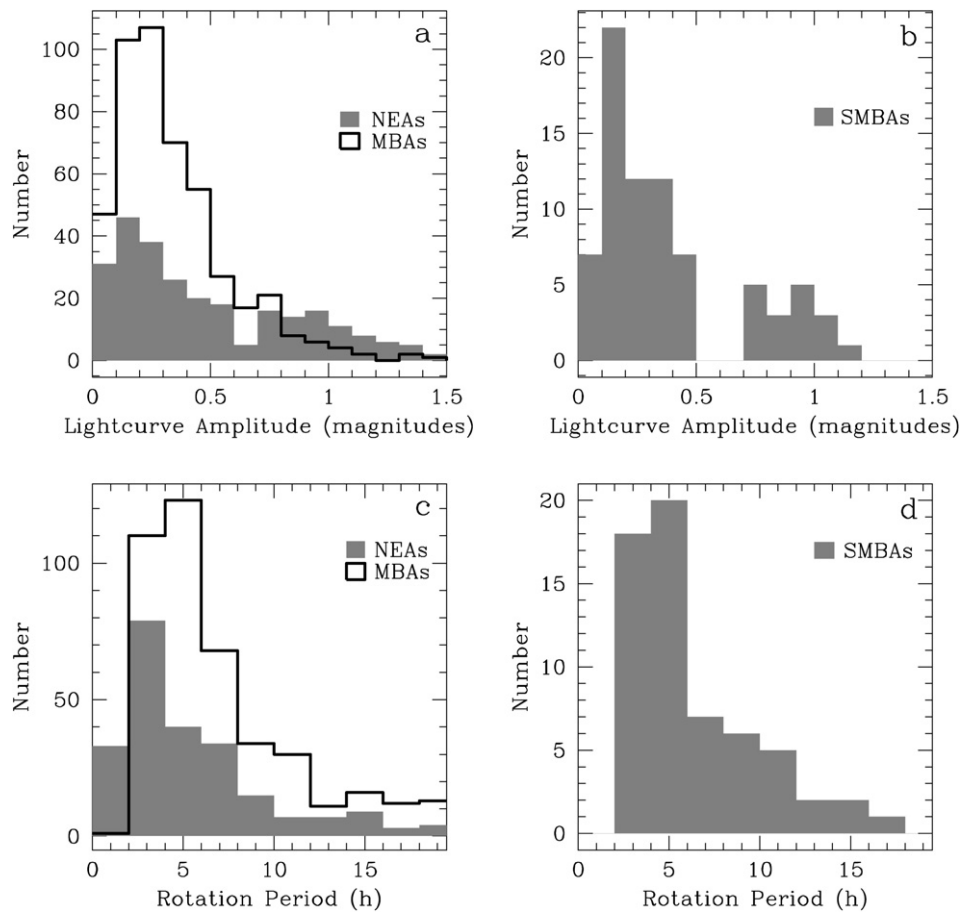


Fig. 1. Comparison of amplitude and rotation period data from asteroid lightcurves: (a) distribution of lightcurve amplitude for all NEAs and MBAs with a diameter less than 20 km; (b) lightcurve amplitude for small MBAs (diameter less than 5 km); (c) rotation period of NEAs and MBAs; and (d) rotation period of small MBAs. The distributions for small MBAs have significantly fewer known lightcurves, hence they are plotted separately. Data compiled from Harris et al. (2005).

tion, so the probability of an encounter for each asteroid was

$$P_{\text{enc}} = (\langle P_{\oplus} \rangle + \langle P \rangle) \pi q^2 \Delta t, \quad (1)$$

where

$$\langle P_{\oplus} \rangle = 1.12 \times 10^{-16} \text{ km}^{-2} \text{ yr}^{-1} \quad (2)$$

and

$$\langle P \rangle = 2.02 \times 10^{-16} \text{ km}^{-2} \text{ yr}^{-1} \quad (3)$$

with  $q$  being the close approach distance and  $\Delta t$  the timestep (Bottke et al., 1994).<sup>1</sup> These encounter probabilities predict a  $3 R_{\oplus}$  encounter with Earth or Venus every  $\sim 3$  Myr. Binary asteroids in the simulation were tested for encounters out to  $24 R_{\oplus}$  at each timestep by the same method.

The encounter velocities were those from the distribution of  $v_{\infty}$  (where  $v_{\infty}$  is velocity at infinity) used in Walsh and Richardson (2006). The distribution of expected encounter

properties was taken from a series of  $N$ -body simulations of NEA migration from major source regions in the Main Belt (Bottke et al., 2002; Bottke, 2004, personal communication; the distribution is similar to the impact speed distribution of Bottke et al., 1994).

Fundamentally the model presented here is a statistical model of asteroid lifetimes and planetary encounters in the NEA population. The next step would include actual integrated orbits of a population of asteroids, tracking the close approaches for each. This method would include resonant encounters, and bodies in long-lived safe orbits. However, in light of the robust results found with the present approach, we do not expect to arrive at significantly different conclusions by increasing the realism in this way. Rather we believe that additional binary formation mechanisms are needed to explain the observed NEA binary population, since even our conservative approach results in very few surviving binaries.

### 2.3. Binary evolution

#### 2.3.1. Basic stability limitations

Two strict limitations were placed on the binaries: their mutual pericenter distance had to be outside the radius of the primary body, and the semi-major axis had to be smaller than

<sup>1</sup> Tests were run with more recent values for encounter probabilities ( $70 \times 10^{-18}$  and  $115 \times 10^{-18} \text{ km}^{-2} \text{ yr}^{-1}$ ; Bottke, 2007, personal communication), which did not affect the NEA binary population, and only slightly increased the number of MBA binaries persisting in the simulations. Overall the largest change was to increase the lifetimes of binaries in the simulation, which was balanced by decreasing the number of binaries formed.

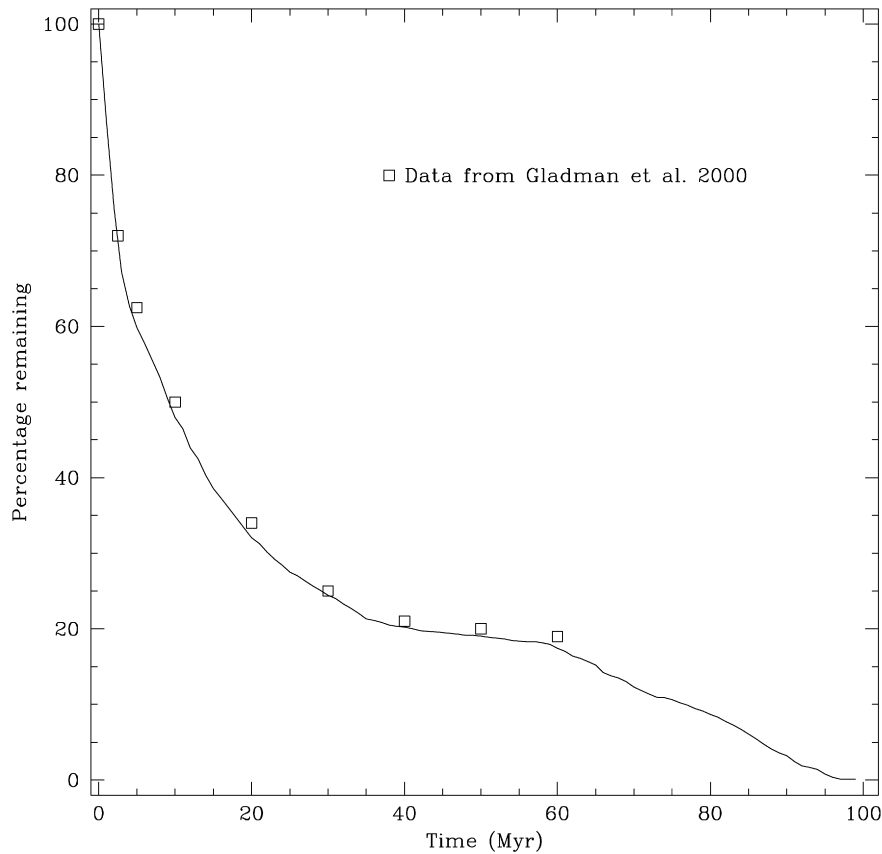


Fig. 2. Percent of surviving NEAs used to assign lifetimes in the steady-state model. The squares are data from Fig. 2 of Gladman et al. (2000).

the mutual Hill sphere. When a binary was formed that violated these requirements, or evolved to a disallowed state, it was immediately removed and replaced by a new asteroid/SMBA binary.

### 2.3.2. Tidal evolution

Tidal forces between the primary and secondary will affect the binary in most cases by: changing the semi-major axis of the secondary's orbit, synchronizing the secondary's rotation with its orbital period, and changing the eccentricity of the secondary. Weidenschilling et al. (1989) published formalisms for the change of the semi-major axis of a tidally evolving asteroid binary, and this formalism was used in the paper of Walsh and Richardson (2006) (Eq. (3)) to estimate evolutionary timescales for the simulated binaries. In this work the formula is applied during each timestep to evolve each binary's semi-major axis.

All but one of the observed binary NEAs with known eccentricities have  $e < 0.1$ . The damping timescales of eccentricity due to tidal interactions used in the paper of Walsh and Richardson (2006) was adapted to recalculate the binary's eccentricity during each timestep in the steady-state model,

$$de = -e \times \frac{1}{\tau_e} \times \Delta t, \quad (4)$$

where  $de$  is the change in eccentricity based on the eccentricity damping timescale  $\tau_e$  over the timestep  $\Delta t$  (Murray and Dermott, 1999). This formalism is for a secondary with a spin period equal to its orbital period and considers only

the effects of the tides raised by the primary on the secondary. Tides raised on the primary by the secondary, which play a greater role for larger mass ratios, can have the effect of raising the secondary's eccentricity (Goldreich, 1963; Margot and Brown, 2003).

The rate at which tidal evolution operates is strongly dependent on the values of  $Q$ , the tidal dissipation factor, rigidity  $\mu$ , and the effective rigidity  $\tilde{\mu}$ . In the paper of Walsh and Richardson (2006), these values were estimated based on the radar measurements of one binary NEA, 2000 DP<sub>107</sub> (Margot et al., 2002). However, those estimated values are based on multiple assumptions, including the binary's age and various asteroidal properties. Due to the uncertainty of  $\mu$  and  $Q$  values for small bodies of different internal structures (rubble piles or fractured interiors), and the potential for tidal evolution to significantly alter the properties of binary asteroids, multiple values are tested in these simulations. The nominal values used in this study were very similar to those from the paper of Walsh and Richardson (2006), namely  $\mu Q = 2.26 \times 10^9 \text{ dyn cm}^{-2}$  and  $\tilde{\mu} = 1.66 \times 10^4 \text{ dyn cm}^{-2}$ . Additional tests were done to explore a range of possible parameters; one with properties similar to that of Phobos,  $\mu Q = 10^{12} \text{ dyn cm}^{-2}$  and  $\tilde{\mu} = 6.75 \times 10^6 \text{ dyn cm}^{-2}$ , and a system designed to have very fast tidal evolution with  $\mu Q = 4.74 \times 10^6 \text{ dyn cm}^{-2}$  and  $\tilde{\mu} = 32 \text{ dyn cm}^{-2}$  (Yoder, 1981). This provided a wide range of possible body properties which can vastly alter the timescales at which tidal evolution will occur. In the text the two cases are referred to as *Phobos* and *Fast* tidal properties.

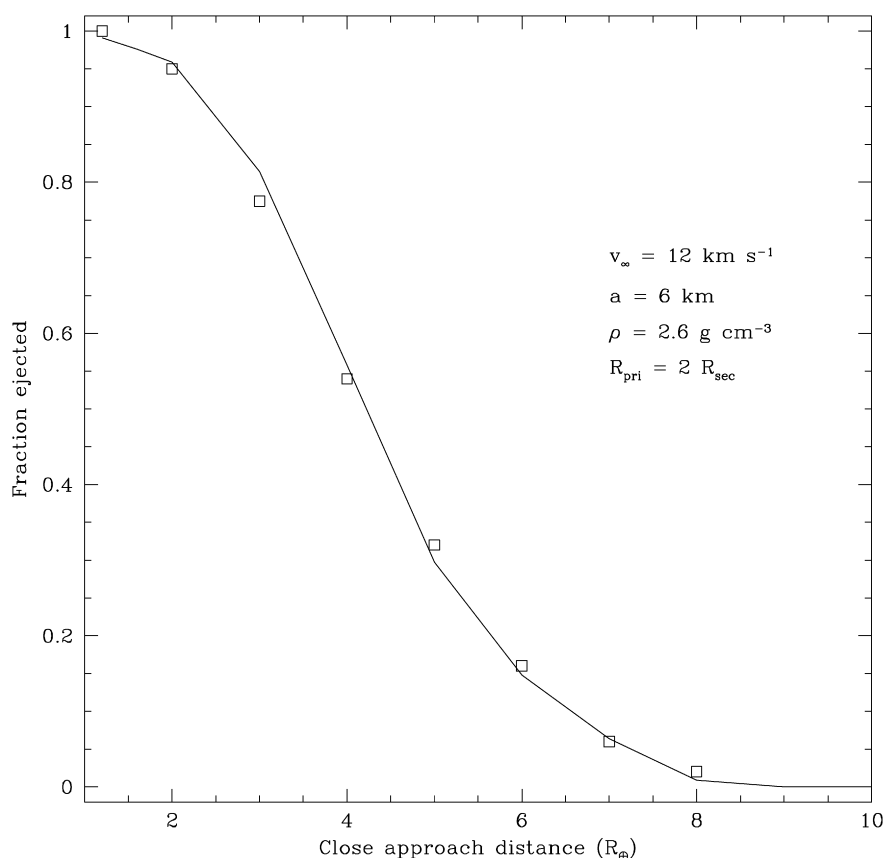


Fig. 3. Comparison of this work, using HNBODY to calculate binary disruption probability (solid line), with results presented in Botke and Melosh (1996a) (squares).

An evolutionary factor not included in this simulation is the binary YORP (BYORP) effect (Cuk and Burns, 2004). Similar to how the YORP effect can change asteroid spin rates and obliquities, BYORP can potentially alter binary eccentricity and semi-major axis on timescales significantly faster than tidal evolution. BYORP, similar to the YORP effect's dependency on obliquity, depends on the binary's inclination. Because binary inclination is not tracked in our model this effect is not included. However, any effect which increases binaries' semi-major axes, as we expect BYORP to do rapidly in many cases, will only decrease the binary fraction in steady-state as their susceptibility to disruption via a planetary encounter increases quickly with increased  $a$ .

### 2.3.3. Binary encounter with a planet

In order to consider the possibly disruptive effects that a planetary encounter could have on a binary asteroid, direct 3-body encounters were simulated and incorporated into the steady-state model. These simulations were done separately and compiled into a look-up table and then used in the steady-state model via interpolation. In a separate test explained below, 3-body encounters were simulated directly within the model.

For integer values of close approach in Earth radii from 1 to 24  $R_{\oplus}$ , and the same speeds used in the tidal disruption simulations of Walsh and Richardson (2006) ( $v_{\infty} = 8, 12, 16, 20, 24 \text{ km s}^{-1}$ ), a series of simulations were run over a range of binary mass ratio ( $M_{\text{sec}}/M_{\text{pri}} = 1.0, 0.5, 0.1$ , and

0.01) and semi-major axis ( $a = 2, 4, 6, 8, 10, 15$ , and  $25 R_{\text{pri}}$ ). The simulations were performed with an  $N$ -body code, HNBODY, using a Runge–Kutta algorithm modified for close encounters (Gültekin et al., 2004; see also Gültekin, 2006). For each set of parameters 1000 simulations were run with orbit orientation randomized assuming zero eccentricity (zero eccentricity is an assumption we make due to the relative quickness of tidal eccentricity damping). Thus each set of parameters assumed a probability for binary disruption, which was then used in the Monte Carlo simulation to determine the fate of binary encounters. This code was tested against the results in Botke and Melosh (1996a, 1996b), for the case of  $v_{\infty} = 12 \text{ km s}^{-1}$ ,  $a = 6 \text{ km}$  and mass ratio of 0.125, yielding very close matches (see Fig. 3). The results also match well with the analytical calculation of Agnor and Hamilton (2006), relating the Hill sphere of the binary and its semi-major axis,

$$r_{\text{td}} \approx \frac{a}{R_{\text{pri}}} R_{\oplus}, \quad (5)$$

where  $r_{\text{td}}$  is the tidal disruption distance for a binary encounter with Earth.

A result of these simulations is a statistical estimate for binary lifetimes against separation due to planetary encounters (these lifetimes are used only for comparisons and do not directly govern interactions in the simulations). This value is calculated for binaries of various semi-major axes (assuming a relatively average binary encounter scenario with  $v_{\infty} = 16 \text{ km s}^{-1}$  and a mass ratio of 0.1). A critical encounter distance is de-

Table 1  
Predicted lifetimes for NEA binaries for systems with encounters of  $v_\infty = 16 \text{ km s}^{-1}$  and a mass ratio of 0.1

Semi-major axis ( $R_{\text{pri}}$ )	Critical $q$ for 50% ejection	Lifetime (Myr)
2	1.6	10.
4	3.0	2.
6	4.0	1.6
8	5.0	1.0
10	6.0	0.7
15	8.0	0.4
25	12.5	0.17

*Note.* The limiting close approach distance in Earth radii,  $q$ , was selected for the distance at which 50% of the randomly oriented binaries were disrupted as a result of the close approach. The lifetime is then how often an encounter at that critical distance is expected to occur.

terminated for each binary semi-major axis as the distance at which 50% of randomly oriented binaries disrupt due to the tidal forces of the close encounter. The statistical encounter probabilities for each critical distance then determine lifetime between binary-disrupting encounters (Table 1). The lifetime for very close binaries of 4 or 6  $R_{\text{pri}}$  is only 2 or 1.6 Myr, suggesting that many of the observed NEA binaries are potentially quite young. The NEA binary Hermes has a semi-major axis of 5  $R_{\text{pri}}$ , which means it should have a critical encounter every  $\sim 1.75$  Myr. This result for Hermes highlights the nature of this calculation, that it is a statistical average, and does not consider resonant encounters, or dynamically stable orbits within the NEA population.

A separate implementation of the code performed 3-body simulations of binary encounters within the code, as they happened throughout the simulation. The advantage of these simulations, run with `pkdgrav`, is that the exact semi-major axes can be given to the system, rather than relying on interpolating previous runs. In this implementation the binary has zero eccentricity on its encounter, but any eccentricity that the system may have gained from the encounter is kept by the system after the encounter. This implementation was run on a small subset of runs, primarily to test the results for the runs which interpolated via a look-up table of previous runs. The results were very nearly identical in both number of binaries and their properties.

#### 2.4. Migrating binary MBAs

A large unknown in the study of binary NEAs is the extent of any migration of binary asteroids from the Main Belt population. If binary asteroids can migrate successfully from MBA orbits into NEA orbits then the numbers and properties of these binaries may be extremely important in shaping the binary NEA population.

In this work binary MBAs were included in the steady-state simulation as a variable percentage of the incoming asteroids. The binary properties were drawn from a previous study of family-forming collisions modeled with SPH code and `pkdgrav` (Durda et al., 2004). The binaries with primaries having  $D < 5 \text{ km}$  were used, however a large portion of these

were EEBs<sup>2</sup> with a mass ratio of unity, where the bound particles were at the resolution limit of the simulation. Durda et al. (2004) presented results with these unit-mass-ratio systems removed, considering them more a function of the numerical resolution rather than actual dynamics. The distribution used in this work also excludes unit-mass-ratio systems, when both components are at the resolution limit. Fundamentally the properties of this set of binaries is limited by the resolution of these simulations, and this may skew the properties towards high-separation binaries, which will be separated more easily by planetary encounters.

Another difficulty with the MBA binary population is the potential for a very large range in binary ages, and hence a wide range of tidal evolution end states. Therefore just using the raw binaries from a collision simulation may represent newly formed binary MBAs that will not account for any post-collision evolutionary effects. Thus a third population of binary MBAs is also considered, those with the basic selection effects applied (pericenter distance  $q > 2.0a/R_{\text{pri}}$  and semi-major axis  $a < R_{\text{Hill}}$ , where  $R_{\text{Hill}}$  is the Hill sphere), and 100 Myr of tidal evolution while the binaries are migrating into the NEA population (see Fig. 4, and Section 2.3.2 for the full description of the tidal evolution formalisms used).

Observations are just starting to constrain the binary fraction among the SMBA population, with current work estimating a fraction close to the NEA fraction of  $\sim 15\%$  (Harris and Pravec, 2006). The survivability during transport from a Main Belt orbit to an Earth-crossing orbit is unknown as these binaries may suffer close encounters with Mars during this transition.

### 3. Caveats

The method we describe is entirely statistical, based on Monte Carlo selection of asteroid lifetimes, asteroid properties, hyperbolic encounter values, time of encounters and binary lifetimes. Some of the parameters included, notably encounter timescales, asteroid lifetimes, close approach distance and velocity, are coupled, which is a nuance not treated within this statistical approach (e.g., low speed encounters with a planet are far more likely for a body with a small semi-major axis, rather than a Main Belt binary which has just evolved to have a perihelion below 1 AU). It is possible that parameters are consistently selected in a way in which orbital subtleties are lost, affecting the results. A treatment which considers the evolution of numerically integrated asteroids migrating from source regions in the Main Belt would provide an additional level of sophistication and accuracy by way of reducing dependence on the Monte Carlo selection of encounter properties, lifetime and frequency of encounters.

<sup>2</sup> Escaping Ejecta Binaries are binaries formed during a catastrophic collision. As some material is ejected from the system entirely, two or more escaping pieces may remain bound to each other. The EEBs differ from the binaries formed around the target body as they can be loosely bound small collisional fragments of similar size to one another.



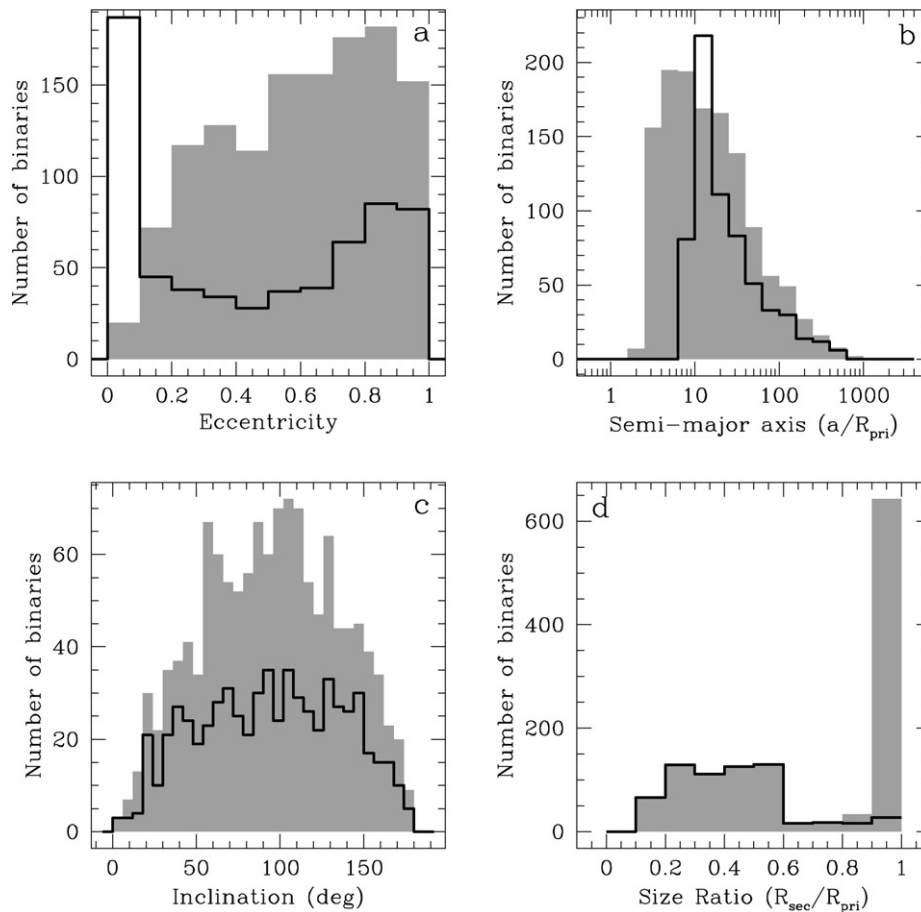


Fig. 4. The total number of binary MBAs for the (shaded) original distribution and (outline) the distribution with 100 Myr of tidal evolution and single particle pairs removed. The original distribution is from Durda et al. (2004) simulations of a 34 km diameter impactor striking a 100 km diameter target at  $3 \text{ km s}^{-1}$  at an impact angle of  $30^\circ$ . Shown are the: (a) eccentricity of the binaries; (b) semi-major axis of the binaries in units of the primary radii; (c) inclination; and (d) size ratio.

For expediency, we used the lifetime curve of Gladman et al. (2000), rather than the more sophisticated treatment of Bottke et al. (2002). The former model was simpler to implement in our code, and an increase in sophistication would rely on linking the asteroid lifetimes with the encounter parameters as mentioned above.

The goal of this work was the determination of the contribution tidal disruption makes to the steady-state fraction of binary NEAs. This goal was achieved through the Monte Carlo model, the results of which are described below.

#### 4. Steady-state results and discussion

We first present the nominal case, which includes the best estimate for each of the many variable parameters included in the steady-state model. In subsequent sections we examine the individual effects of each of the main model parameters, and their overall effect on the results. Our nominal case has the following properties:

1. Progenitors follow the shape and spin distributions discussed above (Section 1), matching estimates for SMBAs;
2. Tidal evolution actively changes binary properties, with parameters from Walsh and Richardson (2006);

3. 10% of Main Belt asteroids entering the simulation are binaries, with properties from Durda et al. (2004) (excluding unit-mass-ratio systems) with 100 Myr of tidal evolution;
4. Binary encounters with Earth are handled via a look-up table of 3-body encounters.

##### 4.1. Nominal case

The nominal case is a 1 Gyr simulation using 2000 asteroids. Fig. 5 shows the evolution of binary number over time for the simulation, showing the quick decline in the number of remaining MBA binaries. This rapid decline is due to the comparatively large semi-major axes of the binary MBAs and their consequent very short lifetimes against disruption. The steady-state number of MBA binaries in the population hovers close to 0.2%. The fraction of binaries formed via tidal disruption is 1.2% and has comparable small fluctuations in numbers throughout the simulation.

The properties of the binaries show strong effects of tidal evolution. There is a very strong peak in eccentricity between 0.0 and 0.1, which shows significant tidal damping of the original eccentricity distribution and matches the observed population well (Fig. 6). The distribution of semi-major axis is mildly dependent on the formation mechanism, tidal disruption, or col-

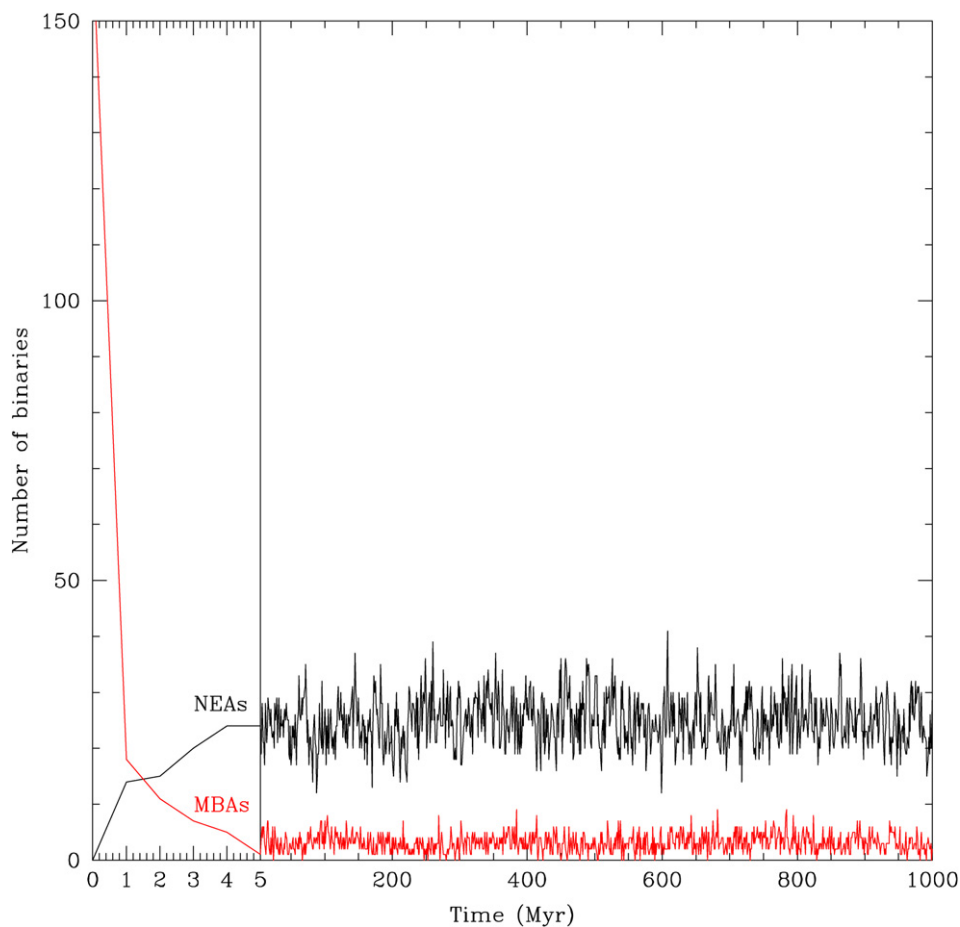


Fig. 5. The number of binaries as a function of time out of a total of 2000 asteroids in the nominal steady-state simulation, showing those formed by tidal disruption (black), and those injected into the simulation from the MBAs (red). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

lisional remnant in the Main Belt. The tidal disruption remnants have semi-major axes almost entirely below  $10 R_{\text{pri}}$ , whereas the binaries from the Main Belt have a number of systems with larger semi-major axis.

The low steady-state percentage of both NEA and MBA binaries is due to short lifetimes against disruption during planetary encounters. The average lifetime before a NEA binary is disrupted is  $\sim 1.2$  Myr, while for an MB-formed binary that time is  $\sim 0.3$  Myr (Fig. 7). While increasing the percentage of MBA binaries will increase the number migrating into the population (see Section 4.2), the properties of the binaries will determine their lifetime, and the sustainable binary fraction. Hence, for the distribution of binary properties used in this nominal case for both the tidal disruption and MBA migrated binaries, the steady-state fraction is dominated by short binary lifetimes against disruption.

#### 4.2. Influence of MBA binary percentage

Tests were run varying the binary percentage of MBA progenitors between 10 (the nominal case), 20, 50, and 80%. The contribution to the NEA binary population from migrated MBA binaries is quite low for the binary properties used, below 1.2% for all four tests values run (Fig. 8).

The binaries that migrate in from the MBA population are disrupted via a close encounter with Earth quite quickly. The average lifetime for a migrated binary in the simulation is  $\sim 0.3$  Myr. The properties of the binary MBAs will drastically affect their lifetime, and also their overall contribution to the steady-state population (see Fig. 7). The properties for the binary MBAs in this work from Durda et al. (2004) represent the results of the most recent asteroid collision simulations. A different formation mechanism for producing smaller-separation binary MBAs could theoretically provide a different set of binary properties to test. As well, detailed observations of binary MBAs could also provide new data to include in the simulations.

#### 4.3. Influence of MBA shape/spin properties

The two different shape/spin distributions used produced similar results. Using the distributions derived for SMBA shape and spin from lightcurve data, as in the Nominal case (see Section 4.1), the steady-state binary fraction for tidal disruption formed binaries was 1.2% (Table 2). This fraction increased slightly when the shape/spin distribution for NEAs was used, increasing to 1.4%. The faster spinning NEAs generally are more effective at producing binaries via tidal disruption, but

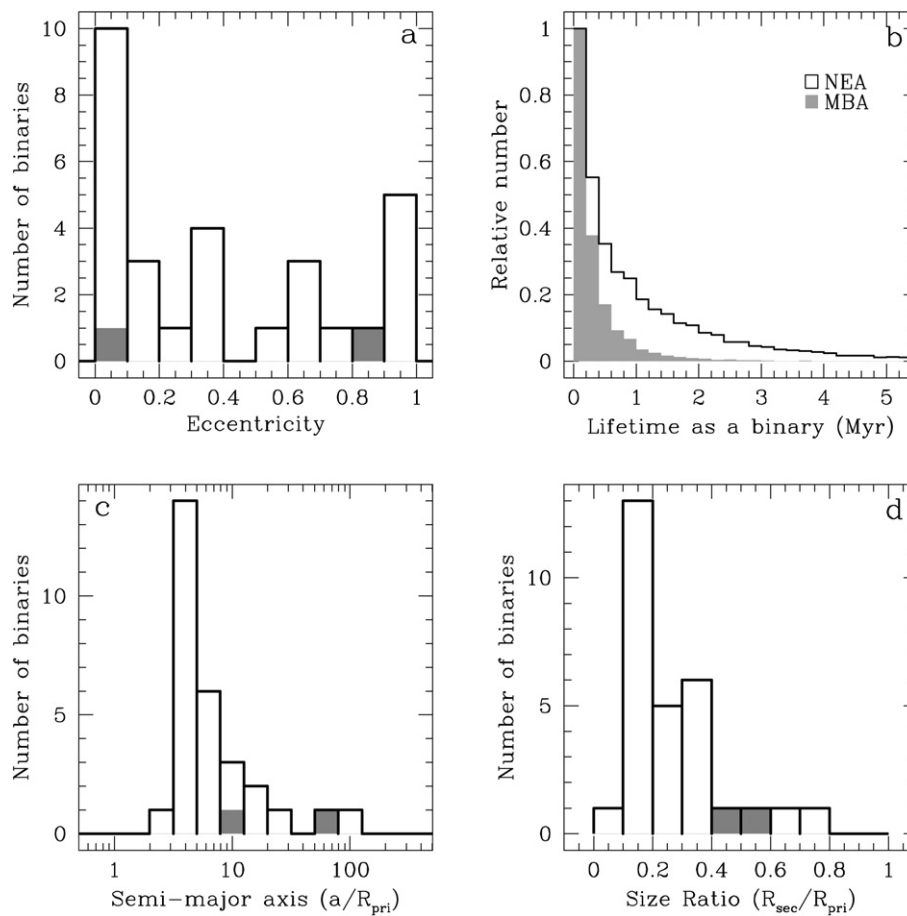


Fig. 6. Properties of the binary population at the end of the 1 Gyr nominal steady-state simulation: (a) eccentricity; (b) cumulative lifetimes of binaries formed by tidal disruption and of those that migrated from the Main Belt; (c) semi-major axis in terms of  $R_{\text{pri}}$ ; and (d) size ratio. The histograms (a), (c), and (d) show just the instantaneous state of the (outline) binaries formed by tidal disruption and those (shaded) that migrated from the MBA population.

within this steady-state model where planetary encounters dominated, the overall affect is minimal.

#### 4.4. Influence of tidal evolution

Tidal evolution of binaries during the simulation strongly changes the eccentricity and semi-major axes of the binaries (Fig. 9). The lifetimes against disruption due to close encounters are greater for closer binaries, and for these binaries the eccentricity damping time-scales are relatively short. The effect of eccentricity damping is quite noticeable with a strong peak of nearly half the binaries at 0–0.1 eccentricity and with small numbers spread out at higher values. This is vastly different than a simulation with no tidal effects where the bulk of the eccentricity values are greater than 0.1.

Similarly the semi-major axis distribution is noticeably increased for the simulation with tidal effects compared to the one without (Fig. 9). The peaks of the distribution are pushed from 3–5  $R_{\text{pri}}$  towards 5–8  $R_{\text{pri}}$ . This does move some bodies out beyond 10  $R_{\text{pri}}$ , though time scales to move any beyond that are very long, and the lifetime of the binary against disruption will decrease rapidly with increasing  $a$ .

The overall binary fraction varies from 0.6 to 1.7% (Table 2), though the size ratio between the two components is essentially

unchanged (Fig. 9). Though the tidal effects noticeably affect  $a$  and  $e$  they are not strong enough to increase  $a$  so rapidly that the lifetimes of binaries decrease dramatically; instead they only slightly lower the binary fraction. Therefore it is essentially a shift in the basic properties only.

The strongest tidal evolution (*Fast*) has the lowest steady-state value, at 0.6%, due to the heavily increased semi-major axes of the systems. However, eccentricities are damped very strongly as well, all into the bin below  $e = 0.1$ , at which nearly all binary NEAs are currently observed. In the case of very slow tidal evolution (*Phobos*), the resulting number of NEA and migrated binaries are identical to a case with tidal evolution.

#### 4.5. Estimates on the properties of NEA binaries formed by tidal disruption

The properties of the steady-state binaries are largely dominated by the preference of planetary encounters to eliminate widely separated systems. This effect is so strong that it may eliminate any significant fingerprint of the formation mechanism that exists in the population (possibly other than primary spin and shape). The eccentricities are damped substantially, with values between 0 and 0.1 dominating the distribution.

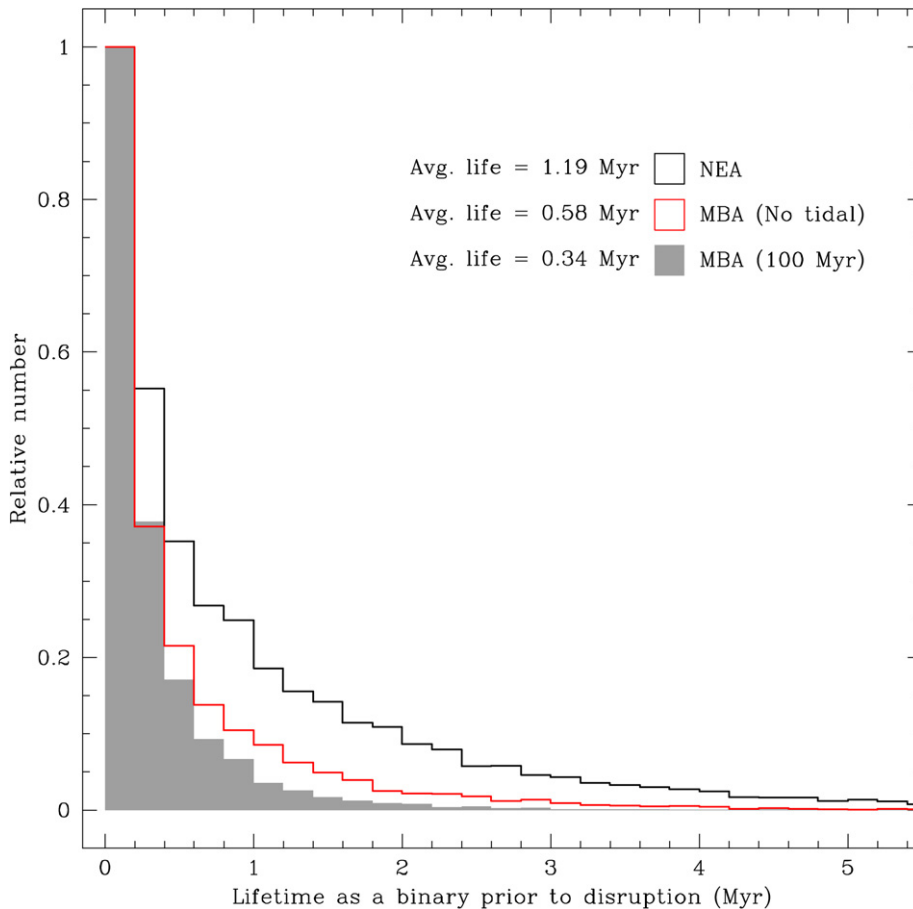


Fig. 7. Time between binary introduction/formation, and its disruption due to a close encounter. The MBAs (shaded) are swapped out after a shorter time than the tidally formed NEA binaries (outline). The binary MBAs which weren't tidally evolved for 100 Myr years before incorporation into the simulation (red), survived longer than the evolved ones as their semi-major axes were generally smaller making them less susceptible to disruption during a planetary encounter. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

The binaries in the steady-state system otherwise retain the fast spinning primary. This potentially diagnostic property may be the only significant observable property not quickly altered, or involved in binary lifetimes. However, with all observed NEA binaries having fast rotation rates it certainly appears that a rotational spin-up origin for most systems is likely. Similarly the observed primaries generally appear quite spherical, which could be a diagnostic feature in future work.

Differentiating between tidally disrupted binaries and those formed from another mechanism will be quite difficult. Any binaries observed are likely to have survived due to selected binary properties, namely a small semi-major axis. Primary spin and shape may also be diagnostic, but currently all the observed binaries look the same with fast-spinning and spherical primaries, both of which are seen in tidal disruption simulations (Walsh and Richardson, 2006).

A topic not covered in this work is the resolution of our progenitor rubble piles, and how this changes binary properties. Tests with resolution ranging from 200 to 2000 particles covering selected encounter parameters have found variance in binary properties at differing resolutions (Walsh, 2006). The noticeable changes to binary properties is a slightly faster primary rotation for lower resolution simulations, and also a slightly

smaller semi-major axis. The number of binaries increases at higher resolution largely due to the large increase in available particles to form satellites. However, when only satellites larger than 1.5% of the progenitor mass are counted, the varying resolution simulations produced similar numbers of satellites.

#### 4.6. Estimates of migrated binaries' numbers and properties

The properties assumed for migrating MBA binaries are such that their lifetime against disruption from a planetary encounter is very short. Thus the steady-state number of binaries having migrated from the MBA population is very small, nearly zero at any given time. However, as mentioned above, this value is highly dependent on the MBA binary properties used in the model. If a formation mechanism is found to create MBA binaries with consistently small separations, their lifetime against disruption would increase dramatically, allowing for a significant presence in the steady-state population.

Another factor which could affect this number is the use of the Monte Carlo model, and the statistical nature of each binary's encounter. As discussed in Section 3, incorporation of integrated orbits for migrating MBAs would provide a more re-

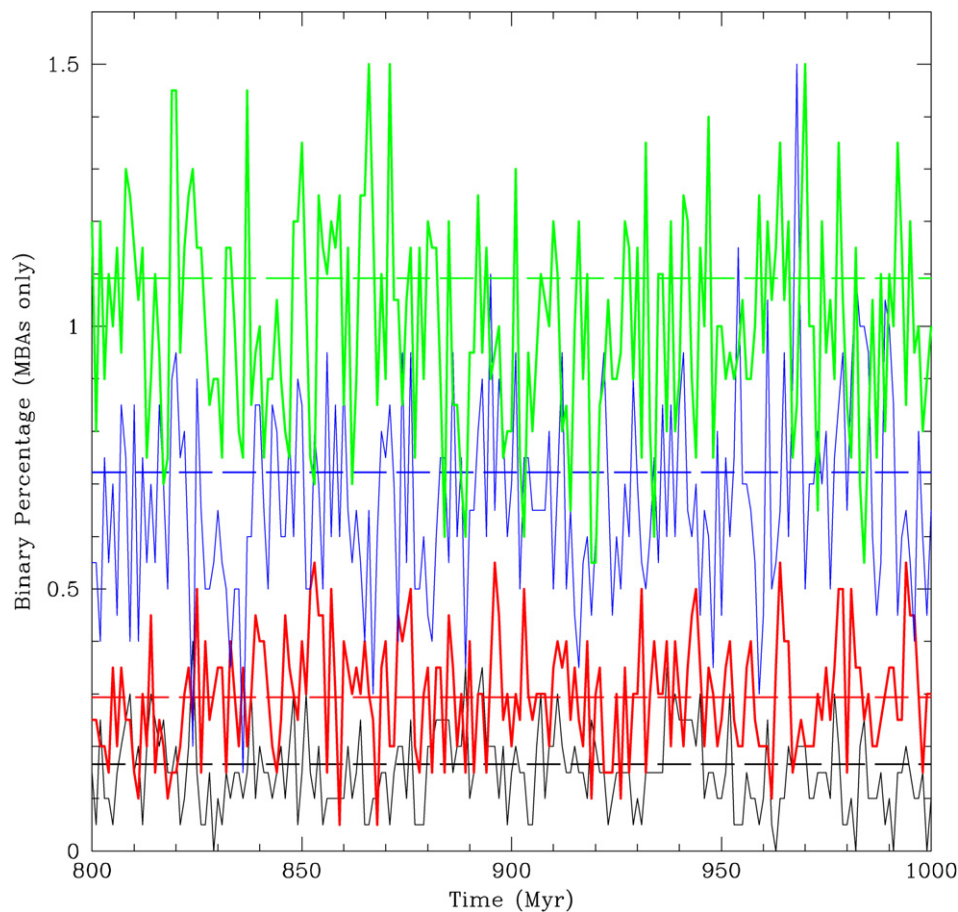


Fig. 8. Binary percentage for migrated MBAs in the simulation for the last 200 Myr, up to the conclusion at 1 Gyr, with the thick dashed line representing the average value for the entire simulation. Varying binary percentages were used for the source population, from top to bottom on the plot: 80% (green); 50% (blue); 20% (red) and 10% (black). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

alistic scenario, and possibly supply enough long-lived binaries to reach noticeable levels.

#### 4.7. Doublet craters

Any encounter of an asteroid or binary at less than  $1 R_{\oplus}$  is flagged with the asteroid or binary parameters, allowing investigation of possible doublet crater formation. While the ratio of doublet to singlet craters would be expected to roughly correspond to the ratio of binary to single asteroids in the NEA population, doublets may still be diagnostic of the binary population.

For a nominal simulation with 20% binary MBAs, approximately 10% of all craters were doublets. When the MBA percentage was increased up to 80%, the doublet percentage increased to 15%. For the second case, about 14% of the impacting binaries had a semi-major axis greater than ten times the combined radii of their components ( $a > 10 \times (R_{\text{pri}} + R_{\text{sec}})$ ) roughly approximates the necessary separation needed for an impacting binary to form two separate identifiable craters). Thus only about 2% of the craters would likely be detectable as doublets.

Tracking ratios of singlet to doublet craters will be significantly more relevant when the steady-state population of bina-

ries is at the observed level around 15%. With a clear deficit of binaries in the NEA population in this steady-state simulation it is not surprising that we find such a low percentage of impacts as possibly observable doublets. Currently it is estimated that at least 10% (3 of 28) of craters on Earth are doublets (Bottke and Melosh, 1996a).

## 5. Conclusion

This study focused on determining how tidal disruption affects the population of NEA binaries. It is clear from the discussions above that tidal disruption provides only a small fraction of the observed binary population. We have shown that these binaries appear similar to those observed, suggesting that some of these systems were in fact formed via tidal disruption. However, the implications of such a small contribution of binaries formed from tidal disruptions as well as the even smaller numbers of surviving MBA binaries, are quite dramatic. We can essentially account for very few of the NEA binaries observed, and require an unknown source or mechanism to create them.

The major constraints on any formation mechanism are a rapidly rotating primary body, as observed for nearly all NEA binaries, and a small semi-major axis to survive planetary encounters. Close planetary encounters are the dominant factor in

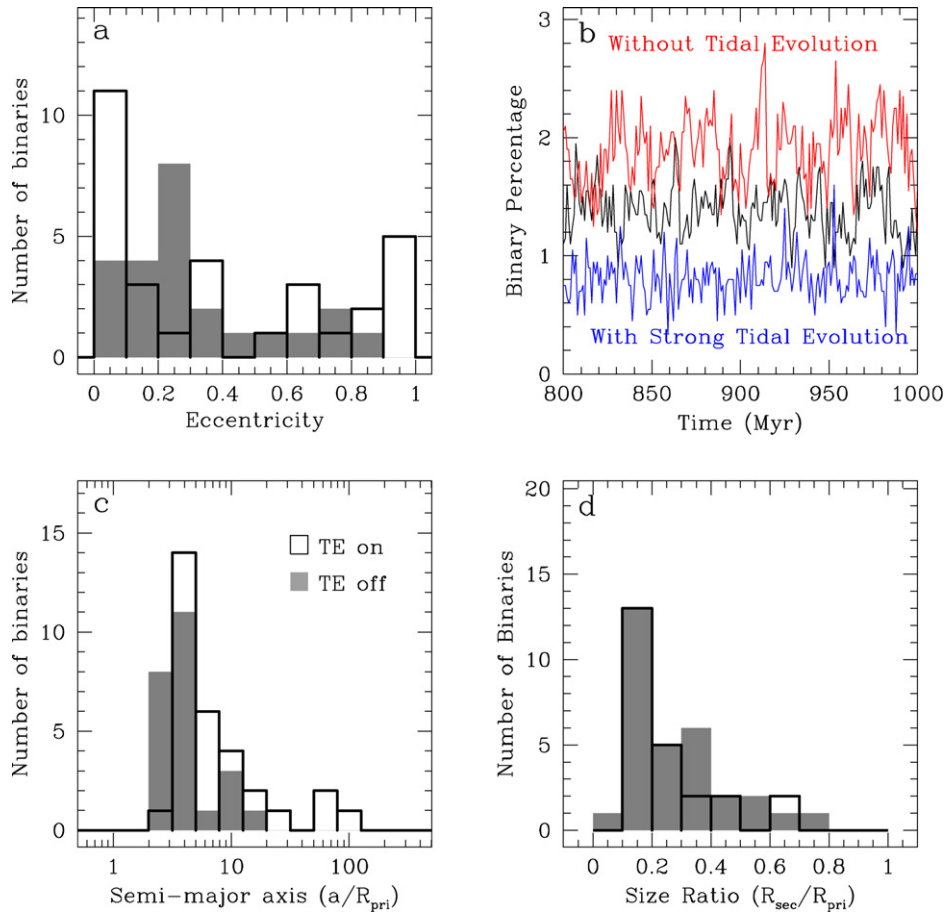


Fig. 9. Effects of tidal evolution on the binaries during the nominal steady-state simulation comparing their properties with tidal evolution (outline, TE on) and with no tidal evolution (shaded histogram, TE off): (a) eccentricity; (b) number of binaries (including the case of *Fast* tidal evolution, where the middle, black line represents the case of nominal tidal parameters, and the uppermost line has no tidal evolution); (c) the semi-major axis; and (d) the size ratios. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 2

Binary fraction of both NEA and MBA binaries are listed with respect to the primary model parameter setting

Progenitor distribution	Tidal evolution	MBA binaries (Myr)	Tidal disruption binaries (%)	Migrated binaries (%)	Total binaries (%)
SMBA (10%)	On	100	1.2	0.2	1.4
NEA (10%)	On	100	1.4	0.3	1.7
SMBA (10%)	On (Fast)	100	0.6	0.2	0.8
SMBA (10%)	On (Phobos)	100	1.7	0.1	1.8
SMBA (10%)	Off	100	1.7	0.1	1.8
SMBA (10%)	On	0	1.2	0.2	1.4
SMBA (20%)	On	100	1.2	0.3	1.5
SMBA (50%)	On	100	1.2	0.7	1.8
SMBA (80%)	On	100	1.1	1.0	2.1

*Note.* The column “Progenitor distribution” refers to the shape and spin distribution of the asteroids being injected into the system, with the percentage of binary progenitors in parentheses. “Tidal evolution” refers to whether or not binary systems would actively be altered tidally throughout the simulation, and the special cases where the tidal evolution parameters were not the nominal case are noted in parentheses as either the *Phobos* or *Fast* simulation. One simulation was run where the binary MBA population did not have any tidal evolution prior to their inclusion in the simulation (compared to the nominal case of 100 Myr); it is listed as 0 Myr under the “MBA binaries” column. The final three columns list the percentage of asteroids in the simulation that were formed from tidal disruption, migrated from the Main Belt, and the total combined percentage of binaries.

the low steady-state fraction of binaries found in this work, and even introducing migrating binaries at an 80% rate was ineffective at increasing the total binary fraction. Thus any new means of introducing binaries into the NEA population must provide significant numbers with small separations.

Lightcurve observations/discoveries of binary MBAs will continue to establish the similarities and differences between the two populations of binary asteroids. This method of binary discovery allows for direct comparison between the NEA population and similar-sized MBAs. With the two populations

having different dynamical, collisional and thermal environments the differences between the two should provide strong constraints on any new binary formation mechanisms proposed. Continued observations will also provide a different set of SMBA binaries that can be used for modeling of binary migration from the Main Belt.

Thermal spin-up (the YORP effect) as a binary formation mechanism could solve many outstanding issues by creating binaries and spinning up primaries in both the NEA and MBA population without the need for close planetary encounters. The YORP effect has been shown to be a potentially important mechanism to modify spin rates and obliquities of asteroids, but no systematic study of YORP as a mechanism for fission and binary creation has yet been carried out (Bottke et al., 2006). Spin-up timescales by YORP depend on the shape and size of an asteroid as well as its distance from the Sun and its axis orientation and therefore might operate at different timescales on the NEA and MBA populations. The scenario for successfully losing mass while retaining some in a stable orbit is unknown, but small separations and fast spinning primaries are likely resulting properties, making this the most promising mechanism to supply the large percentage of observed binaries.

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