ASTR 498N Homework #4 due in class Tu 12 Mar

R Please read O&C §10.5, §10.6, §11.1, §12.1, and §12.2.

P O&C Problem 9.16.

P Using O&C Eq. 9.25 (the result of Problem 9.16 above), the relationship between surface flux and luminosity, and the equation of gravitational equilibrium at the surface to find an expression for the radiative heat flux that can, by itself, support the atmosphere of a star. Show, thereby, that a star of mass M in hydrostatic equilibrium has a maximum luminosity given by

$$L_{\max} = \frac{4\pi c G}{\bar{\kappa}} M$$

Estimate this *Eddington luminosity* by assuming that the surface is hot enough for the opacity to be dominated by electron scattering.

P Explain why the bound-free opacity is proportional to $Z(1+X)$ while the free-free opacity is proportional to $(1-Z)(1+X)$ (O&C Eqs. 9.19 and 9.20).

P O&C Problem 11.2.

P Use the Kramers approximate formula for $\bar{\kappa}_{ff}$ (with unity Gaunt factor) to show that the electron scattering opacity exceeds the free-free opacity when $T > 10^{6.65} \rho^{0.286}$.

P *Bounds on the central pressure of a star*. Show that, in hydrostatic equilibrium, the function

$$P + \frac{GM^2}{8\pi r^4}$$

(where P and M are functions of r) must *decrease* with r. Thus show that the central pressure satisfies

$$P_c > \frac{1}{6}\left(\frac{4\pi}{3}\right)^{1/3} G\bar{\rho}^{4/3}\mathcal{M}^{2/3}$$

where $\mathcal{M}$ is the stellar mass. By assuming that $\rho(r)$ decreases with r, derive a tighter lower bound,

$$P_c > \frac{1}{2}\left(\frac{4\pi}{3}\right)^{1/3} G\bar{\rho}^{4/3}\mathcal{M}^{2/3}$$

and a useful upper bound,

$$P_c < \frac{1}{2}\left(\frac{4\pi}{3}\right)^{1/3} G\rho_c^{4/3}\mathcal{M}^{2/3}$$

where ρ_c is the central density.