

**P** O&C Problem 9.16

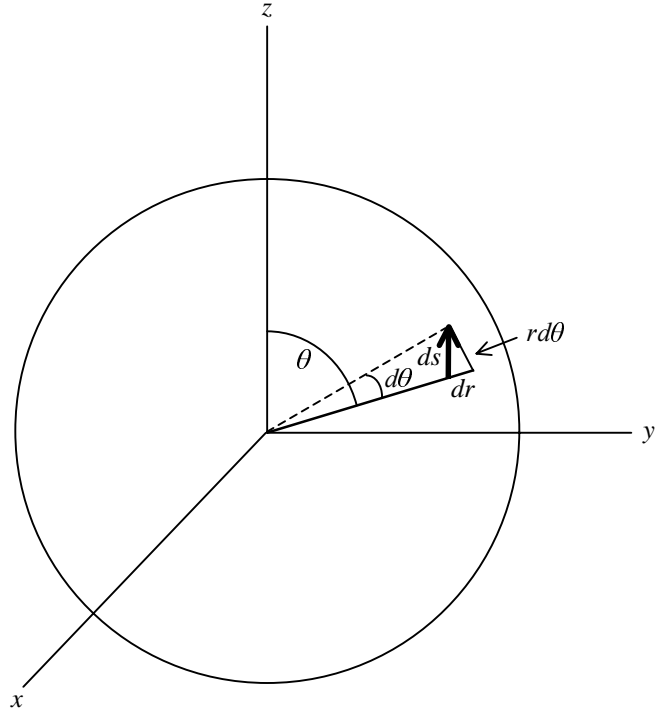
The star is spherically symmetric, so align  $ds$  with the  $z$ -axis. The ingredients are:

$$dr = ds \cos \theta$$

$$r d\theta = -ds \sin \theta$$

$$\frac{dI_\lambda}{ds} = \frac{dI_\lambda}{dr} \frac{dr}{ds} + \frac{dI_\lambda}{d\theta} \frac{d\theta}{ds}$$

This will allow you to write the transfer equation in spherical coordinates [part (a)].



To derive the radiation pressure gradient [part (b)], follow O&C p. 286:

multiply both sides of the transfer equation by  $\cos \theta$  and integrate over all solid angles. Remember that the source function is isotropic and refer to O&C Eqs. (9.6) and (9.7).

**P** Explain why the bound-free opacity is proportional to  $Z(1 + X)$  while the free-free opacity is proportional to  $(1 - Z)(1 + X)$  [O&C Eqs. (9.19) and (9.20)].

Don't worry about the  $Z$  dependences, but do explain physically why both the b-f and f-f opacities depend on the number density of free electrons. Then follow Lecture 6 to derive an expression for the number density of free electrons in terms of the mass fractions  $X$ ,  $Y$ , and  $Z$  (analogous to the expression we derived for the number of free particles) and show that this expression reduces to  $\frac{1}{2}(X+1)$ .