ASTR 498N Homework #6 due in class Tu 23 Apr

R Please review O&C §11.1, §12.1, §12.2 and §13.1, and read §12.3 and §13.2.

P O&C Problems 12.1, 12.2, 13.3

P The Gamow energy defined in Lecture 13 is related to the quantity b defined by O&C [just after Eq. (10.38)] by $b = E_G^{1/2}$. In terms of E_G, O&C Eq. (10.40) for the reaction rate between nuclei A and B can be written

$$r_{AB} = n_A n_B \left(\frac{8}{\pi\mu}\right)^{1/2} \left(\frac{1}{kT}\right)^{3/2} \int_0^{\infty} S(E) e^{-(E_G/E)^{1/2}} e^{-E/kT} dE$$

The product of the two exponential factors in the integrand produces a strongly peaked curve ("fusion window") with a maximum at

$$E_0 = \left(\frac{E_G (kT)^2}{4}\right)^{1/3} \qquad \text{O\&C Eq. (10.41)}$$

(a) By making a Taylor expansion about $E = E_G$, derive the approximation

$$\exp\left[-\frac{E}{kT} - \left(\frac{E_G}{E}\right)^{1/2}\right] \approx \exp\left[-3\left(\frac{E_G}{4kT}\right)^{1/3}\right] \exp\left[-\left(\frac{E - E_0}{\Delta/2}\right)^2\right]$$

where the width of the fusion window is

$$\Delta = \frac{2^{5/3}}{3^{1/2}} E_G^{1/6} (kT)^{5/6}$$

(b) Using this approximation in the reaction rate integral and assuming that $S(E)$ is approximately constant within the fusion window, show that the reaction rate is given numerically by

$$r_{AB} = 6.5\times10^{-24}\,\frac{n_A n_B}{\mu_A Z_A Z_B}\,S(E_0)\left(\frac{E_G}{4kT}\right)^{2/3}\exp\left[-3\left(\frac{E_G}{4kT}\right)^{1/3}\right]\ \mathrm{m^{-3}\ s^{-1}}$$

where μ_A is the reduced mass of the nuclei in a.m.u. and $S(E_0)$ is the fusion factor at the Gamow peak in units of keV barns.

(c) Calculate E_G for each of the fusion reactions $^{2}\mathrm{H}(p,\gamma)^{3}\mathrm{He}$ and $^{12}\mathrm{C}(p,\gamma)^{13}\mathrm{N}$ and compare the exponential inhibition factor in the equation immediately above for the two reactions at a temperature of $2\times10^{7}\ \mathrm{K}$.