ASTR 498N Things to Know & Review

From Within

Here are shorthand reminders of things you should know (or be able to re-derive quickly) without reference to notes. Please refer to the text or lecture notes for definitions of symbols.

Constants

There is no point in remembering any numerical quantity to better than two significant figures.

c	speed of light	(m s^{-1})
pc	parsec	(m)
$\mathcal{M}_\odot$	Mass of the Sun	(kg)
$\mathcal{R}_\odot$	Radius of the Sun	(m)
$\mathcal{L}_\odot$	Luminosity of the Sun	(W)

Magnitudes and such

Magnitude scale	$m_1 - m_2 = -2.5 \log(F_1/F_2)$
Apparent vs. absolute	$m - M = 5 \log(d/10 \text{ pc})$
Effective temperature	$L = 4\pi R^2 \sigma T_e^4$

Radiative transfer

$$\frac{dI_\nu}{ds} = -k_\nu I_\nu + j_\nu \quad \text{or} \quad \frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu \text{ where } d\tau_\nu = k_\nu\, ds \text{ and } S_\nu = j_\nu / k_\nu$$

Cross section	$k_\nu = n\sigma_\nu \qquad [\sigma_\nu] = \text{m}^2$
Mean free path	$\ell_\nu = 1/n\sigma_\nu = 1/k_\nu$

Boltzmann & Saha

Boltzmann	$\frac{n_u}{n_l} \propto \mathrm{e}^{-(E_u - E_l)/kT}$
Saha	$\frac{n_e n_{r+1}}{n_r} \propto \mathrm{e}^{-\chi_r/kT}$ where χ_r is the ionization potential

Mean molecular weight

Number density of free particles $n = \rho / \mu m_H$

Fully ionized hydrogen gas $\mu = 1/2$

Fully ionized helium gas $\mu = 4/3$

Thermodynamics

Fundamental relationship $dU = TdS - PdV + \sum \mu_i dN_i$

Adiabatic processes

For ideal gas $\frac{dP}{P} + \gamma \frac{dV}{V} = 0 \iff PV^{\gamma} = \text{constant}$

From $PV^{\gamma} = \text{constant}$ and the ideal gas law $PV = \nu RT$, you should be able to derive the companion relations $TV^{\gamma-1} = \text{constant}$ and $P^{1-\gamma}T^{\gamma} = \text{constant}$.

Miscellaneous

Characteristic dynamical time $\tau_{\text{dyn}} \sim 1/\sqrt{G\rho}$

Equations of static stellar structure

$\frac{dM}{dr} = 4\pi r^2 \rho$ conservation of mass

$\frac{dP}{dr} = -\frac{GM}{r^2}\rho$ hydrostatic equilibrium

$\frac{dL}{dr} = 4\pi r^2 \rho \left(\varepsilon - T\frac{dS}{dt} \right)$ conservation of energy

$\frac{dT}{dr} \propto L/4\pi r^2$ radiative energy transport

$\frac{dT}{dr} \propto \frac{dP}{dr}$ convective energy transport

You should be able to derive the first three equations using physical arguments. Note that, in the energy conservation equation, $dQ/dt = T\,dS/dt$ is the rate that the subsystem (mass shell) *absorbs* heat, consistent with the thermodynamic sign convention. So the net rate of energy generation in the mass shell is $\varepsilon - dQ/dt$ (all you need to remember).

Other Concepts To Review

Virial Theorem

$2K + U = 0$ nonrelativistic $\qquad K + U = 0$ relativistic $\qquad E = K + U$ always

Three-Fold Way of quasistatic gravitational contraction:

1. The star gets hotter
2. Energy is liberated from the system.
3. The total energy of the system decreases (star is more tightly bound).

Thermodynamics

- In a system with a constant number of particles and no applied fields, we can choose any two state variables (*T, P, V, U, S, …*) to characterize the system.
- The various thermodynamic potentials (*U, F, G, H*) reflect the fact that we can choose the two independent variables as we like. Combinations of more than two variables are always related by equations, such as the fundamental relations (e.g., $dU = TdS - PdV + \sum \mu_i dN_i$), equations of state (e.g., $T = \left(\partial U/\partial S\right)_{V,N}$), or Maxwell relations (e.g., $\left(\partial T/\partial V\right)_{S,N} = -\left(\partial P/\partial S\right)_{V,N}$).

Particle statistics and degeneracy

Occupation index $\quad f(p) = f(\varepsilon_p) = \dfrac{1}{\exp\left[\left(\varepsilon_p - \mu\right)/kT\right] + C}$

$C = 0$ Maxwell-Boltzmann (classical)
$C = +1$ Fermi-Dirac
$C = -1$ Bose-Einstein

Remind yourself of the graphs of the M-B and F-D occupation indices as a function of particle energy, ε_p . Review the criterion for complete degeneracy ($\mu/kT \gg 1$) and the sense in which a degenerate gas is "cold." Be able to explain to define Fermi momentum using words and a diagram. Why does a degenerate electron gas have high thermal conductivity?

Sources of opacity

Be able to describe physically what we mean by bound-bound, bound-free, free-free, and electron scattering opacity. Have a general idea of their temperature dependence (e.g., electron scattering tends to dominate at high temperatures unless the gas is also highly degenerate).