# ASTR430 Homework # 3 – Orbital Motion
Due Wednesday, October 15, 2003

1. Sketch the following orbit at four times, $t = 0, 1/8, 1/4, 3/8$ years, and then describe, without using the names of the orbital elements $a, e, i, \Omega, \omega$, how the orbit is changing in time.

2. a) Estimate the minimum rotation period of a star in hours by equating gravity with the centrifugal force at the star's equator. How does your answer differ for a planet?
b) Calculate the spin angular momentum of the Sun (look up the moment of inertia for a uniform sphere in a physics book), the spin angular momentum of Jupiter, and the orbital angular momentum of Jupiter.
c) If, somehow, Jupiter were absorbed by the Sun (assume that the angular momentum vectors are parallel, that the total angular momentum is conserved, and that the radius of the Sun is not changed), how fast would the Sun spin? What would happen?
d) Now imagine a one solar mass spherical cloud of gas with uniform density and radius 1 light year. How fast does it spin if it has the same angular momentum as the Sun-Jupiter system?

3. In this problem, you'll use conservation of energy and angular momentum to solve for the radial turning points for orbits in central force motion (i.e. you'll derive the blue curves seen in the Central Force Integrator).
a) For a general force, $\mathbf{F}$(r), write down the expressions for angular momentum per unit mass, $h$, and energy per unit mass, $C$, in terms of the position, $r$, and the velocity, $v$.
b) Simply your expression(s) for a central force with the form $\mathbf{F} = -Ar^n\hat{r}$.
c) Use your two equations and the condition of being at a radial turning point to obtain an equation for $r$ in terms of constants.
d) For gravity ($n = -2$), use the specialized formulas for $C$ and $h$ to solve for the radial turning points in terms of the semimajor axis, $a$, and eccentricity, $e$.
e) For the general case in c), argue that there are at most 2 positive real solutions for $r$ (the only ones that matter physically). Interpret the special cases of 0,1, and 2 solutions physically.
f) How many physical solutions are there for the case $n = -3$? What does this tell you about orbits in a $1/r^3$ force field? Test your ideas by investigating circular and near circular orbits with the central force integrator. Show me some plots and summarize what you find!