c. Spin Axis Position

To evaluate whether Saturn could be in the resonance, its spin axis must be located with respect to the $j = 18$ reference frame defined by a z-axis that lies along the pole of the term, and an x-axis along its ascending node on the invariable plane of the solar system. The right ascension and declination of ***s*** with respect to the equator and equinox at epoch J2000.0 are 40.595° and 83.538° respectively (Yoder 1995). Rotating about the vernal equinox by the Earth's obliquity, 23.439° (Yoder 1995), gives ***s*** with respect to the ecliptic and equinox as

TABLE II:

Coordinates of Saturn Spin Axis, $j = 18$ Pole, and Normal to Invariable Plane

Vector	Reference frame	Co-latitude*	Longitude	x	y
s	Ecliptic/equinox	28.049°	79.529°	8.546×10^{-2}	4.624×10^{-1}
k	"	1.579°	17.584°	2.626×10^{-2}	8.322×10^{-3}
k	Invariable plane	0	---	0	0
s	"	27.253°	82.572°	5.920×10^{-2}	4.541×10^{-1}
n	"	0.0644°	-66.476°	4.488×10^{-4}	-1.031×10^{-3}
k	Intermediate	0.0644°	113.524°	-4.488×10^{-4}	1.031×10^{-3}
s	"	27.317°	82.644°	5.875×10^{-2}	4.551×10^{-1}
n	"	0	---	0	0
k	$j = 18$ system	0.0644°	90°	0	1.124×10^{-3}
s	"	27.317°	59.120°	2.355×10^{-1}	3.939×10^{-1}
n	"	0	---	0	0

* For ***s,*** the co-latitude is the obliquity, for ***k, n*** it is the inclination

shown in Table II. The normal $\boldsymbol{k}$ to the invariable plane from Allen is listed as well. Also included in the table are the x and y components of each vector in each system. Since the inclination of the invariable plane is very small, to first order accuracy the coordinate system can be transformed to that plane by subtracting the components of $\boldsymbol{k}$ from $\boldsymbol{s}$. We now introduce the $j = 18$ pole from Applegate *et al.* (1986), who give[1] $\{p, q\} = \sin(I_{18}/2)\{\sin, \cos\}\Omega_{18} = N_8\{\sin, \cos\}\delta_8$, where $N_8 = -10^{-3.25}$, $\delta_8 = 203.518°$. Setting $\sin(I_{18}/2) \approx (1/2)\sin I_{18}$, and recalling that the longitude of the pole is 90° behind its ascending node Ω_{18}, we find the components of $\boldsymbol{n}$ listed in Table II. We can transform again to a system with $\boldsymbol{n}$ at origin by subtracting its components from the other vectors. This gives the vectors in an intermediate system. A final counter-clockwise rotation of the coordinate system by 23.524° puts $\boldsymbol{k}$ in the y-axis. The spin axis lies $\psi = 90° - 59.12° = 30.88°$ from $\boldsymbol{k}$. Figure 4 shows polar views of the $j = 18$ system for $\alpha/g_{18} = -1.16$ along with the separatrix, and $\boldsymbol{s} = (x_s, y_s, x_s) = (0.236, 0.394, 0.888)$. The separatrix is more narrow than the example of Figure 2 because the inclination is an order of magnitude smaller. Since the trajectory does not enclose $\boldsymbol{n}$ at the origin, such motion produces the longitude libration diagnostic of resonance trapping.

[1] In the Applegate *et al.*(1986) notation, our $j = 18$ is their $j = 8$.