

ASTR121 Challenge Problem #1 – (Hamilton)
due Tuesday May 13

1. **Superluminal Motion.** In this problem you'll explore the conditions under which two objects can appear to be separating at greater than the speed of light using algebra, trigonometry, and a little light calculus.

a) Consider an AGN that ejects a dense blob of hot gas at speed v_b . The blob is aimed somewhat toward Earth with the Earth-AGN-blob angle given by α . The blob travels an actual distance d in time $t = d/v_b$. Draw a picture showing the direction to Earth, the AGN, and the blob after it has travelled a distance d . Now think about two pulses of light, one emitted when the blob is ejected (blob and AGN are together) and a second one emitted by the blob when it has already moved a distance d . Find the apparent time t_{app} between the arrival of the two light signals on Earth. The apparent distance that the blob has travelled as seen from Earth is $d_{app} = d \sin \alpha$. Obtain an expression for the apparent speed of the blob on the sky: $v_{app} = d_{app}/t_{app}$.

b) Test your answer in a) for the special cases i) $\alpha = 0$, ii) $\alpha = 90^\circ$, and iii) $v_b = 0$. For each case look at the figure and determine what you expect before doing the math. Do these limits make sense? Are the units correct in your answer to a)? It is an excellent habit to spot check your answers in this way.

c) For $v_{app} = c$, find the angle α that minimizes v_b . What is the minimum v_b ? This will be the slowest possible speed to make the blob appear to be moving at the speed of light.

d) Repeat part c) for $v_{app} = \chi c$ where $\chi > 1$. What range of possible α allow $v_{app} > c$? Find the critical α and show that $v_b/c = \chi/\sqrt{1 + \chi^2} = \cos \alpha$. What sort of geometry gives the maximum possible apparent speed? What is the maximum possible apparent speed?