ASTR330: HOMEWORK #3 SOLUTIONS

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Question 1 a) Collision Energies. The table from HW#1 shows that each factor of ten increase in diameter leads to a factor of 1000 increase in energy released during am impact. This is consistent with

$$E = \frac{1}{2}mv^2$$

because the mass of the impactor

$$m = \frac{4}{3}\pi R^3 \rho$$

(m=mass, R=radius, ρ=density) is proportional to the radius cubed. If you increase the radius or the diameter by a factor of 10, the Mass goes up by a factor of $10^3 = 1000$ and, then from the first equation, the Energy goes up by a factor of 1000 too.

b) The impact energy for a 1km object at 15 km/s is 33,500 MT. For 30 km/s and 60 km/s, the energies are 134,000 MT and 536,000 MT. The first equation predicts that a factor of 2 increase in speed should result in a factor of $2^2 = 4$ increase in energy which agrees with the ratios:

$$\frac{536,000 \text{ MT}}{134,000 \text{ MT}} = 4 \qquad \frac{134,000 \text{ MT}}{33,500 \text{ MT}} = 4$$

Question 2 The problem on page 160 is with Figure 5.4. We do a quick check to see if the figure makes sense. First, less massive gas molecules should move faster and hence escape from a planet most easily. We see than hydrogen moves faster (is lost more easily) than helium as expected. The even more massive gas molecules have curves that indicate lower speeds, as expected. Next, check the escape velocities of the planets. Equation 5.1 indicates that more massive planets have more rapid escape velocities. On the plot, the planets are ordered from the most massive Jupiter near the top to the least massive Pluto near the bottom. This looks OK. Finally, check the temperatures. These look bad! Venus is only 190K (below freezing point for water) while Earth is at 850K - super hot! In general, we expect surface temperatures to decrease with distance from the Sun, and this is just not the case here. Neptune and Uranus are plotted hotter than Jupiter and Saturn. Mars is hotter than Venus. I do not think that the author would have made this obvious a mistake. What she is probably plotting is the temperatures at the outer edge of the atmosphere, where gas molecules have the chance to escape into space. Looking at Fig. 5.17, we see that Venus' upper atmosphere is actually pretty cold. The author's mistake, however, is in not telling the reader what temperatures are being plotted.

Question 3 a) Two reasons for high altitude eastward winds. 1) The moving packet of air drops in closer to the Earth's spin axis when it moves away from the equator. Since angular momentum is nearly conserved, the air packet speeds up to move faster than before. 2) Wind speed is measured relative to Earth's surface. Points on Earth's surface move at different speeds due to Earth's rotation, with the equator moving fastest and the poles not moving at all. When the air packet moves away from the equator, it is above a part of the Earth that rotates more slowly. Both effects lead to a wind that blows to the East.

From the book, between the equator and 30 degrees North latitude, the air packet speeds up by 537m/s - 465m/s = 72m/s, and the Earth slows down by 465m/s-403m/s = 62m/s. The increase in the air parcel's speed is slightly more important.
b) Low-level air parcels moving north from 30 to 60 degrees latitude lead to the "Westerlies", winds which blow from the west. From the book, the speed of Earth's rotation at 30° N is 403m/s. This is also the speed of the air. We start by working out how this speed changes. Use the fact that angular momentum, mvr, remains constant and equate its value at 30° and 60°. Look at the Fig. 5.35 on page 189 to see what to plug in for r, the distance from the spin axis.

$$m\ (403\ \text{m/s})\ R_E\cos(30) = mvR_E\cos(60)$$

So

$$v = \frac{\cos(30)}{\cos(60)} 403\ \text{m/s} = 698\ \text{m/s}$$

Be sure than your calculator is set to use degrees, not radians in the trigonometric functions! Next, we work out how fast the Earth's surface moves at 60°. At the equator, the speed is 465m/s, so at 60 degrees lat it is just 465cos(60) km/s = 233 km/s. The wind speed is the difference of the absolute wind speed minus the Earth's surface speed: 698 km/s - 233 km/s = 465 m/s. This is 3.5 times faster than the book's calculation for the equatorial Hadley cell (134 m/s).
c) Convert to mph.

$$465\text{m/s}\left(\frac{1\text{km}}{1000\text{m}}\right)\left(\frac{1\text{mile}}{1.6\text{km}}\right)\left(\frac{3600\text{s}}{1\text{hr}}\right) = 1050\text{miles/hr}$$

That is a very fast speed! No wind moves this fast. Lots of things will slow down wind: friction of air against the surface and turbulence in the flow. The book also mentions eddy currents which are circular flows like in Jupiter's red spot. These dissipative effects must be fairly strong to lower the average windspeed to the observed few tens of miles per hour.

Question 4 a) The coloration of Jupiter's yellow and brown cloud layers are thought to arise from sulfur compounds. See, in particular, Fig. 6.12 on page 222.
b) A very efficient greenhouse gas absorbs at multiple wavelengths in the infrared. It also absorbs a lot of red light, the visual wavelength nearest to the infrared. Sunlight goes into Neptune's atmosphere, and the red light is preferentially absorbed. The light bounces off the white cloud layers and comes back out; during this time more red light is absorbed. The light that escapes from Neptune is primarily blue which accounts for the planet's coloration. Uranus and Neptune probably have cloud layers that are similar to those of Jupiter and Saturn hidden at depths not visible to us.

Question 5 a) Question 6.4 has done much of the work for you. It tells us that a 4 Earth Mass core puts out 1.1×10^{14} W of power. We divide this by 4 to go back to an Earth mass of material, and then divide by Earth's surface area $4\pi R_E^2$ where R_E = 6171km =6171000m is Earth's average radius. So

$$\text{Internal Heat Loss} = \frac{1.1\times10^{14}\text{W}}{4\times4\pi R_E^2} = \frac{1.1\times10^{14}\text{W}}{16\pi(6171000\text{m})^2} = 0.0575\text{W/m}^2$$

This is much smaller than the average power in sunlight received by a square meter of Earth's surface 342 W/m^2.

b) Since the heating due to sunlight decreases like the inverse square of distance, it will be 4 times weaker when we move to twice Earth's distance from the Sun (e.g. to 2 AU). We need to make it 342 W/0.0575 W = 5950 times weaker, so we will need to move further than that. How about 40 AU, where Pluto is? The sunlight will be 40^2= 1600 times weaker there. Need to go still further - how about 75 times further? $75^2 = 5625$, so we need just a bit more. We can also solve this problem using algebra by equating the strength of sunlight to that of radioactive heating:

$$\frac{342}{r^2} = 0.0575$$

where r is the distance from the Sun in AU. Solving, we get $r^2 = 5950$ and $r = \sqrt{5950} =$ 77AU.

c) All known planets in the Solar system are inside 77AU and hence have surface temperatures controlled by the Sun. Furthermore, objects smaller than Earth have a lot less radioactive material than Earth does which pushes the boundary calculated in b) still further outward. Only comets in Oort cloud are not heated significantly by the Sun.