ASTR330: HOMEWORK #4 SOLUTIONS

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Question 1 a) Venus takes about one and two-thirds orbits around the Sun in one Earth year. Mercury, being closer to the Sun, makes just over four orbits during the time it takes Earth to make one (so it catches Earth 3 times in one year).This demonstrates Kepler's third law, because the closer the planet is to the Sun (the smaller the semi-major axis), the shorter its orbital period.

b) Elliptical orbits with low eccentricities are very nearly circular, with an e=0 orbit being a perfect circle. As the eccentricity increases, the center of the circle becomes offset from the Sun, and for large eccentricities, the orbit starts to look like a partially-squashed circle.

c) As in all elliptical orbits, the asteroid will move fastest when it is closest to the Sun. A drawing should show the equal area swept out in equal time by the asteroid at opposing ends of its orbit (see Fig. 7.3 on page 247). In a given time interval, the asteroid travels a shorter distance far from the Sun, and a larger distance close to the Sun. The semi-major axis of this asteroid is 1 AU, so we can plug into Kepler's third law (equation 7.3 on page 248) to get the period: $P^2 = ka^3$ with the constant $k = 1$ when the period P is measured in years and the semimajor axis a in AU. The asteroid at 1 AU should have a 1-year period. Upon investigation, the asteroid indeed takes the same amount of time to orbit the Sun as the Earth does.

Question 2 As on previous homeworks, we use the equations for the mass of an object ($M = V\rho$, where M is mass, V is volume, and ρ is density) and the volume of a sphere $V = \frac{4}{3}\pi r^3$, with r the radius of the sphere. The satellite's mass is then:

$$M_{sat} = \frac{4}{3}\pi R_{sat}^3 \rho_{sat}$$

and the planet's mass is

$$M_{planet} = \frac{4}{3}\pi R_{planet}^3 \rho_{planet}.$$

So plugging these masses into the given expression for r_{Roche}, we find:

$$r_{Roche} = \left(\frac{3(\frac{4}{3}\pi R_{planet}^3 \rho_{planet})}{\frac{4}{3}\pi R_{sat}^3 \rho_{sat}}\right)^{1/3} = R_{sat}\left(\frac{3(R_{planet}^3 \rho_{planet})}{R_{sat}^3 \rho_{sat}}\right)^{1/3} = R_{sat}\frac{R_{planet}}{R_{sat}}\left(\frac{3\rho_{planet})}{\rho_{sat}}\right)^{1/3}$$

So

$$r_{Roche} = R_{planet}\left(\frac{3\rho_{planet}}{\rho_{sat}}\right)^{1/3} \quad \text{and finally} \quad \frac{r_{Roche}}{R_{planet}} = \left(\frac{3\rho_{planet}}{\rho_{sat}}\right)^{1/3}$$

b) See attached Planetary Calculator output.

c) The Roche limit, measured in planetary radii, is further out for the terrestrial planets than for the giant ones. This is a consequence of the greater densities of the terrestrial planets, which strengthens the destructive power of the tides that they raise on nearby objects. Note that in actual distance units like meters or kilometers, the Roche limit is larger for larger planets. Some of you may have noticed that the "3" in this equation has other values in some other text or websites. The value of this coefficient depends on various assumptions that are made about the satellite's spin period and internal strength.

Question 3 a) This question turned out to be more confusing than I had hoped. Unfortunately, I also cannot make an electronic plot, so I'll have to just describe it.I wanted you to make a plot with time along the x-axis and mass along the y-axis. The plot should 2 curves on it, one for Jupiter and one for Earth. Earth grows to about 1/10 its current mass in 47,000 years (Fig. 8.16). Jupiter grows to about 10 times Earth mass in 390,000 years (Fig. 8.18). This marks the end of the runaway growth phase. Afterwards, both planets grow extremely slowly, as they collide with other larger objects over 100 million years. The curves for both planets should therefore be fairly flat for hundreds of thousands of years after the runaway growth phase. Jupiter grows large enough, quickly enough, that its gravity stirs up the asteroid belt and causes the large objects in that region to collide at high speeds which shatters them into smaller objects again.

b) A hotter solar nebular would have moved the snow line outward (see Fig 8.14) and so Jupiter would have formed further from the Sun, perhaps allowing an extra planet to form in the asteroid belt. A cooler solar nebular would have allowed Jupiter to form closer to the Sun, perhaps preventing Mars from forming and stunting Earth's growth as well.

Question 4 a) Test the equations. In the limit t=0, $2^0 = 1$ and the equations reduce to: 1) ^{87}Sr = ^{87}Sr(t=0) and 2) ^{87}Rb = ^{87}Rb(t=0). The interpretation of this is that at time t=0, the equations predict the initial amounts of Sr and Rb, as expected. The equations pass this test. Next try large time $t >> T$. Here $2^{-\infty} = 0$, so 1) ^{87}Sr = ^{87}Sr(t=0) + ^{87}Rb(t=0) and 2) ^{87}Rb = 0. After a long time has passed, all Rubidium has decayed, and each Rubidium atom has produced a Strontium atom. This is as expected. Finally, try $t = T$. Here $2^{-1} = 1/2$ so 1) ^{87}Sr = ^{87}Sr(t=0) + 0.5 ^{87}Rb(t=0) and 2) ^{87}Rb = 0.5 ^{87}Rb(t=0). Exactly half the Rubidium has decayed after one halflife. The equations pass all of these tests, so we can have more confidence that they are correct.

b) Testing the other equation, we find that at t=0: ^{87}Sr(t=0) = ^{87}Sr(t=0) - ^{87}Rb(t=0). This statement is false! Furthermore, at large time $t >> T$, we find ^{87}Sr(t=0) + ∞, and an infinite number of Strontium atoms! The equation is definitely wrong.

c) At t=4.46 billion years, t/T = 4.46/48.8 = 0.0914 halflives have passed and the number of Rubidium atoms has decreased to ^{87}Rb = ^{87}Rb(t=0) $2^{-0.0914}$ = 0.9386 ^{87}Rb(t=0). So 93.86% of the radioactive Rubidium has not decayed yet. In reality, we measure the 93.85% in our meteorite samples and work backwards to determine that nearly all meteorites are 4.46 billion years old!

Question 5 a) Calcium-Aluminum rich Inclusions (CAIs) are extremely high-temperature compounds found in primitive, undifferentiated meteorites. They probably formed very near the Sun, either by direct condensation from the Solar Nebula, or by melting of the more volatile elements from dust grains that pre-dated the formation of the Solar System.

b) Chondrules are small spheres of silicate material found in the most primitive meteorites, the chondrites. They show evidence for melting followed by rapid cooling before being incorporated into meteorites. There spherical shapes follow from solidification in a weak gravitational field. Both Chondrules and CAI date back to before meteorites formed.

Question 6

The relative dates (the 13 million year timespan in Fig. 9.13) are determined by using a particular radioactive decay system (^{53}Mn decays to ^{53}Cr) with a 3.7 million year halflife. This technique gives very accurate relative dates over a few tens of millions of years. To connect these to an absolute age of 4.5 billion years, a more long-lived radioactive isotope must be used. The two scales on Fig. 9.13 represent two different attempts, each with typical errors of a few million years. This is not bad out of 4.5 billions years (something like 0.1% error), but it leaves us with an accurate set of relative ages and a more poorly determined absolute age.

Planetary Calculator

Results in MKS units

Planet	(3*rho/500)^(1/3)
MERCURY	3.19386846
VENUS	3.15817979
EARTH	3.21141754
MARS	2.87242996
JUPITER	1.99833194
SATURN	1.60570877
URANUS	1.97809427
NEPTUNE	2.14288253
PLUTO	2.30081886

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