ASTR430 Homework # 2 – Orbital Motion
Due Thursday, March 1

Read the rest of the "Building Physical Intuition in Mechanics" handout and try some examples from each section. In the following problems - and in all future homeworks - make sure to check units, limits, symmetry, and your common sense! You will catch many errors this way. Show me that you have thought about your answers rather than just deriving them.

1. Sketch the following orbit at four times, $t = 0, 1/8, 1/4, 3/8$ years, and then describe, without using the names of the orbital elements a, e, i, Ω, ω, how the orbit is changing in time.

2. Atmospheric Drag.
Imagine a satellite orbiting above the Earth's equator with orbital elements $a = 5.2R_E, e = 0.8$ and $i = 0$ where R_E is the Earth's radius and a, e, i are the orbit's semimajor axis, eccentricity, and inclination, respectively.
a) How close does the satellite approach the Earth's surface at pericenter, and what is its speed there? Is it moving faster or slower that the Earth's rotation?
b) The Earth's atmosphere decays exponentially with height with a scale height of approximately 10 km, so it is an excellent approximation to treat atmospheric drag on the satellite as an impulse at pericenter. Which way does this impulse point relative to the velocity vector at pericenter? Show that the orbital semimajor axis decreases in time and find out what happens to the orbital eccentricity and inclination.
c) Sketch the original orbit, and three or four of its future states. What happens in the end?

3. At apocenter, a 10-ton satellite on an $a = 4R_E$, $e = 0.5, i = 0$ orbit uses a large coiled spring

to launch a 1-ton communication satellite onto an $a = 6R_E$, $e = 0.0, i = 0$ orbit (R_E is the Earth's radius). Find the new orbit of the (now 9-ton) parent satellite. Are satellite orbital energy and angular momentum lost, gained, or conserved? Explain your answer.

4. As the interstellar cloud collapsed to form our Solar System, the gas was heated significantly. In this problem, you will estimate how hot the disk was at 1AU assuming that no energy was lost via radiation during collapse. In reality, lots of energy is radiated away so this will be a serious overestimate! Imagine an H_2 molecule which is at rest at infinity and falls inward toward the Sun.
a) Calculate the circular velocity at 1AU around a 1 Solar Mass star, and determine the total (kinetic + potential) energy of an H_2 molecule on a circular orbit at 1AU.
b) Noting that the total energy of the molecule at rest at infinity is zero, calculate the temperature increase of the hydrogen gas assuming that it has not suffered radiative losses.

5. EXTRA CREDIT CHALLENGE PROBLEM (Experts Only): Convert the scalar equation

$$\frac{d^2r}{dt^2} - \frac{h^2}{r^3} = -\frac{GM_\odot}{r^2}$$

that we derived in class into one for $d^2u/d\theta^2$ using the transformation $u = 1/r$ and the expression for angular momentum $h = r^2 d\theta/dt$. Solve for u and show that the solution can be written as

$$r = \frac{h^2/GM_\odot}{1 + e\cos(\theta - \varpi)}$$

where e and ϖ are integration constants. These solutions are ellipses with the Sun at one focus.