ASTR430 Homework # 3 – Orbital Motion
Due Tuesday, March 13

1. a) Estimate the maximum rotation period of a star in hours by equating gravity with the centrifugal force at the star's equator. How does you answer differ for a planet?
b) Calculate the spin angular momentum of the Sun (look up the moment of inertia for a uniform sphere in a physics book), the spin angular momentum of Jupiter, and the orbital angular momentum of Jupiter.
c) If, somehow, Jupiter were absorbed by the Sun (assume that the angular momentum vectors are parallel, that the total angular momentum is conserved, and that the radius of the Sun is not changed), how fast would the Sun spin? What would happen?
d) Now imagine a one solar mass spherical cloud of gas with uniform density and radius 1 light year. How fast does it spin if it has the same angular momentum as the Sun-Jupiter system?

2. Dimensional Analysis. In class, we looked at ballistic trajectories and found an expression for the range of a projectile launched with velocity v at an angle θ from the horizontal: $D = (v^2/g)f(\theta)$, where $f(\theta)$ is an undetermined dimensionless function and g is assumed to be constant.
a) Use dimensional analysis to obtain a similar expression for the maximum height H of the projectile assuming constant g.
b) Use conservation of Energy to solve for the maximum height exactly and check your answer against part a)– what is $f(\theta)$?
c) Now use dimensional analysis to get an expression for H using the true $1/r^2$ gravity of the Earth. Neglect the Earth's spin, treat the Earth as a perfect sphere, and consider a projectile launched at an arbitrary angle. Start by identifying four variables that the solution will depend upon. See if you can form a dimensionless constant from the four variables and remember that the most general solution is the one that is multiplied by the most general dimensionless function.
d) Do the related problem #107 from the Physical Intuition in Mechanics handout (Note that here the projectile is fired vertically with $\theta = 90°$).
e) Finally, solve for the maximum height of a projectile exactly and check your answer against part c). Start by arguing that trajectories will be segments of conic sections. What conservation laws can you use? Your answer will be algebraically messy - simplify it as much as you can. Does your answer make sense? Consider all of the limiting cases that you can think of as tests – including your answers from c) and d). Write these down and use them to argue whether your full answer is reasonable.

3. In this problem, you'll use conservation of energy and angular momentum to solve for the radial turning points for orbits in central force motion (i.e. you'll derive the blue curves seen in the Central Force Integrator).
a) For a general force, $\mathbf{F}(\mathrm{r})$, write down the expressions for angular momentum per unit mass, h, and energy per unit mass, C, in terms of the position, r, and the velocity, v.
b) Simply your expression(s) for a central force with the form $\mathbf{F} = -Ar^n\hat{r}$.
c) Use your two equations and the condition of being at a radial turning point to obtain an equation for r in terms of constants.
d) For gravity ($n = -2$), use the specialized formulas for C and h to solve for the radial turning points in terms of the semimajor axis, a, and eccentricity, e.
e) For the general case in c), argue that there are at most 2 positive real solutions for r (the only ones that matter physically). Interpret the special cases of 0,1, and 2 solutions physically.
f) How many physical solutions are there for the case $n = -3$? What does this tell you about orbits in a $1/r^3$ force field? Test your ideas by investigating circular and near circular orbits with the central force integrator. Show me some plots and summarize what you find!