ASTR430 Homework #7
Due Thursday, May 10, 2001

1. a) Use a symmetry argument to determine how the direction and magnitude of gravity from Saturn's wide flat ring (modeled as a uniform-density flat sheet of infinite extent) varies with height above the ring.

b) Show that, just above the ring, the ring's gravity dominates over the vertical component of gravity from Saturn. The undetermined constant from part 1a) is 2π.

c) Now imagine a ring "atmosphere" - gas molecules above and below the rings of Saturn. There is some evidence that a ring atmosphere of Oxygen, Hydrogen, Water Vapor, and OH actually exists. Use previous handouts and homeworks to help you derive how the density of gas should vary as a function of height above the rings.

2. Consider the 2:1 resonance between Io and Europa which is governed by the following approximate expressions.

$$\frac{da_I}{dt} = 2\beta m_E a_I^2 n_I e_I \sin(\Psi) \qquad \frac{da_E}{dt} = -4\beta m_I a_E^2 n_E e_I \sin(\Psi)$$

$$\frac{de_I}{dt} = -\beta m_E a_I n_I \sin(\Psi) \qquad \frac{de_E}{dt} = 0$$

$$\frac{d\varpi_I}{dt} = -\frac{\beta m_E a_I n_I}{e_I} \cos(\Psi) \qquad \frac{d\varpi_E}{dt} = 0$$

In the above expressions, subscripts E and I stand for Europa and Io, respectively, t is time, β is a constant, m is mass, a is the semimajor axis, $n = \sqrt{(GM/a^3)}$ is the mean motion (the satellite's average angular speed), e is eccentricity, and ϖ is the longitude of pericenter. The resonant angle $\Psi = 2l_E - l_I + \varpi_I$ where l is the satellite longitude and $dl/dt \approx n$.
a) Show that the above expressions conserve energy. You can use $2n_E \approx n_I$.
b) Now, because of their steep dependence on distance, the tides pushing Io outward dominate the tides pushing Europa outward. To a good approximation, therefore, we can add constant tidal terms $\dot{a}_{drag} > 0$ and $\dot{e}_{drag} < 0$ to the Io equations only ($\dot{\varpi}_{drag}$ is also negligible because of the e_I in the denominator of the corresponding resonant term). Set the new da_I/dt equation equal to zero (not a great approximation, but not too bad), solve for $\sin\Psi$ and use it to eliminate $\sin\Psi$ in de_I/dt. Solve for the equilibrium eccentricity. The existence of this equilibrium eccentricity allows tides to dissipate energy in Io.