

ASTR450 Homework # 10 – Perturbations
Due Thursday, December 5

1. **Impulse Approximation to a Drag Force.** Consider a satellite on an inclined elliptical orbit acted on by a drag force of the form $\mathbf{F} = -k\mathbf{v}_{rel}$, where \mathbf{v}_{rel} is the velocity of the satellite relative to the atmosphere, and k is a positive constant. Recall that Earth's atmosphere decays exponentially with height (scale height $\approx 10\text{km}$).

a) Approximate the range of eccentricities for which the drag force can be approximated by an impulse at pericenter.

b) Consider a rotating Earth. Start by making a qualitative estimate of the error made in neglecting rotation. How does the Earth's rotation affect an equatorial orbit ($i = \Omega = 0$)? Describe how the orbital elements a, e, i, Ω, ϖ vary in time.

c) Now imagine orbits with $i \neq 0, \Omega \neq 0$. Using the perturbation equations and other physical arguments, describe qualitatively how these orbits will evolve (i.e. how a, e, i, Ω, ϖ vary in time).

2. **Radial Perturbation Forces.** Consider a radial perturbation force of the form $\mathbf{F} = R\hat{r}$, where R is a function of the distance r .

a) Apply the perturbation equations to this force and obtain simplified expressions for da/dt , de/dt , di/dt , $d\Omega/dt$, and $d\varpi/dt$.

b) A radial perturbation to gravity, which is itself a radial force, is an example of a central force. So angular momentum must be conserved. Show that your equations conserve the angular momentum vector and describe the constraints that this imposes on these orbits.

c) Now let $R = Ar^n$ where A is a constant. Take the time average of your expressions over a single unperturbed Keplerian orbit (this step assumes that the perturbation is small). Show that $\langle r^n \sin \nu \rangle = 0$ and argue, on physical ground, that $\langle \cos \nu \rangle$ is negative (or zero) and that $\langle r^{-2} \cos \nu \rangle = 0$. It can be shown that $\langle r^n \cos \nu \rangle$ is negative for $n > -2$ and positive for $n < -2$. Use this fact to determine how the sign of your time-averaged $d\varpi/dt$ depends on A and n . Use the Central Force Integrator to check your results numerically.

d) Finally, consider the General Relativistic (GR) Perturbation $R = Ar^{-4}$ where A is a small negative constant. The integral $\langle r^{-4} \cos \nu \rangle = a^{-4}e(1 - e^2)^{-5/2}$. Solve, analytically, for the value of A that will give 30 degrees of precession per orbit for $e = 0.5$ (The true effects of GR on Mercury's orbit are almost exactly a million times weaker). Convert your prediction into the proper initial conditions for the Central Force Integrator, and test it! Start your orbit at pericenter and turn in a copy of your plot.