

ASTR450 Homework # 7

Due Tuesday, November 12

1. a) **Orbital Elements.** Find the six orbital elements $(a, e, i, \Omega, \omega, \nu)$ for an asteroid which, at time $t=0$, has $(X, Y, Z) = (2.5AU, 0, 0)$ and $(V_x, V_y, V_z) = (V_E/\sqrt{10}, -V_E/\sqrt{5}, 0)$. Here V_E is the speed of the Earth in its orbit, and an AU is the astronomical unit. Take the reference plane to be the XY plane and the reference direction to be \hat{X} . It is easiest to use dimensionless units; take $V_E = 30\text{km/s}$ to be the unit of velocity, and the AU to be the unit of distance, and $GM = 1$ to define the unit of mass (so in dimensionless units, $V_E = 1, GM = 1$, and the Earth-Sun distance is one).

b) Find the pericenter distance, the apocenter distance (if it exists), and the semilatus rectum of the orbit, and use these quantities, and your orbital elements, to draw a reasonably accurate sketch of the orbit relative to the X and Y axes.

2. Use the “3D Binary Star Integrator” on the class webpage to investigate orbits near the L4 and L5 Lagrange points. Set “Mass of Star 2” to .001, and the “Eccentricity”, “Inclination”, “Argument of Pericenter” and “Longitude of Ascending Node” to zero. Try different True Anomalies to see what range gives Tadpole orbits around L4, Tadpole orbits L5, and Horseshoe orbit (one that surround both L4 and L5). For a Jupiter-mass planet, roughly how many orbital periods does it take to go once around the equilibrium point (This is the Libration Period)? For small Tadpole orbits, try increasing the secondary mass - how does the libration period change, and at what mass ratio is stability lost? Compare this with the value on Danby, page 265. Can you spot Danby’s error? Try some small eccentricities and inclinations - what happens?. Rotating coordinates are most useful, but try some inertial coordinates so that you see what is going on. Write up a page or two discussing your findings and attach some relevant orbits. Explore and have fun!

3. The following program is due in THREE WEEKS (Nov. 19).

Two-Body Problem. Write a two part computer program that translates i) from orbital elements $(a, e, i, \Omega, \omega, \nu)$ to positions and velocities (x, y, z, v_x, v_y, v_z) and ii) from positions and velocities back to orbital elements. Devise your own algorithms, or use the ones given in Danby, Section 6.15. For Danby’s algorithm, note that Eq. 6.15.4 comes from 6.2.5 and that \mathbf{P} is a vector pointing from the mass-occupied focus to pericenter with magnitude e .

Basic Program Your program should work for 2D ($i = \Omega = 0$) elliptical orbits, but does not have to handle unbound (parabolic and hyperbolic) orbits.

Extra Credit Your program should work for 3D elliptical and inclined orbits.

More Extra Credit Your program should handle hyperbolic and parabolic orbits as well as elliptical ones.

Be thorough in testing your program! Test it by translating $(x, y, z, v_x, v_y, v_z) \rightarrow (a, e, i, \Omega, \omega, \nu) \rightarrow (x, y, z, v_x, v_y, v_z)$ and $(a, e, i, \Omega, \omega, \nu) \rightarrow (x, y, z, v_x, v_y, v_z) \rightarrow (a, e, i, \Omega, \omega, \nu)$ for a number of cases. Also test your programs against results from the 2D and 3D orbit viewers and against your intuition for a number of special cases (e.g. circular orbits ought to have $\mathbf{r} \perp \mathbf{v}$).