

ASTR450 Homework # 4 – Central Force Motion  
Due Thursday, September 28

Reading: Start Danby's Chapter 6.

1. Danby: Page 83, Problem 8 (Hard). To start, draw a picture of the elliptical orbit and label the semimajor axis, semiminor axis and latus rectum. The fact that a particle sweeps out equal areas in equal times for all central forces will be useful for this problem. You will need to look up some hairy integrals!
2. Danby: Page 83, Problem 9 (Moderate). Change notation a bit so that the inward force has strength  $GM/r^2$ , the launch is perpendicular to the radius vector at a speed  $v$ . Find the condition on  $v$  for escape, and determine whether launch is at pericenter or apocenter. For bound orbits, find  $a$  and  $e$  in terms of the initial conditions  $r, v$ .
3. Danby: Page 84, Problem 10 (Hard). You can assume that the force is gravity (so set  $\mu = GM$ ). Draw a picture to start and use the equation that describes the velocity of a particle in terms of  $a$  and  $r$ . Under what conditions will the future orbit be a circle? An ellipse? A parabola? A hyperbola?
4. Danby: Page 85, Problem 25 (Moderate). Instead of the question about six cases, describe what happens for orbits with positive energy  $C > 0$ , negative energy  $C < 0$ , and zero energy  $C = 0$ . What are the possibilities on how  $r$  varies with time in each case? Use the Central Force Integrator to show numerically what happens to four types of motion when the force law is changed to  $r^{-2.9}$ : i) inward to  $r = 0$ , ii) outward to  $r \rightarrow \infty$ , iii) inward to a minimum distance then outward, and iv) outward to a maximum distance then inward. Are these types of motion still possible?