ASTR450 Homework # 4 – Central Force Motion
Due Thursday, February 26

Reading: Start Danby's Chapter 6.

1. Danby: Page 83, Problem 8 (Hard). To start, draw a picture of the elliptical orbit and label the semimajor axis, semiminor axis and latus rectum. The fact that a particle sweeps out equal areas in equal times for all central forces will be useful for this problem. You will need to look up some hairy integrals!

2. Danby: Page 83, Problem 9 (Moderate). Change notation a bit so that the inward force has strength GM/r^2, the launch is perpendicular to the radius vector at a speed v. Find the condition on v for escape, and determine whether launch is at pericenter or apocenter. For bound orbits, find a and e in terms of the initial conditions r, v.

3. Danby: Page 84, Problem 10 (Hard). You can assume that the force is gravity (so set $\mu = GM$). Draw a picture to start and use the equation that describes the velocity of a particle in terms of a and r. Under what conditions will the future orbit be a circle? An ellipse? A parabola? A hyperbola?

4. Danby: Page 85, Problem 25 (Moderate). Instead of the question about six cases, describe what happens for orbits with positive energy $C > 0$, negative energy $C < 0$, and zero energy $C = 0$. What are the possibilities on how r varies with time is each case? Use the Central Force Integrator to show numerically what happens to four types of motion when the force law is changed to $r^{-2.9}$: i) inward to $r = 0$, ii) outward to $r \to \infty$, iii) inward to a minimum distance then outward, and iv) outward to a maximum distance then inward. Are these types of motion still possible?