Dynamical stability of multi-planet systems: application to extrasolar systems with mean motion resonances

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The dynamical study of planetary systems has become more and more relevant, now that several hundreds of other systems have been found: understanding their dynamical architecture is a key step for several fields of research such as planet formation and solar system formation, but also in the quest for Earth-like planets. For most of the systems, we know just a few parameters: the number of planets, their masses, their periods, and the mass of the star. That is enough information to allow dynamical modeling of those systems, thus leading to a good understanding of their stability properties. Such knowledge can be used, for example to determine whether or not an Earth-like planet would be stable in the habitability region of this system.

We propose an overview of the different stability criteria that are used in studying the dynamics of planetary systems. We present the Hill-stability criterion for the general three-body problem (Marchal and Bozis, 1982), and its simplification to systems of two planets (Gladman, 1993). We proceed to a derivation of a general stability criterion for two planets equivalent to the one described in Chambers et al. (1996), but using the Tisserand criterion instead of the complete Jacobi integral. A brief description of the Lagrange criterion is also proposed.

The recent increase in computing power, along with the development of new integration algorithms, have allowed to push dynamical analyses much further. We provide a introduction to the principles of symplectic integrators and their characteristics (Wisdom and Holman (1991), Levison and Duncan (1994), Chambers (1999)). We show some general results that were found for multi-planet systems (Chambers et al. (1996), Smith and Lissauer (2009)).

Simulations can be used to study dynamical stability within known extrasolar systems. We explore a few results on this topic (Barnes and Greenberg (2006)) and explain the method that is used. Although the Hill criterion previously discussed applies in most cases, it seems that some configurations allow dynamical stability even if the Hill criterion is not satisfied: we describe how mean motion resonances can create additional stability regions (Barnes and Greenberg, 2007).

Finally, we incorporate a symplectic integrator called SWIFT developped by Levison and Duncan (1994) into a Python wrapper code, in order to proceed to some custom simulations. In particular, we test several stability criteria and limits in the two-planet case, and demonstrate the impact of a 2:1 mean motion resonance in case of a system that does not satisfy Hill stability.

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