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My chosen article is "On the Convergence of the Theory of Figures," by Hubbard et al. (2014) In it, the authors discuss how to solve for the gravitational potential of a Maclaurin spheroid, which is defined to have constant density with equatorial radius a and polar radius b. They begin by pointing out that the "traditional" Laplace expansion in powers of r'/r (to use the paper's notation) diverges when solving for the potential in the region $b < r < a$. However, they go on to substantiate the claim made by Zharkov and Trubitsyn (1978) that this divergent series is valid, but introduce the caveat that the rotational distortion must be sufficiently small. To show this, the authors derive the geophysical expansion for the surface potential as an expansion in the small parameter l^2, where $l^2 = (a^2/b^2) - 1$. In their derived expansion, it is clear that the expansion converges if and only if $l^2 < 1$, making $l^2 = 1$ a critical value. They also note that Saturn and Jupiter are far less oblate than a Maclaurin spheroid at this critical value (at which $a = \sqrt{(2)}*b$).

Next, the authors discuss how many terms should be included in the expansion for the geophysical surface potential. They introduce a dimensionless Δ, defined as the difference between the exact surface potential at the point r = b (the "North Pole," which they also call an "audit point") and the geophysical surface potential evaluated for a finite number of terms. The motivation for this particular location is that Δ is at a maximum there. They conclude that, "expansion to ~degree 12 suffices to keep Δ below the Juno detection limit," referencing the (en route) Jupiter orbiter Juno. Finally, the authors generalize to concentric Maclaurin spheroids by using a set of audit points with one point at the pole of each. They find that for a standard polytrope model of Jupiter, the Δ values are $\sim 10^{-13}$, only a few orders of magnitude higher than the floating-point precision of a 64-bit computer. This leads to the conclusion that the standard Jupiter models are not sufficiently oblate to cause concern about the convergence or otherwise of the Laplace expansion used to calculate their gravitational potentials.

My project will focus on testing this paper's results. I will write code to numerically calculate the potential for Maclaurin spheroids with various l^2 values and then compare the numerical calculation to this paper's prediction. Since lack of computational time is not a significant problem for this project, I can afford to calculate out to many more terms than their suggested ~12 degrees. In addition, for each spheroid I test, I can see how many degrees are necessary to get below the Juno detection limit, which the authors seemed to implicitly endorse as the standard to which their expansion should be held. I can also do a survey of spheroids in the Solar System whose a and b values are known to reasonably high precision and see if any of them surpass the critical $l^2 = 1$ value. Hubbard et al. note in their last paragraph that, "Small, rapidly-rotating bodies such as asteroids may enter a parameter space where TOF convergence may be of concern." It would be interesting to see if calculations for the gravitational potentials of such bodies needed to be revised in light of this paper's analysis.

Hubbard, W., Schubert, G., Kong, D., Zhang, K.. 2014. On the convergence of the theory of figures. Icarus **242**, 138-141.

Zharkov, V.N., Trubitsyn, V.P., 1978. Physics of Planetary Interiors Pachart, Tuscon, pp. 221-295.