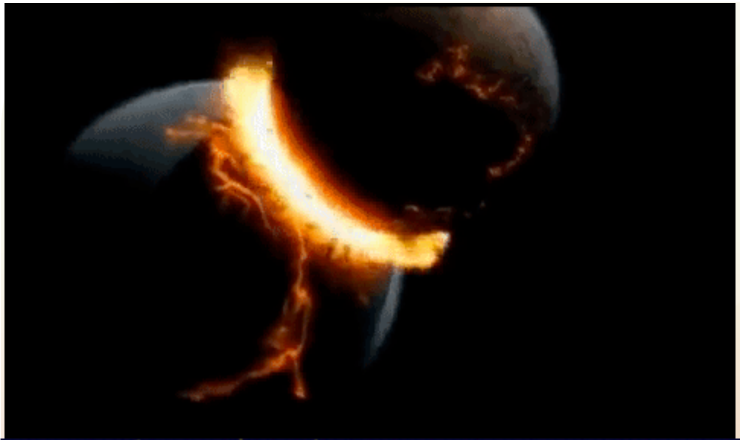

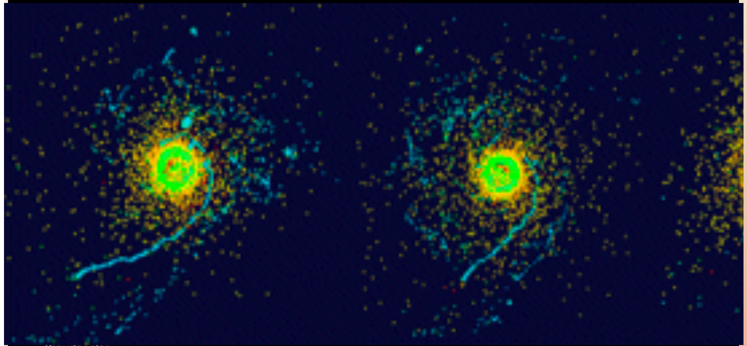

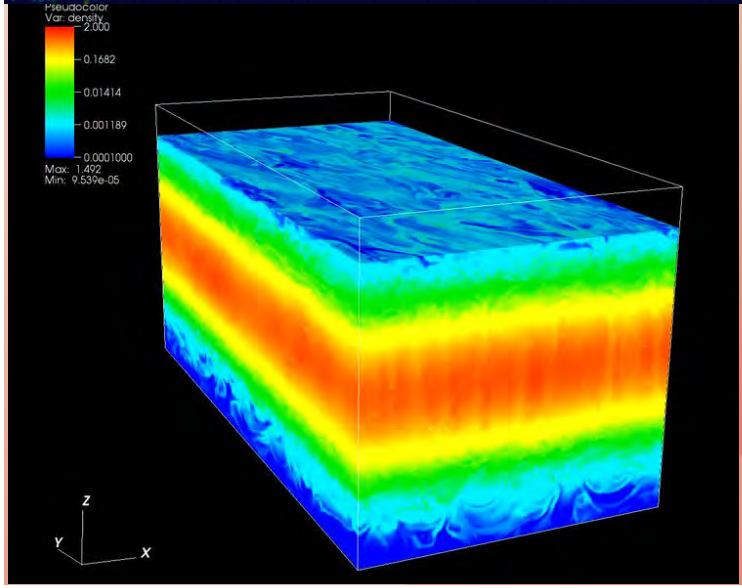

Magneto-rotational Instability in the Protolunar Disk

Sara Frederick
ASTR 630
Thursday, Dec 8, 2016

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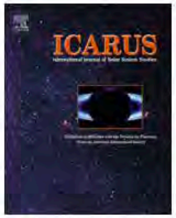

Magneto-rotational instability in the protolunar disk

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ABSTRACT

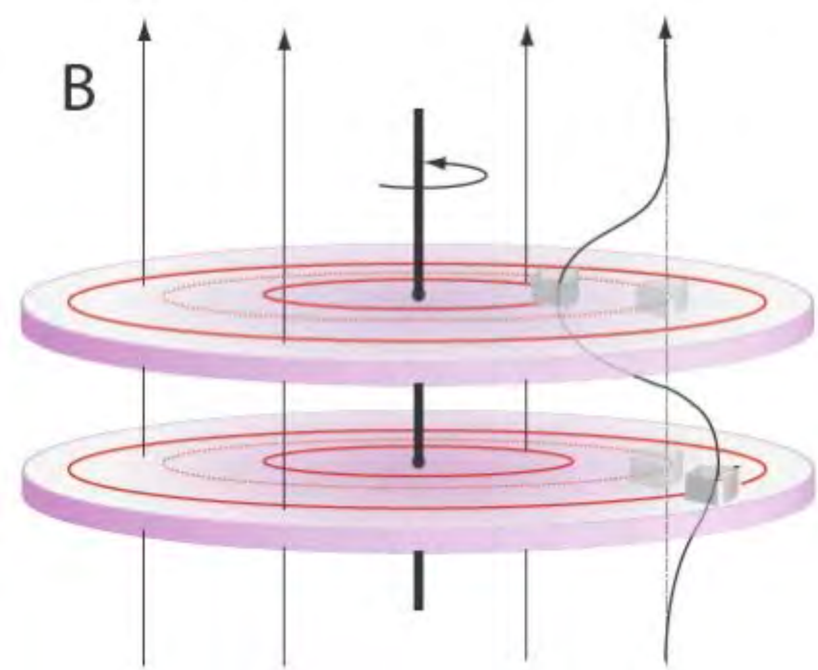
We perform the first study of magnetohydrodynamic processes in the protolunar disk (PLD). With the use of published data on the chemical composition of the PLD, along with existing analytical models of the disk structure, we show that the high temperatures that were prevalent in the disk would have led to ionization of Na, K, SiO, Zn and, to a lesser extent, O_2. For simplicity, we assume that the disk has a vapor structure. The resulting ionization fractions, together with a relatively weak magnetic field, possibly of planetary origin, would have been sufficient to trigger the magneto-rotational instability, or MRI, as demonstrated by the fact that the Elsasser criterion was met in the PLD: a magnetic field embedded in the flow would have diffused more slowly than the growth rate of the linear perturbations. We calculate the intensity of the resulting magnetohydrodynamic turbulence, as parameterized by the dimensionless ratio α of turbulent stresses to gas pressure, and obtain maximum values $\alpha \sim 10^{-2}$ along most of the vertical extent of the disk, and at different orbital radii. This indicates that, under these conditions, turbulent mixing within the PLD due to the MRI was likely capable of transporting isotopic and chemical species efficiently. To test these results in a conservative manner, we carry out a numerical magnetohydrodynamic simulation of a small, rectangular patch of the PLD, located at 4 Earth radii (r_E) from the center of the Earth, and assuming once again that the disk is completely gaseous. We use a polytrope-like equation of state. The rectangular patch is threaded initially by a vertical magnetic field with zero net magnetic flux. This field configuration is known to produce relatively weak MRI turbulence in studies of astrophysical accretion disks. We accordingly obtain turbulence with an average intensity $\alpha \sim 7 \times 10^{-6}$ over the course of 280 orbital periods (133 days at $4r_E$). Despite this relatively low value of α, the effective turbulent diffusivity $D \sim 10^{10} - 10^{11}$ cm^2 s^{-1} of a passive tracer introduced in the flow is large enough to allow the tracer to spread across a radial distance of $10r_E$ in ~ 13–129 yr, less than the estimated cooling time of the PLD of ~ 250 yr. Further improvements to our model will need to incorporate the energy balance in the disk, a complete two-phase structure, and a more realistic equation of state.

Outline of this Talk

- ~~Part 1: Conditions for MRI~~
- Part 2: Simulations of portion of the PLD
- Future Directions

Magneto-rotational Instability (MRI)

- Requirements:
  - Magnetic Field Threading Disk
  - Differential Rotation
  - Ionization of Disk



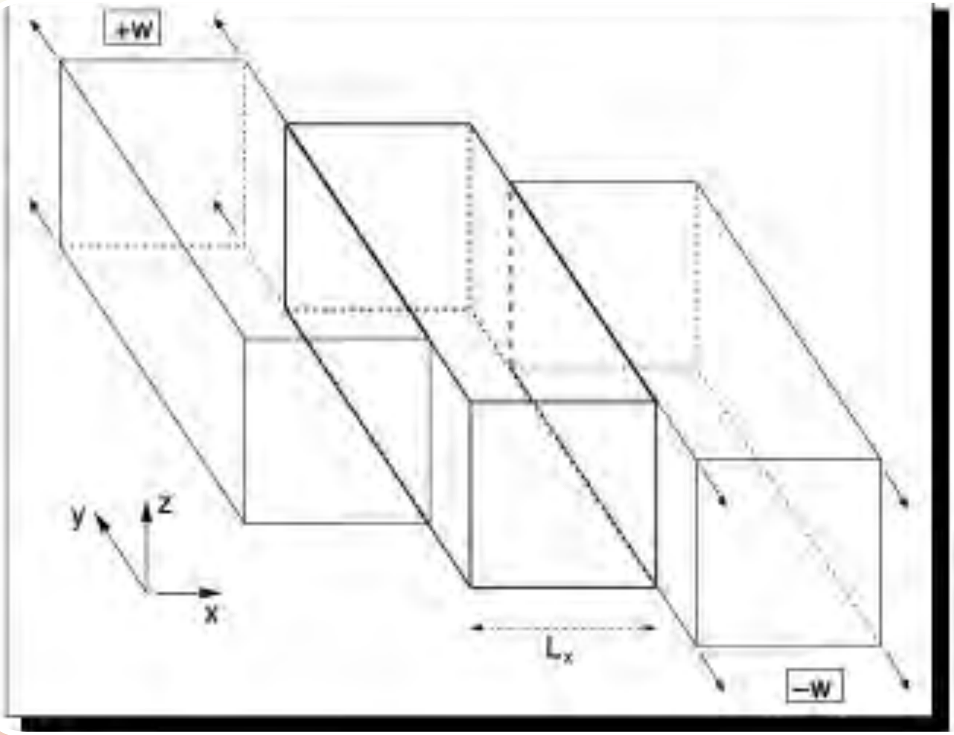
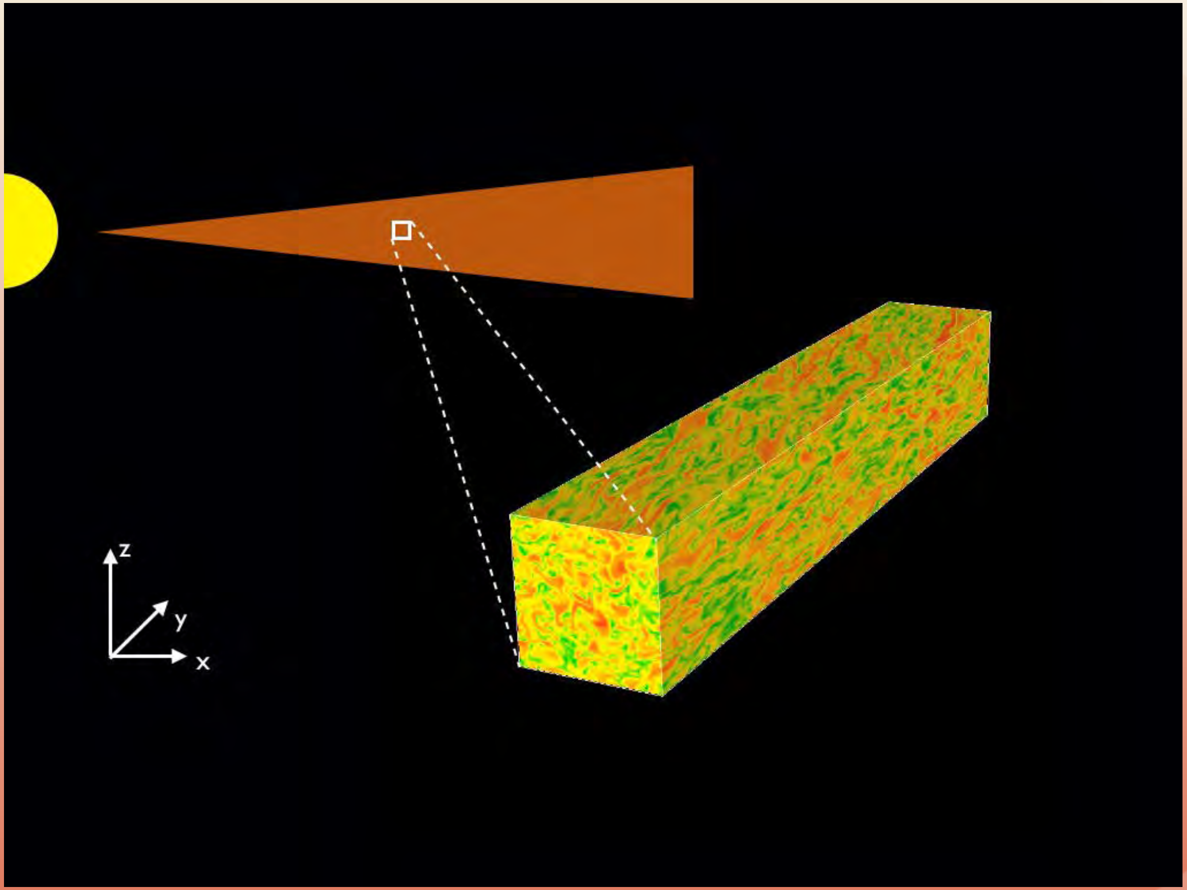


Figure 10.1: Schematic representation of the shearing boundary condition. The computational domain (central box) is assumed to be surrounded by identical boxes sliding with constant velocity $w = |q\Omega_0 L_x|$ with respect to one another.

MHD Simulations

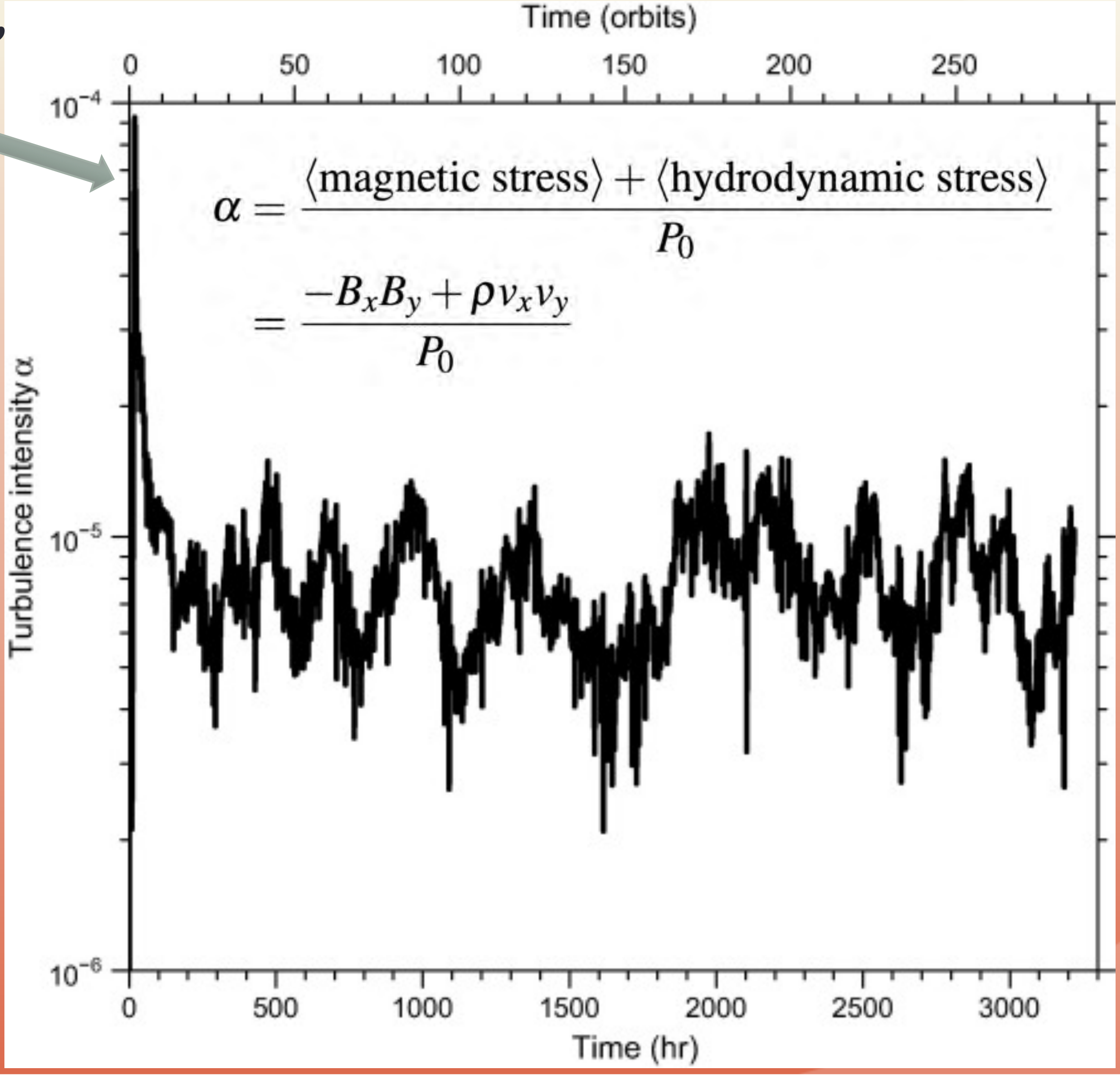
Shearing Box Approximation



credit: Jake Simon

Disk Turbulence

“Channel Flow”



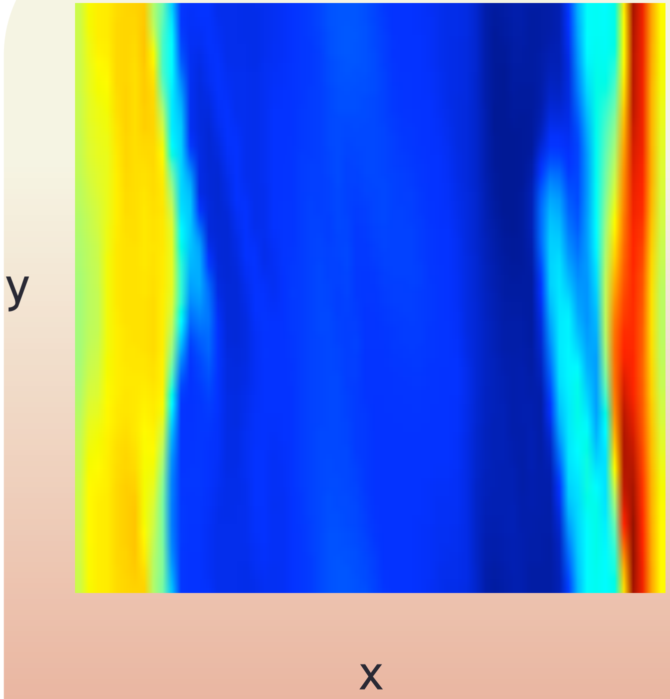
$$\alpha = \frac{\langle\text{magnetic stress}\rangle + \langle\text{hydrodynamic stress}\rangle}{P_0} = \frac{-B_xB_y + \rho v_x v_y}{P_0}$$

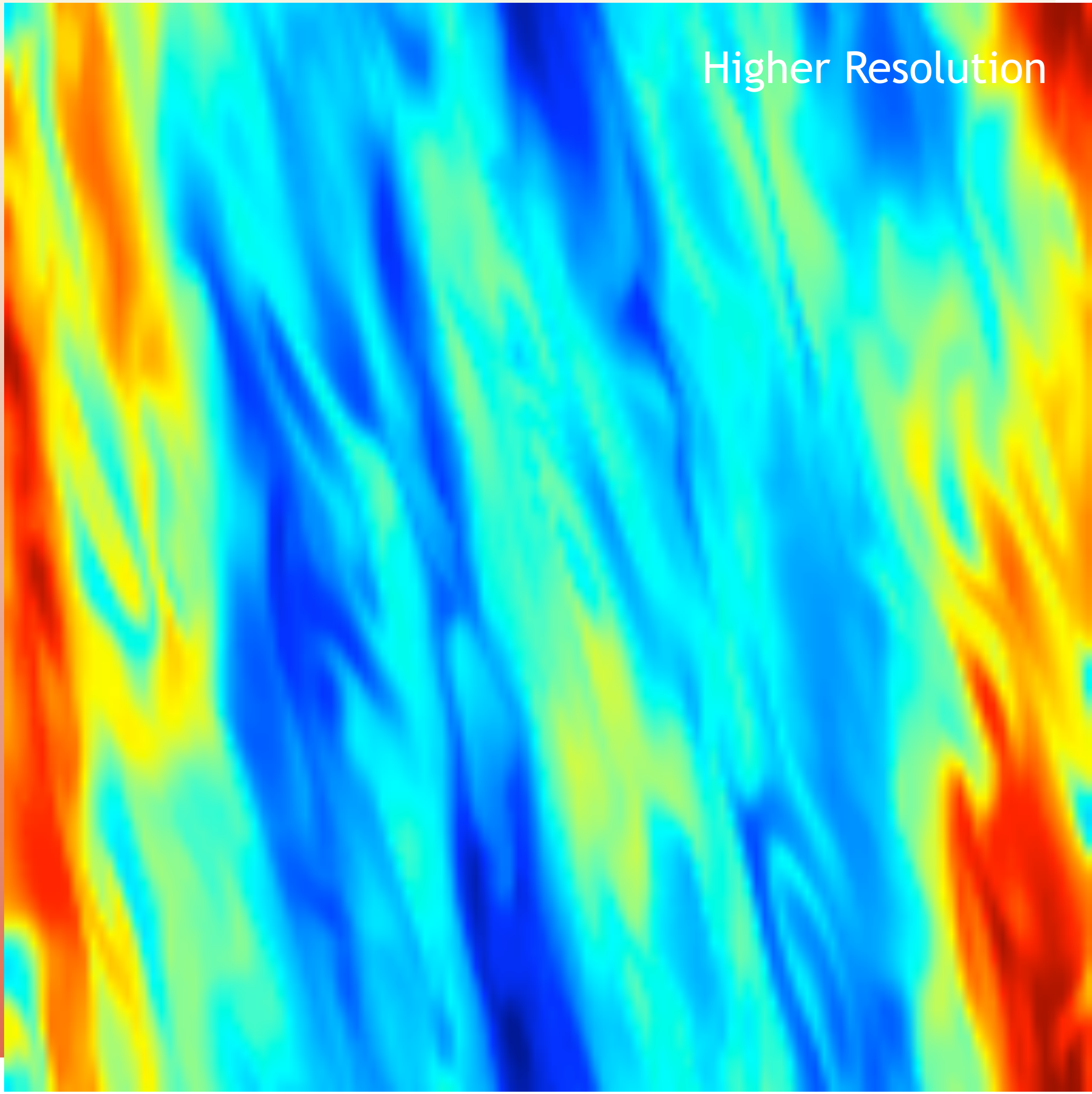
3D MHD Simulation of MRI

- Similarities
  - Initial magnetic field $B=B_z \sin(x)$
  - No stratification (density change with z)
  - Shearing Box

- Differences
  - PLUTO Code (Zeus)
  - Box grid dimensions
  - Ideal E.O.S. (Ideal + Polytrope)

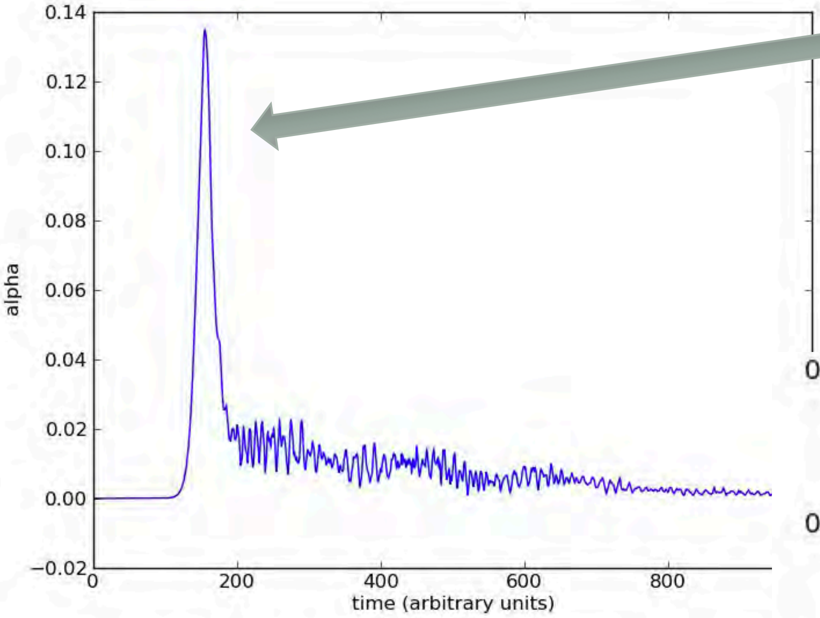
Density over time



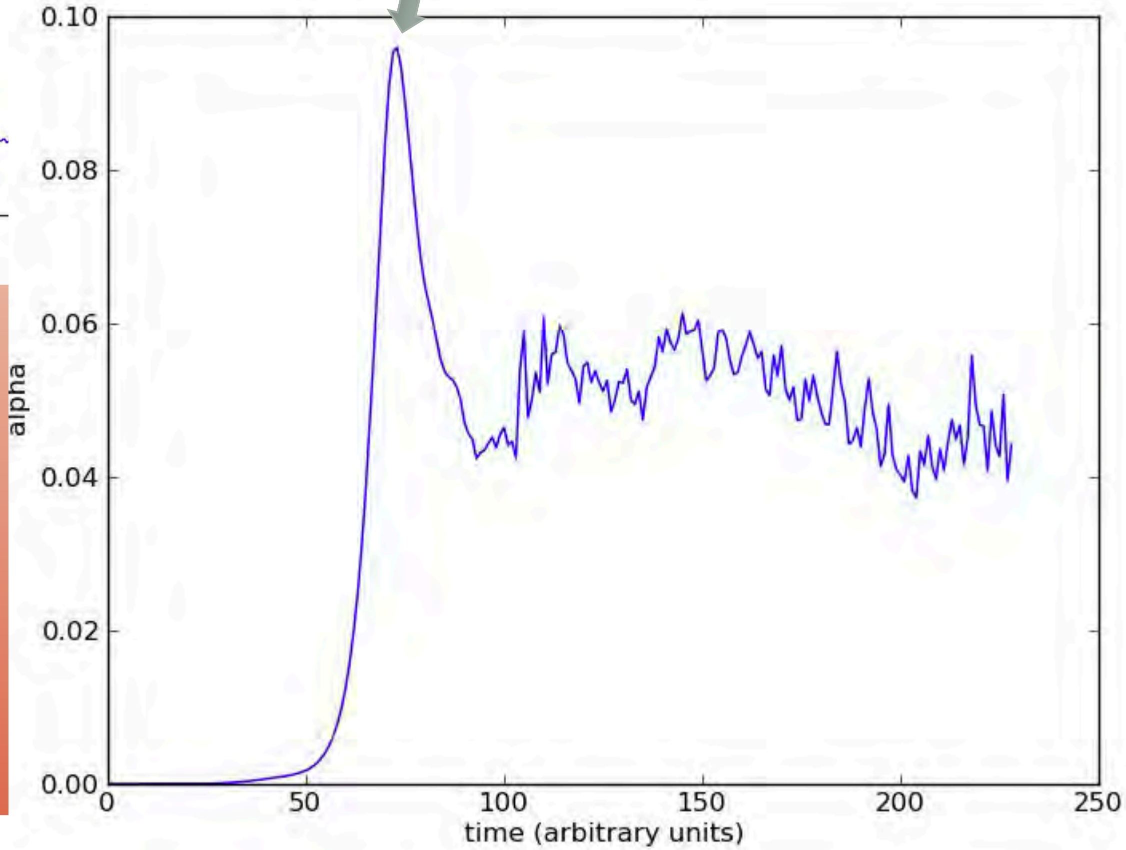


Turbulence over time

“Channel Flow”

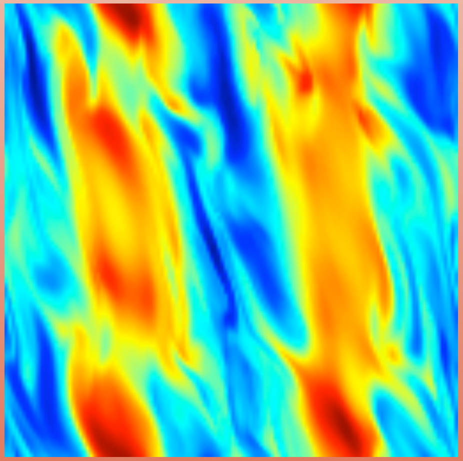


Turbulence dies with smaller grid



Future Work

- Measure Diffusion following tracers in disk
- Different magnetic field configurations and equations of state
- Run longer simulation to see if turbulence persists



For instance, with stronger magnetic field…

Acknowledgments

- Doug
- The Grand Poobah
- CTC @ UMD, YORP

Questions?

References

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Extra Slides

Chemistry of PLD

- Saha Equation

$$\xi^2 \approx f\,\frac{2.41\times10^{15}}{n_\mathrm{n}}\,T^{3/2}\exp\left(-\frac{\chi}{k_\mathrm{B}T}\right)$$

$$\xi_\mathrm{crit} \approx \left(1.49\times10^{12}\ \mathrm{cm}^{-2}\right)\left(\frac{\Omega}{4.38\times10^{-4}\ \mathrm{s}^{-1}}\right)\left(\frac{T}{2000\ \mathrm{K}}\right)^{1/2}\sigma.$$

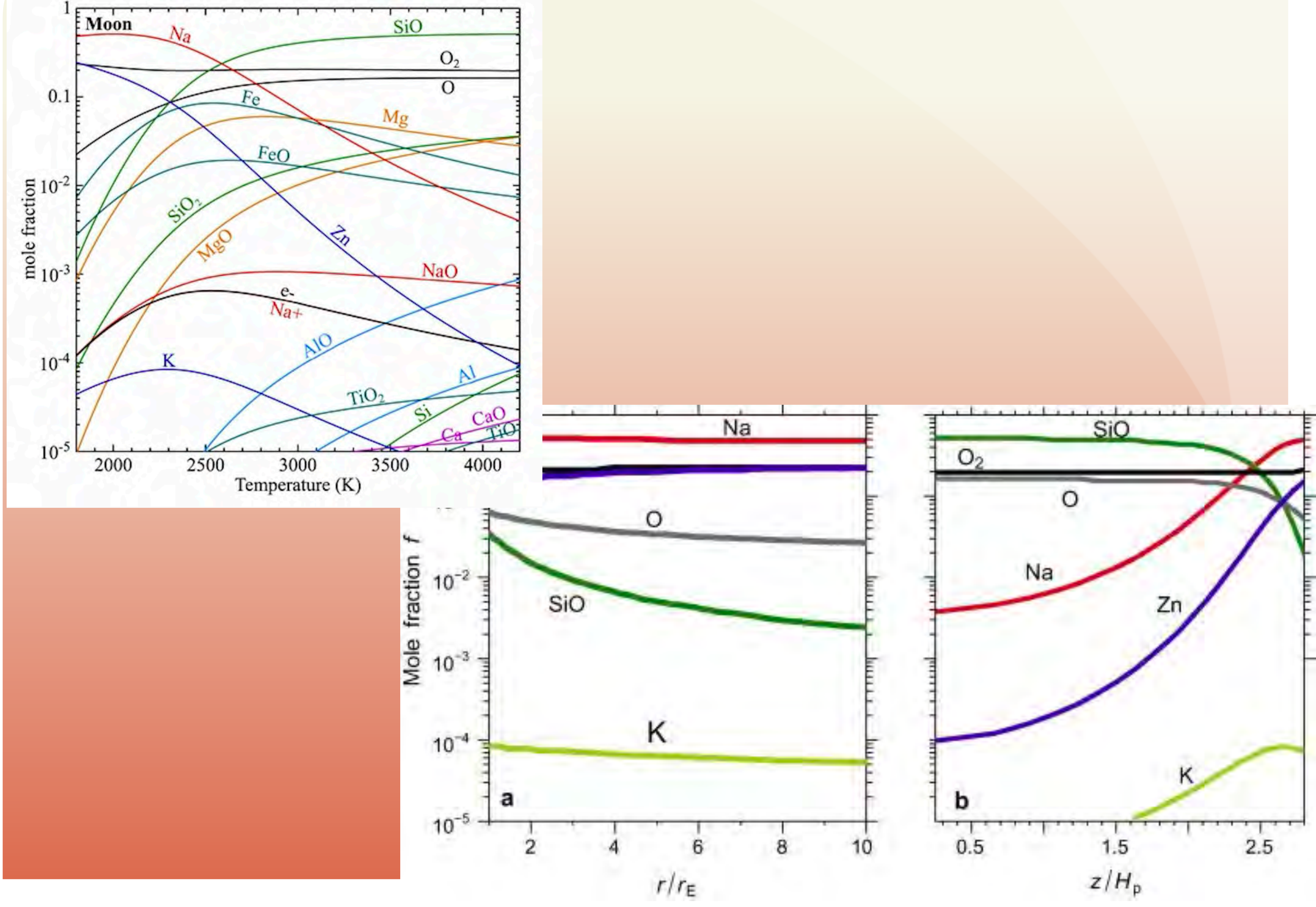
- Assuming Model: T(z) from Ward 2012 and T(r) from Thompson and Stevenson 1988

$$T \approx T_\mathrm{c}\left[1-\frac{1}{x_\mathrm{c}}\left(\frac{z}{H_0}\right)^2\right]$$

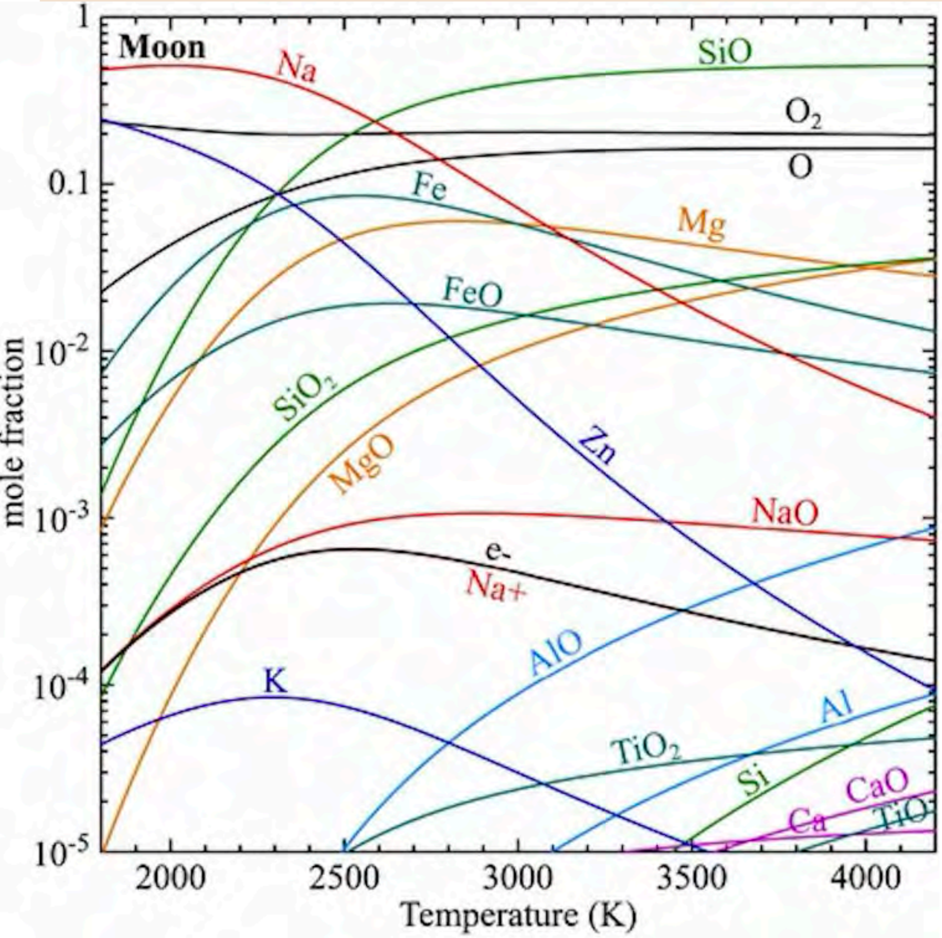
$$T_\mathrm{phot} = T_0\left[\ln\left(\frac{P_0}{P_\mathrm{phot}}\right)\right]^{-1} \approx 2200$$

$$\mathrm{K}\left\{1+0.036\ln\left(\frac{\kappa}{1\ \mathrm{cm}^2\ \mathrm{g}^{-1}}\right)+0.036\ln\left[\frac{H_\mathrm{phot}}{r_\mathrm{E}}\left(\frac{r}{r_\mathrm{E}}\right)^3\right]\right\}^{-1}$$

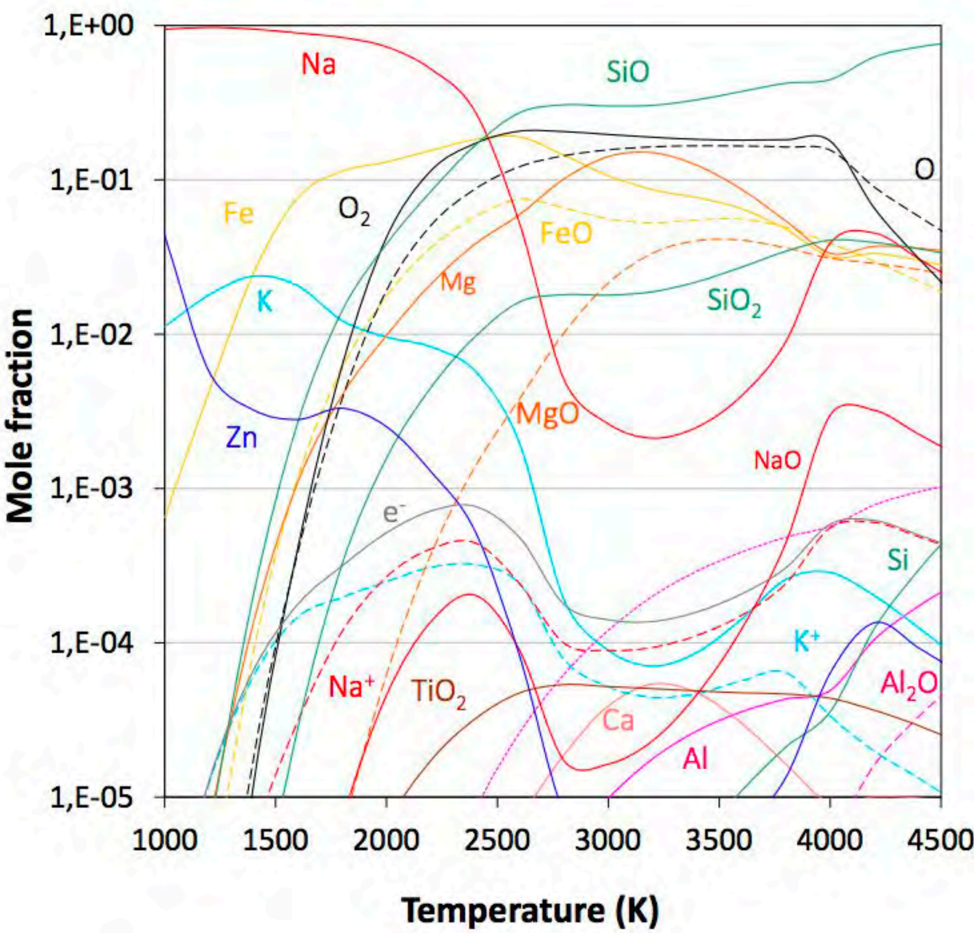
Visscher & Fegley 2013



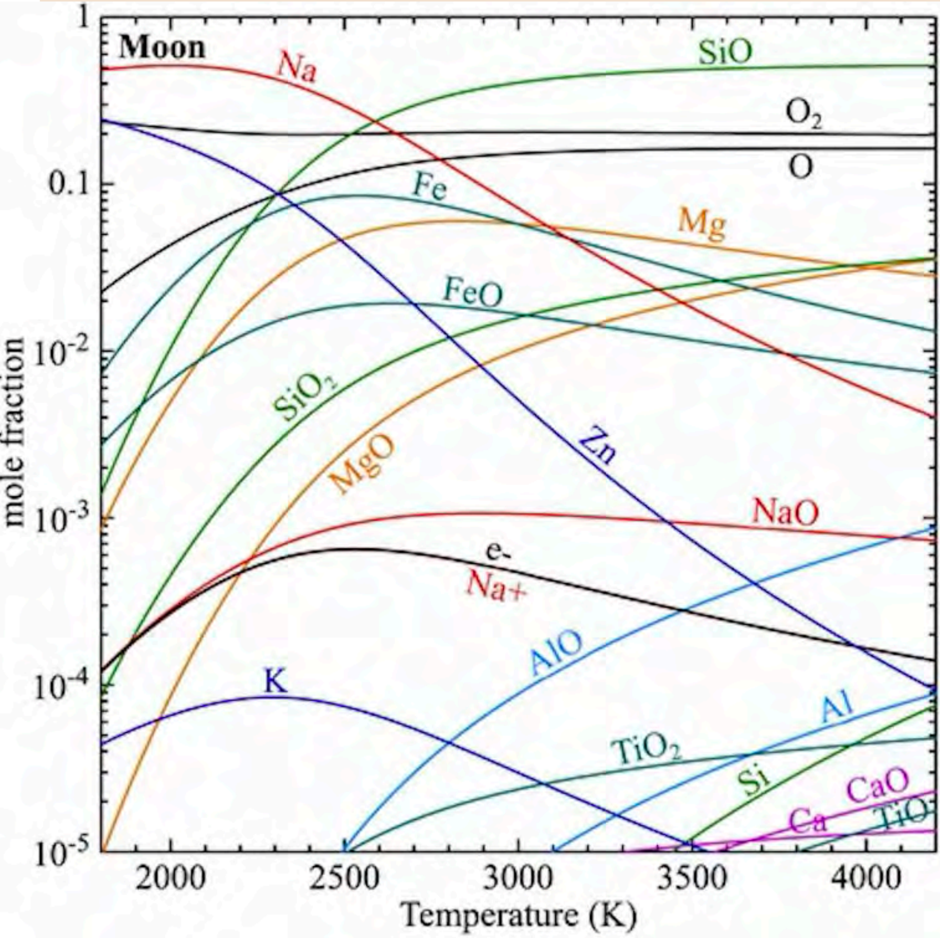
Visscher & Fegley 2013



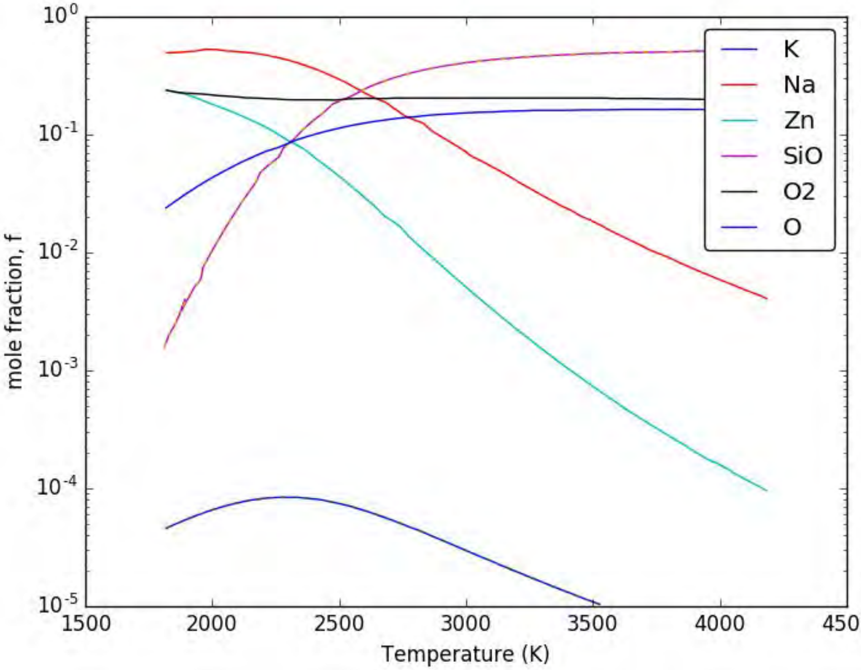
Brugger 2015



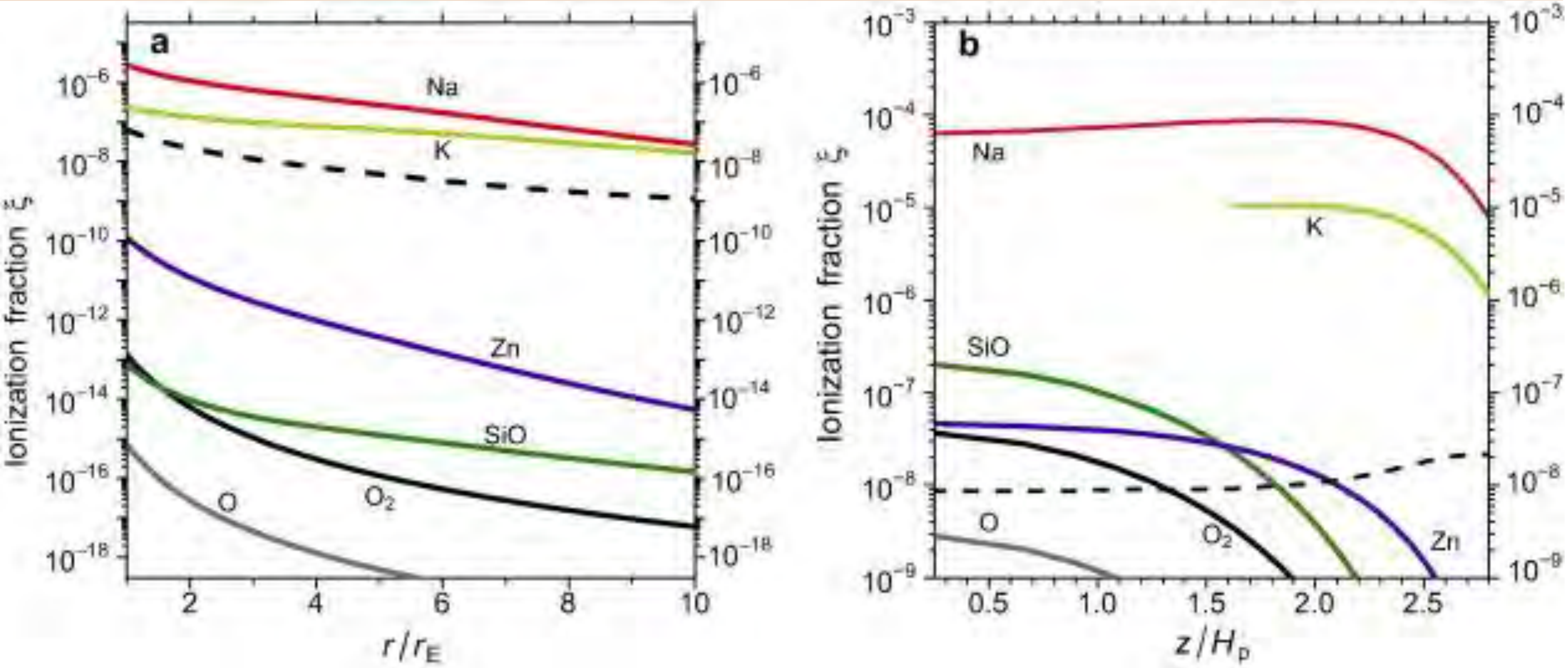
Visscher & Fegley 2013



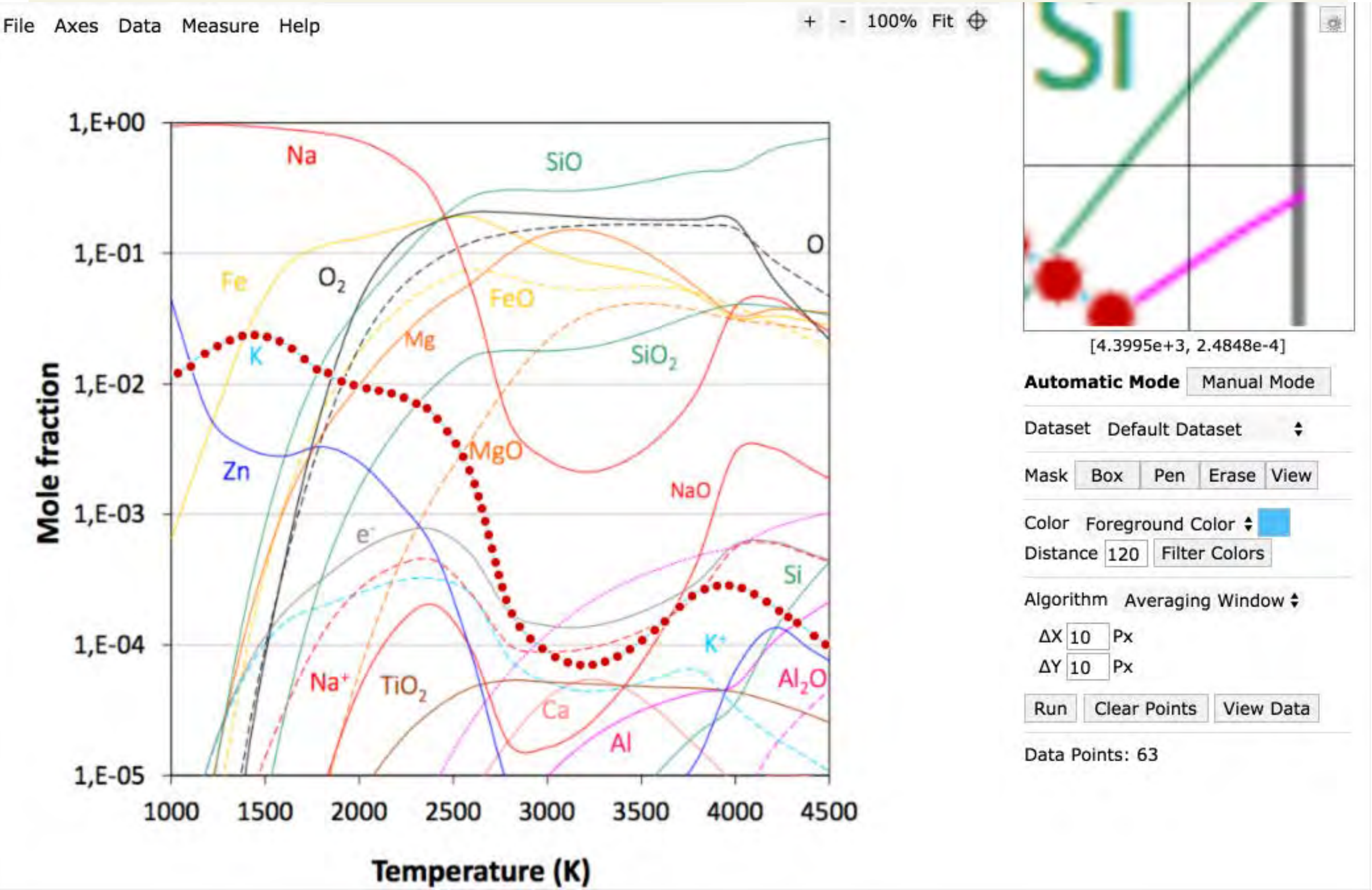
Re-plotted data



Certain elements were above threshold for ionization to occur



Digitizing Chemistry Data



[4.3995e+3, 2.4848e-4]

Automatic Mode

Dataset: Default Dataset

Color: Foreground Color

Distance: 120

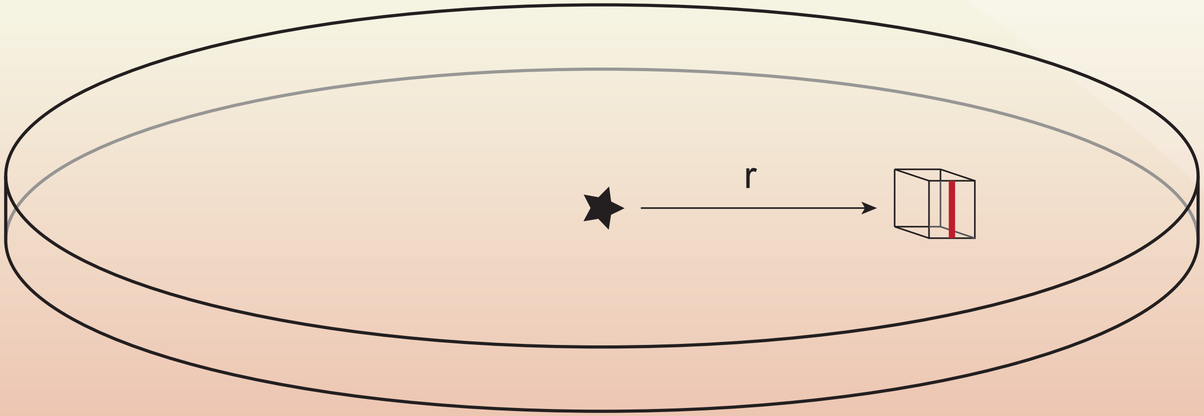
Algorithm: Averaging Window

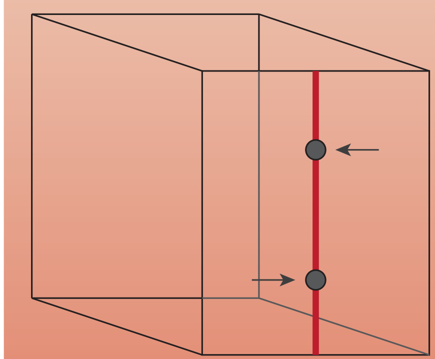
ΔX 10 Px

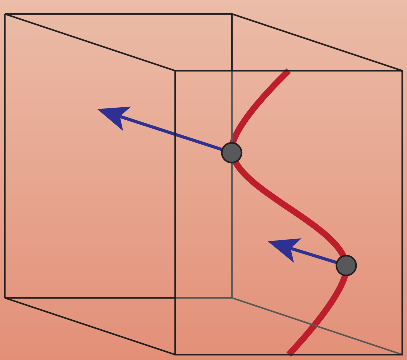
ΔY 10 Px

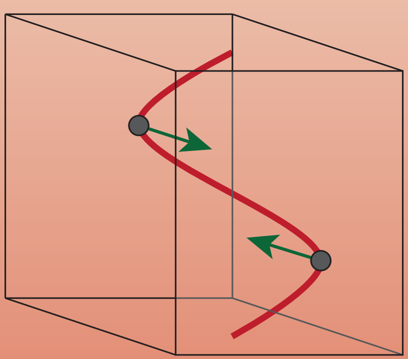
Data Points: 63

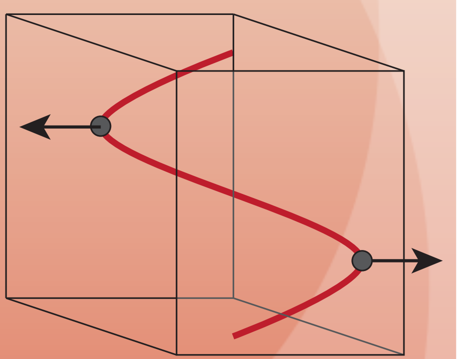
MRI



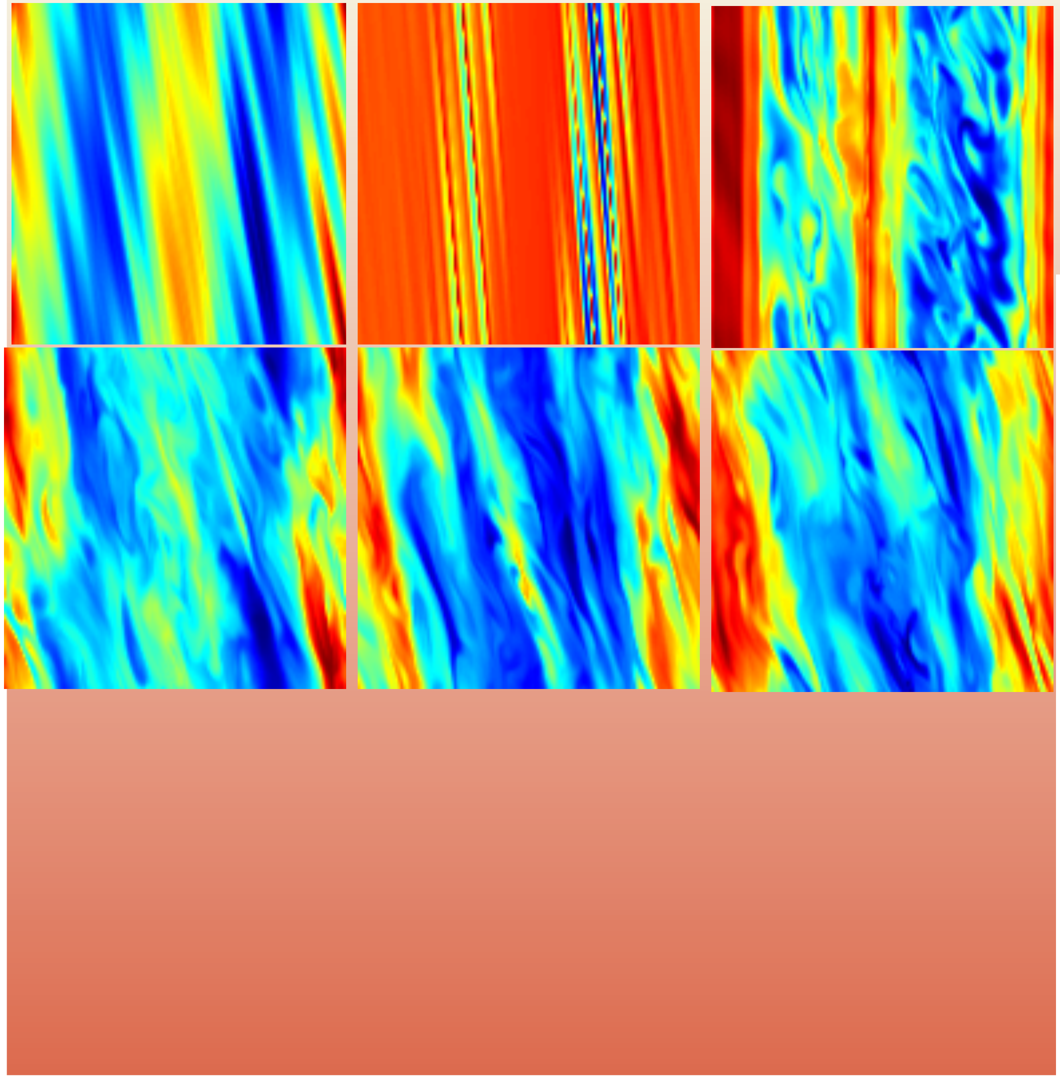

perturb initially vertical magnetic field

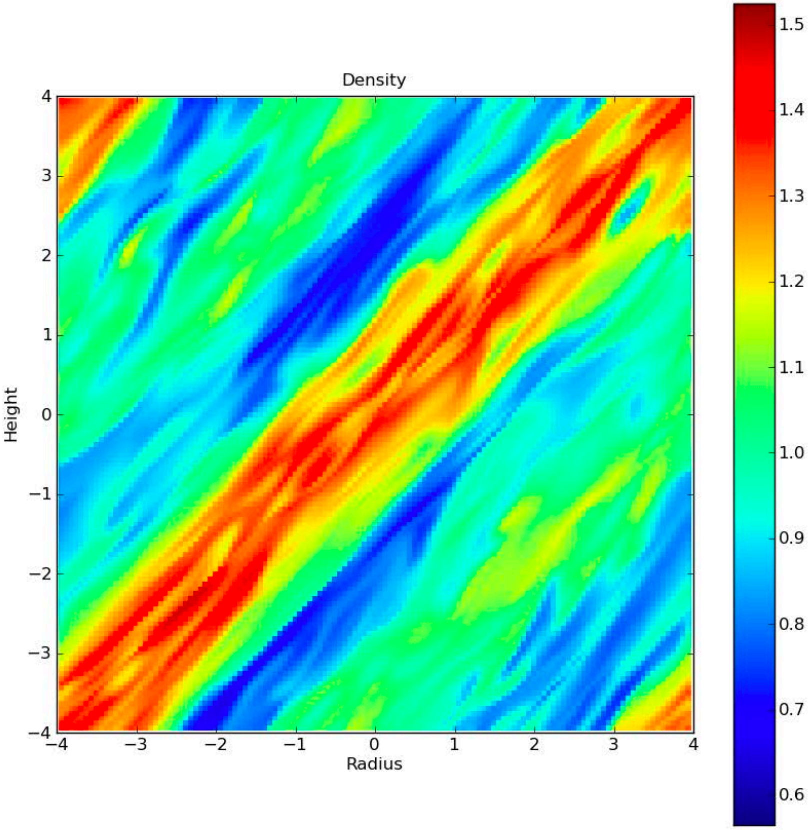

shear leads to azimuthal displacement


magnetic tension transfers angular momentum

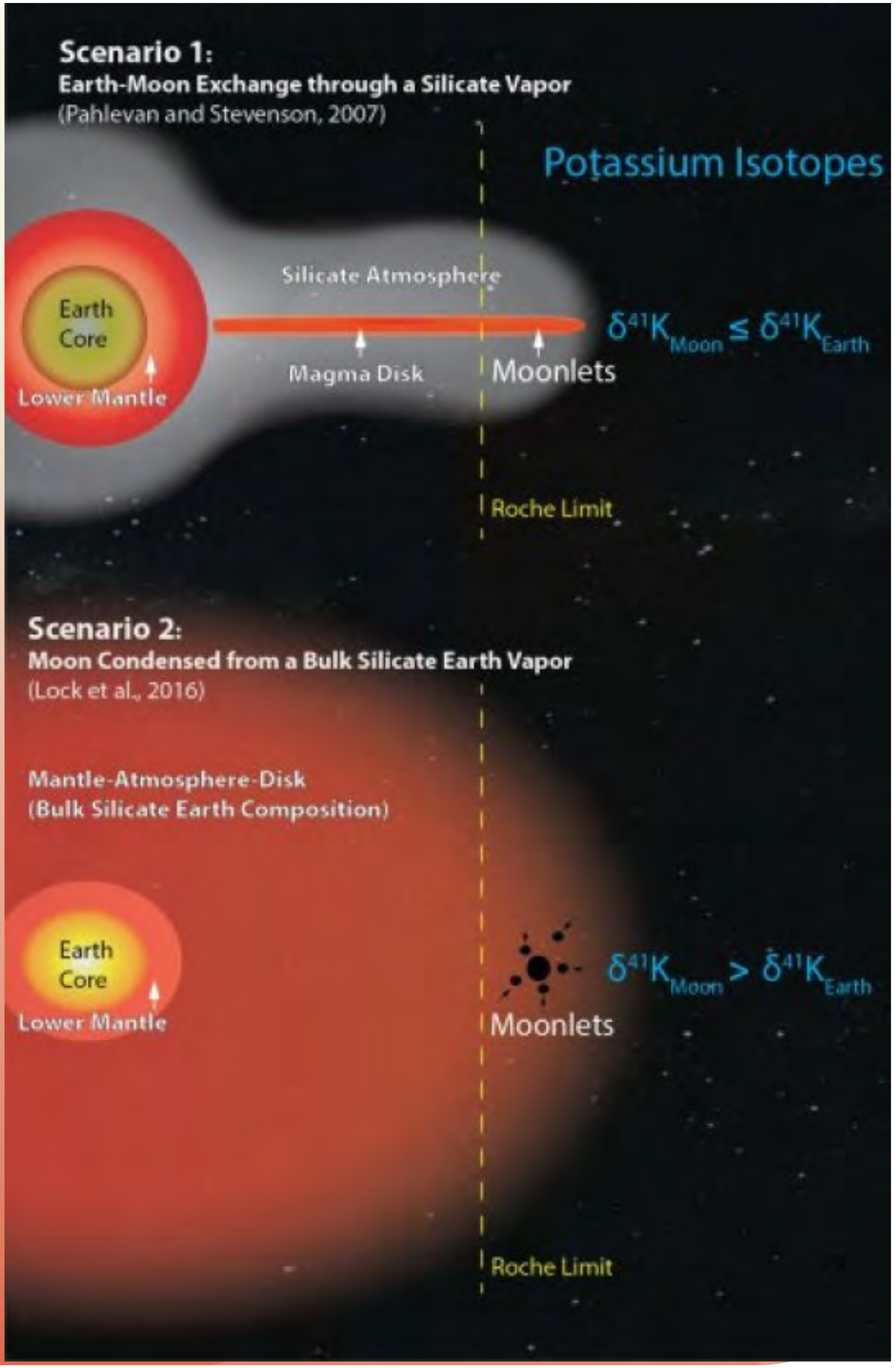

fluid elements separate radially → instability

Turbulence snapshots (density)

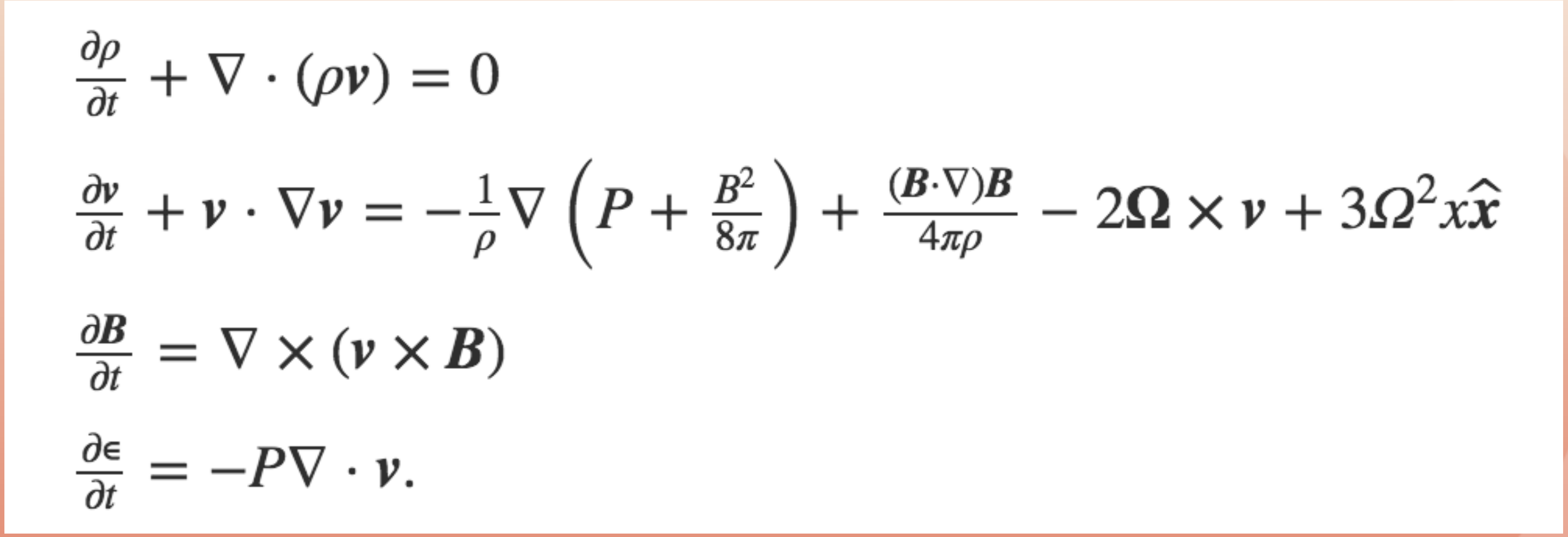




Model


Scenario 1:
Earth-Moon Exchange through a Silicate Vapor
(Pahlevan and Stevenson, 2007)
Potassium Isotopes
Silicate Atmosphere
Earth
Core
Lower Mantle
Magma Disk
Moonlets
$\delta^{41}K_{Moon} \leq \delta^{41}K_{Earth}$
Roche Limit
Scenario 2:
Moon Condensed from a Bulk Silicate Earth Vapor
(Lock et al., 2016)
Mantle-Atmosphere-Disk
(Bulk Silicate Earth Composition)
Earth
Core
Lower Mantle
Moonlets
$\delta^{41}K_{Moon} > \delta^{41}K_{Earth}$
Roche Limit

Equations of MHD



$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\boldsymbol{v}) = 0$$

$$\frac{\partial\boldsymbol{v}}{\partial t} + \boldsymbol{v}\cdot\nabla\boldsymbol{v} = -\frac{1}{\rho}\nabla\left(P + \frac{B^2}{8\pi}\right) + \frac{(\boldsymbol{B}\cdot\nabla)\boldsymbol{B}}{4\pi\rho} - 2\boldsymbol{\Omega}\times\boldsymbol{v} + 3\Omega^2 x\hat{\boldsymbol{x}}$$

$$\frac{\partial\boldsymbol{B}}{\partial t} = \nabla\times(\boldsymbol{v}\times\boldsymbol{B})$$

$$\frac{\partial\epsilon}{\partial t} = -P\nabla\cdot\boldsymbol{v}.$$