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EVOLUTION OF ORBITS AT THE 2:3 RESONANCE WITH NEPTUNE

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Abstract— Results of numerical investigations of the evolution of orbits at the 2:3 resonance with Neptune are presented. The gravitational influence of four giant planets was taken into account. For identical initial values of semimajor axes, eccentricities and inclinations but for different initial orbital orientations and initial positions in orbits, we obtained various types of variations in the difference $\Delta\Omega = \Omega - \Omega_N$ in the longitudes of the ascending node of the body and Neptune and the argument of perihelion ω. If $\Delta\Omega$ decreases and ω increases during evolution, then most of bodies leaves the resonance in 20 Myr. In the case of an increase of $\Delta\Omega$ and a decrease of ω, bodies stay in the resonance for much longer time. Regions of eccentricities and inclinations, for which some bodies were in the η_{18} secular resonance ($\Delta\Omega \approx$ const) and the Kozai resonance ($\omega \approx$ const) were obtained to be larger than those predicted for small variations in the critical angle. Some bodies can be at the same time in both these resonances.

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INTRODUCTION

89 trans–Neptunian objects were known at the end of 1998. Semimajor axes a of orbits of 88 of them are in the interval from 35 to 49 AU. Diameters of these objects are between 100 and 400 km, and their characteristic masses $m \sim 5 \cdot 10^{-12} M_\odot$, where $M_\odot$ is the mass of the Sun. Star magnitudes of objects are $\sim 22 - 24$. The trans-Neptunian belt is often called as the Kuiper belt or the Edgeworth–Kuiper belt in the name of scientists, which for the first time predicted its existence. Now the number of objects in the trans-Neptunian belt with masses of the order of the masses of observed objects ($d > 100$ km) is considered to be equal to 70000 (Jewitt *et al.,* 1996; Levison and Duncan, 1997). Jewitt *et al.,* (1996) valued a total mass of the present belt as $M_\Sigma \sim (0.06 - 0.25) m_\oplus$ at $a \sim 30 - 50$ AU. They considered that the actual width, which is proportional to the mean inclination, must exceed the observed width by a factor of 3. As Pluto, about 35% observed trans–Neptunian objects are in the 2:3 resonance with Neptune. They are called Plutinos. Jewitt *et al.* (1998) consider that, due to observational selection, this portion is overestimated and actually only $10 - 20\%$ of trans-Neptunian objects in the zone of 30–50 AU are Plutinos.

Results of investigations of the orbital evolution of some bodies located in the 2:3 resonance with Neptune are presented in the papers by Morbidelli *et al.* (1995), Malhotra (1995), Levison and Stern (1995), Morbidelli (1997), Gallardo and Ferraz–Mello (1998). Morbidelli (1997) considered three types of orbits located in the 2:3 resonance (regular, strongly chaotic, and weakly chaotic orbits). Elements of weakly chaotic orbits can change regularly for a long time (up to 1 billion years and more) and then begin to change chaoticly. Morbidelli considered that weakly chaotic orbits are the main present deliverer of bodies to the orbit of Neptune. During the age of the Solar System about a half of bodies, including $4.5 \cdot 10^8$ short-period comets, left the neighborhood of the 2:3 resonance. Due to overlapping of the secular resonances and the Kozai resonance, the 2:3 resonance is stable only at a small amplitude of libration $\sigma = -2\lambda_N + 3\lambda - \pi$, where $\lambda = \omega + \Omega + M$ is the mean longitude, $\pi = \omega + \Omega$ is the longitude of perihelion, ω is the argument of perihelion, Ω is the longitude of the ascending node, M is the mean anomaly, the value for Neptune is marked by N.

Gallardo and Ferraz–Mello (1998) investigated the evolution of some orbits, which are in the 2:3 resonance with Neptune and obtained that $\Delta\Omega = \Omega - \Omega_N$ (where Ω_N is the longitude of ascending node of Neptune) librates (the ν_{18} secular resonance) at eccentricities $e < 0.05$ and $e > 0.28$. At $0.05 < e < 0.3$ the transitions between a libration and a circulation are possible. The argument of perihelion ω decreases (i.e., moves clockwise) at $e < 0.15$ and increases at $e > 0.3$. For $e \sim 0.24$ the argument of perihelion librates around 90° (the Kozai resonance). Investigating the evolution of orbits of bodies in the asteroid belt, Kozai (1962) obtained that for large inclinations ω can librate around 90° or 270° and therewith the variations in eccentricity e and inclination i can be large. The period of variations in $\Delta\pi = \pi - \pi_N$ in the trans-Neptunian belt can be less than 3 Myr. The simultaneous libration of $\Delta\pi$ and $\Delta\Omega$ (this resonance $\nu_8 + \nu_{18}$ was predicted by Morbidelli *et al.* (1995)) can cause highly inclined and eccentric orbits. Thomas and Morbidelli (1996) noted that the Kozai resonance doesn't influence on the motion of bodies from the Kuiper belt, but affect on the evolution of long-period comets.

COMPUTER RUNS

We investigated the orbital evolution of trans-Neptunian bodies by numerical integration of the six-body problem (the Sun — the four giant planets — a test body). In order to take into account a gravitational influence of planets, we used the symplectic method, i.e., the RMVS2 algorithm (Regularized Mixed Variable Symplectic) from the Swift integration package worked out by Levison and Duncan (1994). This algorithm provides such speed of calculations (often at the same accuracy of calculation) that is greater by an order of magnitude than the methods of numerical integration, which were worked out earlier. The initial positions and velocities of planets were taken from the test of the SWIFT integration package.

In order to evaluate the accuracy of calculations for some typical investigated orbits, we compared the results of calculations obtained with use of the RMVS2 integrator with those obtained with the BULSTO method by Bulirsh and Stoer (1966). For one variant of calculations, the plots of time variations in orbital elements obtained with the use of the BULSTO integrator are presented in Fig. 1a and the analogous plots obtained with the use of the RMVS2 integrator are presented in Fig. 1b. In particular, the obtained results show that for quasi-periodical variations in orbital elements the limits of variations in semimajor axes during 1 Myr differed by less than 5% for these two integrators and the differences between the limits of variations in eccentricities and inclinations were much smaller.

The considered time span T of integration was not less than 20 Myr. Examples of the time variations in orbital elements for $T = 20$ Myr are presented in Figs. 2a-h. For many runs, the initial value of a semimajor axis of an orbit is $a_\circ = 39.3$ AU. For such $a_\circ$, the resonant value of a was always reached during evolution. At present this value equals to 39.6 AU, but it slightly varies with variation in the semimajor axis of Neptune.

TYPES OF EVOLUTION

Depending on the character of time variations in $\Delta\Omega = \Omega - \Omega_N$ and the argument of perihelion ω, Ipatov and Henrard (1996, 1997) considered several types of variations: *ID* (Fig. 2a), *DI* (Fig. 2b), *LI* (Fig. 2c), *II* (Fig. 2g), et al. Here, the first letter corresponds to the variation in $\Delta\Omega$ and the second letter corresponds to the variation in ω; a letter "*I*" corresponds to an increase, a letter "*D*" corresponds to a decrease; "*L*" is presented for the case of libration, and "*S*" is for relatively small variations (not more than 360° during 20 Myr). For example, the type *IL* corresponds to the case when $\Delta\Omega$ increases and ω librates (Fig. 2h). In contrast to the type *L*, for the type *S* the time variations don't look like a sinusoid. For some runs, the types of variations in $\Delta\Omega$ and ω change with time (Figs. 2d-e). The letter *L* in the first position corresponds to the circular resonance ν_{18}, and that in the second position corresponds to the Kozai resonance.

For some initial data, the variations in an eccentricity e, an inclination i, and a semimajor axis a were relatively small. For other initial data, they were much larger, and bodies left the resonance, and, therewith, some of them were ejected into hyperbolic orbits. Even variations in initial orbital orientations and initial positions in orbits can highly influence on limits and a character of variations in e, i, and a. For example, at $a_\circ = 39.3$ AU, $e_\circ = 0.15$, and $i_\circ = 5^\circ$, we considered 14 runs with different values of $\Omega_\circ$, $\omega_\circ$, and $M_\circ$ (where M is the mean anomaly, and starting values for a body are designated by zero) and obtained that a body left the resonance for 8 runs, i.e., for more than a half cases. Among these eight orbits, which became nonresonant during 20 Myr, there were six of the type *DI* (Fig. 2b), one of the type *SI*, and one of the type *LI* (Fig. 2c). The type *ID* was obtained for three resonant orbits (Fig. 2a), and for the other three resonant orbits the type changed during evolution: $SI \rightarrow IL$, $SD \rightarrow SI \rightarrow ID \rightarrow IS$, and $SS \rightarrow SI \rightarrow SS \rightarrow SI \rightarrow IL$ (Fig. 2d). Some bodies were in the Kozai resonance (i.e., $\omega \approx$ const) for several Myr. For the above series of 14 runs at $T = 20$ Myr, the maximum eccentricity was smaller than 0.2 for five runs and exceeded 0.3 for three runs. At $\Omega_\circ = \omega_\circ = 0$ and $M_\circ = 30^\circ$, a body was ejected into a hyperbolic orbit after 37 Myr.

At $\Omega_\circ = \omega_\circ = M_\circ = 60^\circ$, $e_\circ = 0.15$, and $i_\circ = 5^\circ$, we made runs for different values of $a_\circ$. For $38.8 \leq a_\circ \leq 39.7$ AU, a body moved during some time (therewith, for more than 20 Myr at $39.0 \leq a_\circ \leq 39.5$ AU) in the 2:3 resonance with Neptune. At $a_\circ$ equal to 37.5, 40.5, 41, and 41.5 AU, we obtained the libration of $\Delta\Omega$ about 180° (the ν_{18} resonance). The type ID was obtained for $39.1 \leq a_\circ \leq 39.3$ AU, the type DI was for $38.5 \leq a_\circ \leq 38.9$ AU and $39.6 \leq a_\circ \leq 39.9$ AU, and changes in types ($SI \rightarrow DI$, $ID \rightarrow SD$, and $DI \rightarrow SD$) were for 39.0, 39.4, and 39.5 AU. At $i_\circ = 5^\circ$, $a_\circ = 39.3$ AU, and $\Omega_\circ = \omega_\circ = M_\circ = 60^\circ$, for $e_\circ$ from 0 to 0.3 (with a step equaled to 0.05), we obtained the following types: $SI \rightarrow SD$, $LI \rightarrow LD$, LD, ID, ID, II, and ID. At $e_\circ = 0.15$ and $a_\circ = 39.3$ AU for the above values of $\Omega_\circ$, $\omega_\circ$, and $M_\circ$, we had the type ID for $0 \leq i_\circ \leq 15^\circ$ and the types IL, ID, and II or some combinations of these types for $30^\circ \leq i_\circ \leq 90^\circ$. For most of the runs, variations in the critical angle $\sigma = 3\lambda - 2\lambda_N - \pi$ exceeded 180°. For nonresonant orbits we usually obtained the types DI, II, or LI. The types DD and DL were not obtained in our runs. For objects 1993SC, 1993SB, 1993RO, the orbital evolution considered by Morbidelli *et al.* (1995) had the following types: ID, II, $ID \rightarrow IL \rightarrow II$, respectively.

VARIATIONS IN ORBITAL ELEMENTS

For orbits with a small amplitude of libration of σ, the regions of i and e corresponding to the secular resonance η_{18} ($\Delta\Omega \approx$ const) and the Kozai resonance are presented by Morbidelli *et al.* (1995) in Fig. 5. These regions are located far from each other: $e < 0.03$ and $i < 10^\circ$ for the η_{18} resonance and $e > 0.2$ at $i < 10^\circ$ for the Kozai resonance. In one of our runs at $a_\circ = 39.3$ AU, $e_\circ = 0.05$ and $i_\circ = 5^\circ$, the values of $\Delta\Omega$ librated around 180° with an amplitude $\sim 180^\circ$ and at the same time ω librated around 270° with an amplitude $\sim 100^\circ$ during 6 Myr (Fig. 2e). The amplitude of σ-libration was large (close to 360°) in this run. For other values of $\Omega_\circ, \omega_\circ$, and $M_\circ$ at $e_\circ = 0.05$ and $i_\circ = 5^\circ$, we usually obtained the type LI, but sometimes also the type DI. According to Fig. 5 in the paper by Morbidelli *et al.* (1995), ω decreases at $e < 0.2$ and $i < 10^\circ$ for a small amplitude of σ-libration. For large variations in σ, we often obtained orbits with increasing ω at these values of e and i (for example, at $e = 0.15$ and $i = 5^\circ$). The region of values of e and i, for which the η_{18} resonance was obtained for some orientations of orbits, in our runs was much larger than that in this figure. This resonance was obtained for most runs at $e_\circ = 0.05$ and $e_\circ = 0.1$.

For many runs, i varies quasi-periodically with time with a period equal to several million years, and $\Delta\Omega$ changes by 360° during this period. In this case, if $\Delta\Omega$ decreases during evolution, then $\Delta\Omega = 0$ when i reaches its maximum value, and $\Delta\Omega = 180^\circ$ when i reaches its minimum value (Fig. 2b). If $\Delta\Omega$ increases, then maximum and minimum values of i are reached at $\Delta\Omega$ equal to 180° and 0, respectively (Fig. 2a). In some runs $\Delta\Omega$ librates, a width of the range of variations in $\Delta\Omega$ exceedes 180°, and this range includes the value of $\Delta\Omega = 180^\circ$. For example, $\Delta\Omega$ varies between 0 and 250°, between 70° and 260°, and between 90° and 250° in the runs presented in Figs. 2c,e,f, respectively.

In many runs we often obtained variations in e and i with a period equal to $T_\omega/2$, where T_ω is the time interval during which ω decreases or increases by 360° (it is a period of libration in the case of the Kozai resonance). The amplitude of these variations in i is usually smaller by a factor of 10 or more than the range of main variations in i. Investigations of the three-body problem (the Sun — a planet — a body) show (Lidov, 1961; Ipatov, 1992) that in the case of the fixed orbit of a planet and $a =$ const we have $\max i_r$ and $\min e$ at $\omega = 0$ or $\omega = 180^\circ$ and $\min i_r$ and $\max e$ at $\omega = \mp 90^\circ$, where i_r is the inclination of a body relative to the orbit of the planet and $i_r < 90^\circ$. Such relationships between variations in ω and i were obtained for all runs for the types DI (Fig. 2b) and LD (Fig. 2f) and for some runs for the types ID and SI (Fig. 2d). For some runs for the types ID (Fig. 2a), LI (Fig. 2c) and SI, we have $\min i$ at $\omega = 0$ or $\omega = 180^\circ$. Note that for the same type ID at $\omega = 0$ or $\omega = 180^\circ$ we have $\min i$ for large variations in the critical angle σ and $\max i$ for small ($< 180^\circ$) variations in σ. For smaller values of σ, the intervals of variations in orbital elements a, e, and i are smaller.

For the 2:3 resonance with Neptune, $\Delta\pi = (\omega + \Omega) - (\omega_N + \Omega_N)$ usually decreases during evolution

(Figs. 2a,f,g). Sometimes $\Delta\pi$ mainly decreases (Fig. 2b). The η_8 resonance ($\Delta\pi \approx$ const) was usually obtained for the type LI. In Fig. 2c for $13 < t < 16.5$ Myr, $\Delta\pi$ varies between 160° and 360°. For two other runs, $\Delta\pi$ varies in the ranges $240 - 360°$ and $180 - 400°$ during 7 and 10 Myr, respectively. Note that for the orbits located outside mean motion resonances a libration of $\Delta\pi$ was often obtained at $a_\circ \approx 42$ AU (more rare at $a_\circ \approx 40$ AU), and for some runs it took place during all considered time interval.

It is known that most of the observed trans-Neptunian objects with $a \leq 42$ AU are located near the 2:3 resonance with Neptune. In our runs, the orbits, which had the type ID with a small libration of σ and were in this resonance, were the most stable. Orbits of the observed trans-Neptunian objects are not well known. In some cases for small variations in initial orbital elements, the type of variations in $\Delta\Omega$ and ω can change and the limits of variations in orbital elements can differ highly. Therefore, one must consider initial data from some vicinity of initial orbital elements (the dimensions of this vicinity depend on the errors of the determination of orbits), when he evaluates the lifetimes of actual trans-Neptunian bodies.

For all considered runs of the evolution of resonant orbits, the maximum values of e and i exceeded 0.07 and 3°, respectively. The interval $\Delta a = a_{max} - a_{min}$ of variations in a for the 2:3 resonance with Neptune is about 1 AU. It may change by a factor of 1.8 for variations in $\Omega_\circ$, $\omega_\circ$, and $M_\circ$. For example, for $a_\circ = 39.3$ AU, the values of Δa varied from 0.6 to 1.07 AU at $e_\circ = 0.05$ and $i_\circ = 5°$ and from 0.68 to 1.24 AU at $e_\circ = 0.15$ and $i_\circ = 5°$. For $\Omega_\circ = \omega_\circ = M_\circ = 60°$ and $e_\circ = 0.15$, the values of Δa equaled to approximately 1.1–1.2 AU at $0 \leq i_\circ \leq 10°$ and were 0.84–0.88 at $60° \leq i_\circ \leq 90°$. In the case of $i_\circ = 5°$, the values of Δa increased from 0.86 to 1.24 AU for an increase of $e_\circ$ from 0 to 0.3. The above values of Δa are presented for resonant orbits. If a body left the resonance, then the values of Δa could be much larger.

CONCLUSION

Results of numerical investigations showed that a character and limits of variations in orbital elements for the 2:3 resonance with Neptune can differ significantly for various initial orbital orientations and initial positions of bodies in orbits. If the difference $\Delta\Omega = \Omega - \Omega_N$ in the longitudes of the ascending node of the body and Neptune decreases and the argument of perihelion ω increases during evolution, then most of bodies leave the resonance in 20 Myr. In the case of an increase of $\Delta\Omega$ and a decrease of ω, bodies remain in the resonance for much longer time. The regions of eccentricities and inclinations, for which some bodies were in the secular resonance ν_{18} ($\Delta\Omega \approx$ const) and the Kozai resonance ($\omega \approx$ const), were obtained larger than the regions predicted for small variations in a critical angle. Some bodies were simultaneously in both these resonances.

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FIGURE CAPTIONS

Fig. 1. Time variations in the semimajor axis a, eccentricity e, inclination i of a body's orbit, the difference $\Delta\Omega = \Omega - \Omega_N$ in the longitudes of the ascending node of the body and Neptune, the argument of perihelion ω, and the difference $\Delta\pi = \pi - \pi_N$ in the longitudes of perihelion of the body and Neptune, and the critical angle $\sigma = 3\lambda - 2\lambda_N - \pi$, where $\lambda = \omega + \Omega + M$. Results were obtained by numerical integration of motion equations of the six-body problem (the Sun — the giant planets — a body) with the use of the BULSTO integrator (a) and the RMVS2 integrator (b). Elements of the initial orbit of a body are: $a_\circ = 39.3$ AU, $e_\circ = 0.15$, $i_\circ = 5°$, $\Omega_\circ = \omega_\circ = M_\circ = 60°$.

Fig. 2. Time variations (in Myr) in the semimajor axis a (in AU), eccentricity e, aphelion and perihelion distances $Q = a(1 + e)$ and $q = a(1 - e)$ (in AU), inclination i (in degrees), the difference $\Delta\Omega = \Omega - \Omega_N$ in the longitudes of the ascending node of the body and Neptune, the argument of perihelion ω, and the difference $\Delta\pi = \pi - \pi_N$ in the longitudes of perihelion of the body and Neptune, and the critical angle σ (all angles are presented in degrees). For all variants, $a_\circ = 39.3$ AU. Results were obtained with the use of the RMVS2 integrator. Influence of the giant planets was taken into account. (a) $e_\circ = 0.15$, $i_\circ = 5°$, $\Omega_\circ = \omega_\circ = M_\circ = 180°$, type ID; (b) $e_\circ = 0.15$, $i_\circ = 5°$, $\Omega_\circ = 30°$, $\omega_\circ = 0$, $M_\circ = 30°$, type DI; (c) $e_\circ = 0.15$, $i_\circ = 5°$, $\Omega_\circ = 60°$, $\omega_\circ = 0$, $M_\circ = 60°$, type LI; (d) $e_\circ = 0.15$, $i_\circ = 5°$, $\Omega_\circ = 0°$, $\omega_\circ = M_\circ = 60°$, types $SS \to SI \to SS \to SI \to IL$; (e) $e_\circ = 0.05$, $i_\circ = 5°$, $\Omega_\circ = \omega_\circ = M_\circ = 60°$, types LL and LI; (f) $e_\circ = 0.1$, $i_\circ = 5°$, $\Omega_\circ = \omega_\circ = M_\circ = 60°$, type LD; (g) $e_\circ = 0.3$, $i_\circ = 10°$, $\Omega_\circ = \omega_\circ = M_\circ = 60°$, type II; (h) $e_\circ = 0.3$, $i_\circ = 45°$, $\Omega_\circ = \omega_\circ = M_\circ = 60°$, type IL.