ASTR 300   Stars and Stellar Systems   Spring 2011

Homework Set No. 1   **Solutions**

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1. If the Earth did not rotate, could you define the celestial poles and celestial equator? (Chapt. 2, Review Question 8)

   No, you could not define the celestial pole or equator. The direction to the pole is defined by the Earth's rotational axis.

2. As the earth turns on its axis, an observer on the earth's surface sees the sun, moon and stars (except for *circumpolar* stars) rise in the east, traverse the sky, and set in the west. All these objects reach their highest point above the horizon when they cross the **meridian**. For an observer in the northern hemisphere, the meridian is the arc that passes from the north point on the horizon, up through the north celestial pole, through the zenith overhead, and on down to the south point of the horizon. The angular distance along the meridian from the northern horizon up to the north celestial pole (NCP) is equal to the latitude, ϕ, of the observer.

   (a) For an observer in College Park, the latitude is $\phi = 39^o$. The celestial equator is 90° from the NCP. On March 20 (the *vernal equinox*) the sun will be on the celestial equator. How high above the southern horizon will the sun be when it crosses the meridian on March 20?

   (b) On June 21 (the *summer solstice*) the sun is 23.5^o above the celestial equator, while on December 22 (the *winter solstice*) the sun is 23.5^o below the celestial equator. How high is the sun above the southern horizon when it crosses the meridian on June 21? How high is it when it crosses on December 22?

   The whole arc of the meridian (horizon to horizon) is 180°. The part of the arc from the north horizon to the NCP is 39°, and from the NCP to the celestial equator is 90°. The part that remains, $180^o - (39^o + 90^o) = 51^o$, is thus the height of the celestial equator above the southern horizon.

   On June 21 it is just $51^o + 23.5^o = 74.5^o$. Likewise, on December 22, the height above the horizon will be $51^o - 23.5^o = 27.5^o$.

3. If a beam of light with a cross-section of one square meter makes an angle of θ with a surface, then that beam will be spread out over an area of $1/\sin(\theta)$ square meters when it strikes the surface. Thus the heating of the surface will be reduced by a factor of $\sin(\theta)$ compared to the heating that would be produced by a beam shining straight down ($\theta = 90^o$).

   (a) Using your results from Question 2 above, how much is the heating by the sun reduced on March 20 compared to a point on the earth's surface where the sun is directly overhead?

   (b) By what factor is the sun's heating on June 21 greater than the heating on December 22 at College Park?

(a) On March 20, the angle is 51°, so the reduction is by a factor of $\sin(51^o) = 0.7771$.

(b) On June 21, the factor is $\sin(74.5^o) = 0.9636$, while on December 22 the factor is $\sin(27.5^o) = 0.4617$. Thus there is $0.9636/0.4617 = 2.087$ times more heating by the noontime sun in June than in December.

4. The apparent visual magnitude of the sun is -26.8 (Table A-9). The magnitude of Sirius, the brightest star, is -1.47. (Also, see Figure 2-6 on p 15.) By what factor is the sun brighter than Sirius? (Hint: use the equation on page 16 of the text.)

The magnitude difference is $(-1.47) - (-26.8) = 25.33$.

We can now apply the equation

$$\frac{I_{sun}}{I_{Sirius}} = (2.512)^{(m_{Sirius} - m_{sun})} = (2.512)^{25.33} = 1.355 \times 10^{10} \, .$$

Thus the sun appears over 13 billion times brighter than Sirius.

5. After the α Centauri system, the nearest star to our solar system is Barnard's Star, which is 5.9 ly distant (see Table A-9, p 424). But the apparent visual magnitude of Barnard's star is $m_V = 9.5$, much too faint to be seen with the naked eye. If a star must have a magnitude of 6.5 to be seen with the naked eye, by what factor would the brightness of Barnard's star have be increased to become visible? Considering the inverse square dependence of brightness on distance, how close would Barnard's star have be to become visible?

The difference between its actual magnitude and what it would need to be visible is just $9.5 - 6.5 = 3.0$. Applying our formula again, we see that this corresponds to a brightness ratio of just

$$\frac{I_{visible}}{I_{actual}} = (2.512)^{(m_{actual} - m_{visible})} = (2.512)^{3.0} = 15.85 \, .$$

Since a star's brightness varies as the inverse square of the distance, the distance would have to decrease by a factor of $\sqrt{15.85} = 3.981$. (I.e., $3.981^2 = 15.85$.) Since Barnard's star is now at 5.9 ly, we would have to move it to $5.9/3.981 = 1.48$ ly.

Barnard's star is actually moving towards us at 107 km/s, and in about 10,000 years it will pass within 3.8 ly of us; after that its distance will increase again. When it is 3.8 ly away it will be the nearest star. Will it be visible to the naked eye then?

No. We just saw above that it would only reach naked-eye visibility if it were to come closer than 1.48 ly.

6. Both the Earth and the Moon orbit their common center of mass, but since the Earth is much more massive, its orbit is much smaller that that of the Moon. Look up the mass of the Earth and of the Moon in Table A-5 (p 422). How much more massive is the Earth than the Moon? (I.e., what is the ratio of their masses?)

From Table A-5, $M_{Earth} = 5.976 \times 10^{24}$ kg, $M_{Moon} = 7.350 \times 10^{22}$ kg. Thus

$$\frac{M_{Earth}}{M_{Moon}} = \frac{5.976 \times 10^{24}}{7.350 \times 10^{22}} = 81.31$$

Look up the distance from the Earth to the Moon in Table A-11. Calculate the distance of the Earth's center from the center of mass of the Earth-Moon system. Express this distance in units of the Earth's radius.

From Table A-11, the Earth-Moon distance is $a = 384400$ km. Referring to slide 12 from "lecture 2 (continued)", you see that the distance a_1 of the Earth's center from the center of mass is given by

$$a_1 = \frac{M_{moon}}{M_{earth} + M_{moon}} a = \frac{7.350 \times 10^{22}}{5.976 \times 10^{24} + 7.350 \times 10^{22}} 384400 = 4670 \text{ km}$$

Since the Earth's radius is $R_\oplus = 6378$ km, $a_1 = 0.73 R_\oplus$. Thus the center of mass of the Earth-Moon system is *inside the Earth*, about 3/4 of the way to the Earth's surface.

7. We saw that the true form of Kepler's third law can be written

$$P^2 = \frac{a^3}{m_1 + m_2}$$

if we express the period P in years, the semi-major axis a in astronomical units (AU), and the masses of the two bodies, m_1 and m_2, in solar masses. (If the orbit is circular, then $a = r$, the distance between the two bodies.)

Consider a satellite orbiting the Earth. What is the mass of the Earth in solar masses? You can neglect the mass (m_2) of the satellite.

From Table A-5, $M_\oplus/M_\odot = 5.976 \times 10^{24}/1.989 \times 10^{30} = 3.00 \times 10^{-6}$. Here, $M_\odot$ stands for the mass of the sun.

The Hubble Space Telescope (HST) is in a relativity low orbit, 560 km above the Earth's surface. What is the radius of the HST's orbit? What is this radius in AU (1 AU = 1.496×10^8 km)? Now use the equation to find the period P of the HST's orbit in years. Finally, convert your answer to minutes (there are 525949 minutes in a year).

$a_{HST} = R_\oplus + 560 = 6378 + 560 = 6938$ km

$a_{HST} = 6938/1.496 \times 10^8 = 4.638 \times 10^{-5}$ AU

$$P^2 = \frac{a^3}{m_1 + m_2} = \frac{(4.638 \times 10^{-5})^3}{3.00 \times 10^{-6}} = \frac{9.975 \times 10^{-14}}{3.00 \times 10^{-6}} = 3.32 \times 10^{-8}$$

so that the period is $P = \sqrt{3.32 \times 10^{-8}} = 0.0001822$ years

Finally, the period in minutes is $P = 525949 \times 0.0001822 = 95.83$ min.

Suppose we could triple the height of the Hubble's orbit to 1680 km above the Earth's surface. What would it's new orbital period be?

Now, $a_{HST} = 6378 + 1680 = 8058$ km $= 5.386 \times 10^{-5}$ AU

We then find $P^2 = 5.209 \times 10^{-8}$ yr^2, $P = 0.0002282$ yr,

so that in this case, $P = 120.04$ min.