ASTR 300 Stars and Stellar Systems Fall 2009

Homework Set No. 1 **Solutions**

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1. Give two reasons why winter days are colder than summer days.
   (Chapt. 2, Review Question 9)

   (1) Winter days are shorter than summer days, so the sun shines fewer hours.
   (2) The sun in lower in the sky in winter, so the sunlight is spread out over more area.

2. After Sirius, the second brightest star in the sky is Canopus (α Carinae). We don't see this star because it is located in the southern sky, 57^o below the celestial equator.

   Mexico City is located at a latitude of about 19^o N. Can Canopus be seen from there? If so, how high in the sky can it be? If not, how close can it come to the horizon?

   In Mexico City, the North Celestial Pole (NCP) will be 19^o above the north horizon, since that is the observer's latitude. If we draw an arc from the north point of the horizon through the NCP and through the zenith and on through Canopus when it is due south, we have the following angles: 19^o from the horizon to the NCP, 90^o from the NCP to the celestial equator, and finally 57^o from the equator to Canopus. The total is 19+90+57 = 166^o, which is less than 180^o, so Canopus is above the horizon by 180^o-166^o= 14^o. Canopus is visible from Mexico City.

3. Answer Question 3 of Chapter 2, Learning to Look.

   Looking at the diagram on p 32, we see from the rotation arrows that the rising stars are on the right, so this must be east and thus we are looking due north, with the horizon at the bottom of the figure. We note that "the big dipper" is directly below the north star, Polaris.

   If you now look at the sky charts for the northern hemisphere (p 446-451), you see that the one that matches is the chart for November (turn it upside down so you are looking at the northern horizon, and you see the big dipper directly below Polaris). This chart corresponds to the sky in mid-November at 8:00 PM.

   But the problem states that the figure on p 32 represents the sky in mid-September. Turning the September chart upside down, you see that at 8:00, the big dipper is off to the left (the west of north). As the night goes on, the sky will turn counter clockwise, so we see that later in the night, the dipper will be directly below Polaris, as in the figure. The question then is, how many hours must elapse for this much rotation?

   Note that the charts correspond to 9 PM early in the month and 7 PM at the end of the month. Thus *two hours shifts the sky the equivalent of one month.* **Since the November chart matches the figure (taken in September), that is two months difference, the equivalent of 4 hours. Thus the photo was taken at 8 + 4 = 12. I.e., it was taken at midnight.**

4. The eccentricity of the orbit of Mars is $e = 0.0934$ (Appendix A-10). What is the ratio of the aphelion distance to the perihelion distance for this planet? Use the inverse square law to compute the ratio of the intensity of solar radiation on Mars at aphelion to the intensity at perihelion.

   Look at the equations on slide 10 of lecture 3:
   The aphelion distance is $(1 + e)a = (1 + 0.0934)a = 1.0934a$, where a is the semi-major axis of Mars' orbit. The perihelion distance is $(1 - e)a = 0.9066a$. Thus the ratio is $(1.0934a)/(0.9066a) = 1.20604$.
   Since the solar intensity goes as the inverse square of the distance, the ratio of intensity will be $1/1.20604^2 = 1/1.45454 = 0.687503$. Thus Mars gets only about 69% the solar radiation at aphelion that it gets at perihelion.
   (Since $e = 0.0167$ for the Earth, the aphelion/perihelion ratio is 94%, a much smaller effect.)

5. The **absolute magnitude** of the Sun is 4.83. This means that if the Sun were at a distance of 10 parsecs, it would have an apparent magnitude of 4.83. Assuming that the limit of naked-eye visibility under perfect conditions is 6.5 magnitudes, what is the increase in magnitude Δm needed to make a 4.83 magnitude star invisible? To what ratio of intensities does this Δm correspond? (See the first equation on p 17 of the text.) Finally, from the inverse square law, calculate the factor by which we would we have to increase the star's distance to get this reduction in intensity.

   An in increase of $\Delta m = 6.5 - 4.83 = 1.67$ magnitudes would be needed to take a 4.83 magnitude star to the edge of visibility.

   From the equation in the text (or slide 11 of lecture 2) the intensity factor corresponding to this magnitude change is:

   $$I_{4.83}/I_{6.5} = (2.512)^{6.5-4.83} = (2.512)^{1.67} = 4.65621$$

   Since the intensity varies as the square of the distance and $\sqrt{4.65621} = 2.15783$, an increase in the distance by a factor of 2.15783 would decrease the intensity by a factor of $2.15783^2 = 4.65621$, and this will make the star 1.67 magnitudes fainter.

   Now you can answer the question: "At what distance would the Sun cease to be visible to the naked eye?" Give your answer in both parsecs and light years.

   Since the Sun would have a magnitude of 4.83 at a distance of 10 parsecs, we see that if we move it to $2.15783 \times 10 = 21.5783$ parsecs, its magnitude would become 6.5, the edge of visibility.
   Further, since 1 parsec = 3.26 light years, the Sun would be invisible to the naked eye beyond a distance of $3.26 \times 21.6 = 70.4$ light years.

6. Both the Earth and the Moon orbit their common center of mass, but since the Earth is much more massive, its orbit is much smaller that that of the Moon. Look up the mass of the Earth and of the Moon in Table A-5 (p 437). How much more massive is the Earth than the Moon? (I.e., what is the ratio of their masses?)

   From Table A-5, $M_{Earth} = 5.976 \times 10^{24}$ kg, $M_{Moon} = 7.350 \times 10^{22}$ kg. Thus

$$\frac{M_{Earth}}{M_{Moon}} = \frac{5.976 \times 10^{24}}{7.350 \times 10^{22}} = 81.31$$

   Look up the distance from the Earth to the Moon in Table A-11. Calculate the distance of the Earth's center from the center of mass of the Earth-Moon system. Express this distance in units of the Earth's radius.

   From Table A-11, the Earth-Moon distance is $a = 384400$ km. Referring to slide 12 from lecture 3, you see that the distance a_1 of the Earth's center from the center of mass is given by

$$a_1 = \frac{M_{moon}}{M_{earth} + M_{moon}}\, a = \frac{7.350 \times 10^{22}}{5.976 \times 10^{24} + 7.350 \times 10^{22}}\, 384400 = 4670\,\textbf{km}$$

   Since the Earth's radius is $R_\oplus = 6378$ km, $a_1 = 0.73\, R_\oplus$. Thus the center of mass of the Earth-Moon system is *inside the Earth*, about 3/4 of the way to the Earth's surface.

7. We saw that the true form of Kepler's third law can be written

$$P^2 = \frac{a^3}{m_1 + m_2}$$

   if we express the period P in years, the semi-major axis a in astronomical units (AU), and the masses of the two bodies, m_1 and m_2, in solar masses. (If the orbit is circular, then $a = r$, the distance between the two bodies.)

   Consider a satellite orbiting Mars. What is the mass of Mars in solar masses? You can neglect the mass (m_2) of the satellite. (Look up the needed data in appendices A5 and A10 of your text.)

   From Table A-10 we see that the mass of Mars is 0.1075 Earth masses. From Table A-5, The Earth's mass is 5.976×10^{24} kg.
   Thus the mass of Mars is $0.1075 \times 5.976 \times 10^{24} = 6.4242 \times 10^{23}$ kg.

   Further, the Sun's mass is 1.989×10^{30} kg, so we see that the mass of Mars is $m_1 = (6.4242 \times 10^{23})/(1.989 \times 10^{30}) = 3.23 \times 10^{-7}$ solar masses.

Suppose you want your satellite to orbit Mars in the same length of time that Mars takes to turn once on its axis, so that your satellite will be fixed in the Martian sky. Mars turns on it's axis in 24.623 hours. What is this period in years? From Kepler's law (above), find the radius of the orbit a in AU. What is this expressed in kilometers? What is the radius of the orbit expressed in terms of the radius of Mars? (1 AU = 1.496×10^8 km)?

A year is 365.25 days = 24(365.25) = 8766 hours. Thus the period of rotation of Mars is $P = 24.623/8766 = 0.002809$ years.

Then from Kepler's law we find
$a^3 = m_1 P^2 = 3.23 \times 10^{-7} \times (0.002809)^2 = 2.54825 \times 10^{-12}$
and taking the cube root, $a = \sqrt[3]{2.54825 \times 10^{-12}} = 1.366 \times 10^{-4}$ **AU.**

Now $1AU = 1.496 \times 10^8$ **km, so the radius of our orbit is**
$a = (1.496 \times 10^8) \times (1.366 \times 10^{-4}) = 20433$ **km**

Further, the radius of Mars is 3396 **km, so the radius of a synchronous orbit about Mars is** $20433/3396 = 6.017$ **Martian radii.**

Compare this result to the case of a geosynchronous satellite in orbit about the Earth; such satellites orbit at 6.61 Earth radii.