ASTR 300 Stars and Stellar Systems Fall 2009

Homework Set No. 2 **Solutions**

1. Why do optical astronomers often put their telescopes at the tops of mountains, while radio astronomers sometimes put their telescopes in deep valleys? (Chapt. 6, Review Question 4)

   Optical astronomers hope to get above as much of the atmosphere as possible. For radio astronomers, interference from man-made radio signals is a major problem. Putting a radio telescope in a deep valley may shield it from such interference.

2. Why must telescopes observing in the far-infrared be cooled to low temperatures? (Chapt. 6, Review Question 13)

   Objects at room temperatures radiate most strongly in the far infrared. Thus the telescope must be cooled or its own radiation will overwhelm the astronomical signal we wish to observe.

3. An astronomer wants to put a telescope in space that will have a resolving power of 0.01 seconds of arc at visible wavelengths. What must the diameter of the mirror be to achieve this resolution?

   Your text (p 108) gives a formula for the resolution at visible wavelengths: $\alpha = 13.8/D$, where the resolution α is in seconds of arc, and D is the telescope diameter in centimeters. Thus we have $0.01 = 13.8/D$, and we see that $D = 1380$ cm $= 13.8$ meters. That's about 45 feet – a big telescope!

4. Infrared observations of a star show that it is most intense at a wavelength of 2000 nm. What is the temperature of the star's surface? (Chapt. 7, Problem 3)

   Using the Wien displacement law (p 134) we have the relation: $\lambda_{max} = 2,900,000/T$. Here, λ_{max} is in nanometers. (Your book gives the number as $3,000,000$, but this is more accurate. It is *not* the speed of light.) Putting in $\lambda_{max} = 2000$ nm, we find $T = 1450$ K, quite cool for a star.

5. We discussed the Stefan-Boltzmann law which gives the energy radiated by a surface at temperature T:

   $$E = \sigma T^4 \text{ Joule/s/m}^2, \text{ where } \sigma = 5.67 \times 10^{-8} \text{ Joule/s/m}^2\text{/degree}^4$$

   Suppose a space station has a panel one square meter in area exposed to space. The panel is at a temperature of 22 C (71.6 F).

   (a) What temperature units must be used in the Stefan-Boltzmann equation and what is the temperature of the panel in these units?

   This equation requires that the temperature be measured on the Kelvin scale. Thus we add 273 to the Centigrade value: 22 + 273 = 295 K. (See Table A-4, p 436)

(b) How much energy/s is radiated into space by the panel (in Joule/s)? (One Joule/s is a power of one Watt).

Since the area is the standard one square meter, the energy radiated is just $E = \sigma(295)^4 = 429.4$ Joule/s, or 429.4 Watts.

(c) At what wavelength does the radiation from the panel peak?

Using Wien's law as in Problem 4, we have
$\lambda_{max} = 2,900,000/T = 2900000/295 = 9831$ nm.
This is in the far-infrared, as expected for an object a room temperature.

6. Near the end of their lives, stars like the sun become very luminous but have low surface temperatures. Consider such a star with a luminosity of $L = 10^4\ L_\odot$ and a surface temperature of $T = 3000$ K.

(a) Calculate the radius of this star. Give your answer in solar units (units of $R\odot$).

The equation we will use to find the radius is (Lect 5. slide 17):

$$\frac{R}{R_\odot} = \sqrt{\frac{L}{L_\odot}} \left(\frac{5800}{T}\right)^2 .$$

This gives us $\sqrt{10^4} \times [5800/3000]^2 = 10^2 \times 3.738 = 373.8\ R_\odot$.

(b) Convert your radius into AU. Where would the surface of this star be located if you superimposed it on our solar system?

Since $R_\odot = 6.955 \times 10^5$ km, while 1 AU $= 1.496 \times 10^8$ km, we see that the radius of this star is

$$R = \left[\frac{6.955 \times 10^5 \text{ km}/R_\odot}{1.496 \times 10^8 \text{ km/AU}}\right] 373.8\ R_\odot = 1.74 \text{ AU}$$

The distance of Mars from the Sun is 1.52 AU, while Jupiter's distance 5.2 AU (p 440, Table A-10). Thus the surface of this star would extend past the Earth and Mars! (But not to the orbit of Jupiter.)

7. We saw that the energy of the nth level of the hydrogen atom is

$$E_n = \frac{E_1}{n^2} \quad \text{where} \quad E_1 = -2.178 \times 10^{-18} \text{ Joule}$$

is the energy of the ground state (it is negative since the the energy is taken as zero if the electron is completely removed – we have to add energy to free a bound electron).

(a) What is the energy of the $n = 109$ state of the H atom? What is the energy of the $n = 110$ state? If an atom is in the $n = 110$ level and jumps to the $n = 109$ level, what is the energy of the photon emitted? What other levels might the electron jump to from the $n = 110$ level?

The energy of n=109 is $E_{109} = E_1/109^2 = -1.83318 \times 10^{-22}$ J. The energy of the n=110 state is $E_{110} = E_1/110^2 = -1.8000 \times 10^{-22}$ J. The energy emitted in a jump from n=110 to n=109 is $\Delta E = E_{110} - E_{109} = 3.318 \times 10^{-24}$ J. The electron could also jump to levels 1,2, ..., 107, or 108.

(b) Recall that the photon energy is related to the frequency f by $E = hf$ where $h = 6.626 \times 10^{-34}$. What is the frequency of the photon emitted by the $n = 110$ to $n = 109$ transition? How is the frequency related to the wavelength, and what is the wavelength of this photon?

The frequency will be $f = \Delta E/h = 5.007 \times 10^9$ cycles/s. The wavelength is given by $\lambda = c/f$, where c is the speed of light: $c = 3 \times 10^8$ m/s. We thus have $\lambda = 3 \times 10^8/5.007 \times 10^9 = 0.05991$ m. Since 1 m = 100 cm, we have $\lambda = 5.991$ cm.

(c) Astronomers actually observe this radiation. What part of the electromagnetic spectrum is it in? (See Fig. 6-3) What type of telescope would the astronomer use?

This wavelength (6 cm) is toward the short end of the radio window. the astronomer would use a radio telescope.