ASTR 300 Stars and Stellar Systems Fall 2009

Homework Set No. 4 **Solutions**

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1. How do the spectra of HII regions differ from the spectra of reflection nebulae? Why? (Chapter 10, Review Question 3)

   Reflection nebulae show an absorption line spectrum while the spectra of HII regions consist of emission lines with little continuum. This is because a reflection nebula is just starlight reflected by dust, and thus just shows a stellar spectrum. An HII region is a cloud of hot gas that has been ionized by ultraviolet radiation, and the recombining atoms produce emission lines.

2. Why do distant stars look redder than their spectral types suggest? (Chapter 10, Review Question 6)

   The dust in the interstellar medium scatters more blue than red light. Thus distant stars have more blue than red light removed from their energy distribution, making them appear redder than the original energy distribution.

3. Why can the 21-cm radio emission line of neutral hydrogen be observed in the interstellar medium but not in the laboratory? (Chapter 10, Review Question 9)

   An average hydrogen atom in the upper level of the (split) ground state will not radiate a 21 cm photon for about 11 million years. In the laboratory, collisions are constantly disturbing the atoms. It is only in the vast, low density regions of the interstellar medium that enough transitions occur to be detectable.

4. The dust in a molecular cloud has a temperature of about 50 K. At what wavelength does it emit the maximum energy? (Hint: Consider black body radiation, Chapter 7.) (Chapter 10, Problem 2)

   Remember the formula for Wien's displacement law? (e.g., Lect 5, slide 8)

   $$\lambda_{max} = 2,900,000/T$$

   Now for the dust, $T = 50$ K. So we find $\lambda_{max} = 58000$ nm $= 58\mu$m — this is a far infrared wavelength.

5. The density of air in a child's balloon 20 cm in diameter is roughly the same as the density of air at sea level, 10^{19} particles/cm^3. To how large a diameter would you have to expand the balloon to make the gas inside the same density as the interstellar medium, about 1 particle/cm^3? Give your answer in km. (Hint: The volume of a sphere is $\frac{4}{3}\pi R^3$.) (Chapter 10, Problem 5)

   The number of particles in the balloon is the number/cm^3 times the volume of the balloon. The diameter of the balloon is $d = 20$ cm, so the radius of the balloon is $R = d/2$ and the total number of particles is $N = 10^{19} \times \frac{4}{3}\pi(20/2)^3$. After expansion, the total number is again the number/cm^3, which is now 1, times the volume, $\frac{4}{3}\pi(d/2)^3$. But expansion does not change the total number, so we have the equality $10^{19} \times \frac{4}{3}\pi(20/2)^3 = \frac{4}{3}\pi(d/2)^3$. Canceling gives $10^{19} \times 20^3 = d^3$, or, taking the cube root, $d = 20 \times \sqrt[3]{10^{19}}$. Thus $d = 20 \times 2.15 \times 10^6 = 4.31 \times 10^7$ cm. One km=10^5cm, so the expanded diameter is 431 km!

6. What factors resist the contraction of a cloud of interstellar matter?
   (Chapter 11, Review Question 1)

   Your text lists four factors:
   (1) thermal motion (i.e., gas pressure)
   (2) magnetic fields
   (3) rotation
   (4) turbulence

7. What evidence is there that (a) star formation has occurred recently? (b) Protostars really exist? (c) The Orion region is actively forming stars? (Chapter 11, Review Question 3)

   (a) We find regions with stars so young they must have formed recently. *T Tauri* **stars are still in the process of contracting. (p 226)**
   (b) In the youngest clusters, we see stars that when plotted on the H-R diagram, lie above the main sequence: they are protostars that haven't reached the main sequence yet. We also see the jets ejected by the disks around the hidden protostars – jets that make *Herbig-Haro objects***.**
   (c) The Orion nebula has very hot, luminous blue stars that have such short lifetimes that they must have formed recently. Also, with infrared telescopes, we can see many young stars embedded in the dust and gas of Orion.

8. How does the energy of a 0.35 solar mass star reach the surface? What fraction of this star's hydrogen fuel is available during the star's lifetime? Why?

   According to your text (page 265), stars less massive than about 0.4 solar masses are totally convective. This means (1) that convection transports the energy generated in the core to the surface, and (2) since they are completely mixed by convection, they can burn <u>all</u> their hydrogen.

9. If a protostellar disk is 200 AU in radius, and the disk plus the forming star contain 2 solar masses, what is the orbital velocity at the outer edge of the disk in kilometers per second? (Chapter 11, Problem 4)

You can consider the material orbiting at the outer edge of the disk as if it were a planet orbiting a star of 2 solar masses (the matter at the edge sees the material inside its orbit as if it were concentrated at the center). First, use Kepler's 3rd law to find the period of the orbit:

$$(M_1 + M_2)P^2 = a^3 .$$

Let M_2 be the orbiting material and M_1 the mass inside the orbit, so that $M_1 = 2M_\odot$ and M_2 is much less than M_1 and can be neglected. The radius of the orbit is just $a = 200$ AU. Plugging in these values we find

$$(2)P^2 = 200^3, \quad P^2 = 4 \times 10^6 \text{ so that } P = 2000 \text{ years.}$$

To find the velocity of the material, we reverse the procedure we used with the velocity of the double stars. The circumference of the orbit is $2\pi a$, and that is the distance the material travels in one period of 2000 yr. Since

$$\text{distance} = \text{velocity} \times \text{time} , \quad v = \frac{2\pi a}{P} .$$

Now we have to be careful with units: we want v in km/sec, so we need a in km and P in seconds (not yr). Thus $a = 200 AU(1.5 \times 10^8 km/AU) = 3 \times 10^{10}$ km. Also $P = 2000 \times 365.24 \times 24 \times 60 \times 60 = 6.311 \times 10^{10}$ sec. So finally,

$$v = \frac{2\pi a}{P} = \frac{2\pi\ (3 \times 10^{10})}{6.311 \times 10^{10}} = 2.99 \text{ km/sec.}$$

Actually, it's simpler to use the formula for circular velocity given on p 86: $V_c = \sqrt{GM/r}$. With $G = 6.6726 \times 10^{-11}$ m^3/s^{2}kg, $M_\odot = 1.989 \times 10^{30}$ kg, and one AU = 1.496×10^{11} m, we get

$$V_c = \sqrt{\frac{(6.6726 \times 10^{-11}) \times 2 \times (1.989 \times 10^{30})}{200 \times (1.496 \times 10^{11})}} = 2979 \text{ m/sec} = 2.98 \text{ km/sec}$$

10. The CNO cycle is very sensitive to temperature; the temperature dependence of the CNO energy generation rate is approximately T^{16} for the relevant temperatures (~20 million K). By what factor would the energy generation increase if the temperature were to increase from 20 million to 25 million K?

The increase is a factor of $25/20 = 1.25$, so the energy generation would increase by a factor of $(1.25)^{16} = 35.5$.