ASTR 498N Stellar Evolution Spring 2014

Problem Set No. 1 Due Feb 6, 2014

1. We derived the free-fall time, t_{FF}, of a spherically symmetric mass distribution (equation 1.4 of the text).
   (a) Find an expression for the velocity of the collapsing material when it has reached the radius of $r_0/2$. Evaluate this velocity (in km/s) for the case of the sun (i.e., $r_0 = R_\odot$ and $m_0 = M_\odot$).
   (b) What is the velocity when the material reaches the center $r = 0$? Explain.
   (c) Find the time t it takes for the material to reach the half-way point, $r = r_0/2$. Express this as a fraction of the free-fall time; i.e., find c where $t = c \cdot t_{FF}$.
2. Do problem 1.2 of the Phillips text.
3. We saw that the expression for the gravitational potential energy of a star is

$$E_{GR} = -\int_{m=0}^{m=M} \frac{G\, m(r)}{r}\, dm \ .$$

   (a) Show that for a star of mass M and radius R this can be written as

$$E_{GR} = -f\, \frac{GM^2}{R}$$

   where f is a dimensionless quantity depending on the internal density distribution of the star:

$$f = \int_0^1 \frac{(m/M)}{(r/R)} d(m/M) \ .$$

   (b) Evaluate f for a star of uniform density: $\rho(r) = \bar{\rho} =$ constant. (This turns out to be the smallest value f can have.)
   (c) A more realistic distribution of density that is still simple enough to allow evaluation is the linear expression $\rho(r) = \rho_c\,(1-x)$, where $x = (r/R)$ and ρ_c is the central density.
   (i) Find the mean density $\bar{\rho} = 3M/4\pi R^3$ in terms of ρ_c for this linear distribution.
   (ii) Evaluate the parameter f for this linear distribution. Compare the values of f for the uniform and linear density distributions. Comment on the variation of potential energy with the degree of central condensation in stars which otherwise have the same mass and radius.
4. "As the sun evolved towards the main sequence, it contracted under gravity while remaining close to hydrostatic equilibrium, and its internal temperature changed from about 30 000 K, Eq.(1.23), to about 6×10^6 K, Eq.(1.31). ... Find the total energy radiated during this contraction." (This is the first part of problem 1.3 in Phillips.) This problem is simple if you use the stellar radii given in the text and the virial theorem.