ASTR 498N Stellar Evolution Spring 2014

Problem Set No. 2 Due Feb 25, 2014

1. We saw that the general expression for the pressure of highly degenerate electrons is given by the rather formidable expression

$$P \; = \; A \, f(x) \quad \text{where} \quad f(x) \; = \; x(2x^2 - 3)(1 + x^2)^{1/2} + 3 \sinh^{-1}(x) \;\; ,$$

where x is the following function of the Fermi momentum:

$$x \; = \; \frac{p_F}{mc} \; = \; \left(\frac{3}{8\pi} \right)^{1/3} \left(\frac{h}{mc} \right) \, n^{1/3}$$

Consider the limit where $x << 1$. Expand $f(x)$ and find the leading term in the limit of small x. Show that this leads to the non-relativistic result that $P \propto \rho^{5/3}$.

2. We showed that the Saha equation for the case of pure hydrogen in terms of the ionized fraction $y = n^+/(n^0 + n^+)$ is given by the equation

$$\frac{y^2}{1-y} \; = \; \frac{4.01 \times 10^{-9}}{\rho} \, T^{3/2} \, e^{-\frac{157,800}{T}} \; = \; F(\rho, T)$$

which is the following quadratic equation for y:

$$y^2 \; + \; Fy \; - \; F \; = \; 0$$

   (a) Use the quadratic formula to write the solution of this equation for y in terms of F. What does this solution become in the limits $F >> 1$ and $F << 1$? (For the $F >> 1$ case, keep enough terms in the expansion so that you are not left with just $y = 1$.)

   (b) Make a plot of the ionized fraction y and the corresponding neutral fraction $(1 - y)$ for temperatures $5000K \leq T \leq 15000K$. Do this for three values of the density ρ: 10^{-11}, 10^{-10} and 10^{-9} g cm^{-3}.

3. Do problem 2.4 of the Phillips text.

4. In the *Computer Exercise* you were asked to make a **zero age main sequence model** of a one solar mass star with a chemical composition of $X = 0.70$, $Y = 0.28$ and $Z = 0.02$. For this problem, I would like you to make a sequence of ZAMS models of that same composition but varying in mass from 0.5 to 2.5 solar masses.

   (a) Make models for a closely spaced sequence, e.g., 0.5,0.6,0.7,0.8,0.9,1.0, 1.1, 1.2, 1.3, 1.4, 1.6, 1.8, 2.0, 2.2, and 2.5 solar masses. *Write down the values of M, Pc, Tc, R, and L for each model.* These values can be used as first guesses to construct the next model, provided its mass is not too far from the current model. Also, write down the value of ρ_c for each model.

   (b) Plot the values of $\log(L/L_\odot)$ against $\log(T_{eff})$ for your models. Make the plot as an H-R diagram (i.e., let the $\log(T_{eff})$ axis run from right to left).

(c) Make a plot of the mass-luminosity relation: plot $\log(L/L_\odot)$ against $\log(M/M_\odot)$. Fit your results to a mass-luminosity relation of the form

$$\frac{L}{L_\odot} = \mathcal{C}\left(\frac{\mathcal{M}}{\mathcal{M}_\odot}\right)^\nu \ .$$

Here I have added a constant $\mathcal{C}$ since the value of L at $M = M_\odot$ is not exactly $L_\odot$ (Why not?). What is your best fitting value of ν?

(d) Make a nice plot of the values of $\log(\rho_c)$ vs. $\log(T_c)$ for your models.

(e) Finally, look at the second table of the computer output and note that the column ”Lc/Ltot” is the fraction of the flux carried by convection. If it's not zero, you're in a convective region. Where does convection in each model begin and end in terms of the scaled radius $x = (r/R)$? Make a plot of the location and extent of the convective zone(s) as a function of M.