ASTR 498N   Stellar Evolution   Spring 2014

Problem Set No. 4   Due Apr 8, 2014

1. (a) Show by direct substitution that the function

$$\theta(\xi) \;=\; \frac{\sin\xi}{\xi}$$

is a solution of the Lane-Emden equation for a polytropic index of $n = 1$. Further, show that it satisfies the boundary conditions $\theta = 1$ and $d\theta/d\xi = 0$ at $\xi = 0$.

(b) The power series expansion of a polytrope of index n about the origin is

$$\theta(\xi) \;=\; 1 \;-\; \frac{1}{6}\,\xi^2 \;+\; \frac{n}{120}\,\xi^4 \;-\; \frac{n(8n-5)}{15120}\,\xi^6 \;+\; \cdots$$

Use the expansion of $\sin(\xi)$ to show that $\theta(\xi)$ given in (a) agrees with this series expansion.

(c) The first zero of θ is clearly at $\xi_1 = \pi$. What is the value of $d\theta/d\xi$ at $\xi = \xi_1$?

(d) It can be shown that the mean density of a polytrope, $\overline{\rho}$, is related to the central density by

$$\rho_c \;=\; D_n\,\overline{\rho} \quad \text{where} \quad D_n \;=\; -\left[\frac{3}{\xi_1}\left(\frac{d\theta}{d\xi}\right)_{\xi_1}\right]^{-1}$$

Evaluate D_n for this $n = 1$ polytrope.

2. (a) Do problem 5.1 of the Phillips text.
Hints: Use the unnumbered equation between 1.5 & 1.6 and also equation 5.32. Also, the integral

$$\int_0^\infty x^4\, e^{-x^2}\, dx \;=\; \frac{3\sqrt{\pi}}{8} \quad .$$

(b) If the sun is fit by a model with $(R/a) = 5$, what is the polytropic index n for a polytrope with the same value of E_{GR} as that obtained from the expression you derived in (a)?

3. Do problem 5.4 of the Phillips text.

4. Do problem 6.1 of the Phillips text.