ASTR 606 Stellar Structure and Evolution Spring 2008

Problem Set No. 1 Due Feb 7, 2008

1. Consider a spherical star of radius R which we observe from a great distance. The intensity of the radiation which emerges from the star's atmosphere, $I(0,\mu)$, will not be isotropic, but will vary with the angle θ between our line of sight and the local normal to the atmosphere. See the figure on the next page. Since we do not resolve the star's surface, what we see is an average of $I(0,\mu)$ over the visible hemisphere, weighted by the area seen at the various values of $\mu = \cos\theta$. Find an expression for this average over intensity which the distant observer sees. How is this related to the flux F which emerges from any point on the star's surface?

2. Suppose the intensity is given by the simple expression $I(\tau,\mu) = \tau + \mu$ (or some constant times this function), where τ ranges over $0 \le \tau < \infty$ and μ over $-1 \le \mu \le +1$. What is the value of the mean intensity $J(\tau)$ for this $I(\tau,\mu)$? Show that this expression for $I(\tau,\mu)$ is an *exact* solution of the grey transfer equation

$$\mu \frac{dI(\tau,\mu)}{d\tau} = I(\tau,\mu) - J(\tau) .$$

Explain why this solution cannot apply to a real physical situation. (Hint: look at the values of the downward intensity at small optical depths.)

3. Using the formal solution of the transfer equation, compute expressions for the emergent intensity $I(0,\mu)$ for the cases where the source function has the following simple forms:

(a) $S(\tau) = 1.0$ for $0 \le \tau < \infty$.

(b) $S(\tau) = 0.5$ for $0 \le \tau \le 0.3$,
$S(\tau) = 1.0$ for $0.3 < \tau < \infty$.

(c) $S(\tau) = 3.0$ for $0 \le \tau \le 0.1$,
$S(\tau) = 1.0$ for $0.1 < \tau < \infty$.

(d) $S(\tau) = \tau^2$ for $0 \le \tau < \infty$.

For the functions (b)-(d), evaluate the intensity for at least 10 points for $0 \le \theta \le \pi/2$, and graph the results. For (d), also evaluate the (astrophysical) flux $F(0)$.

4. From the definition of the astrophysical flux:

$$F(\tau) = 2\int_{-1}^{1} I(\tau,\mu)\,\mu\,d\mu$$

and the formal solution to the transfer equation for the intensity at $\tau = 0$, prove that

$$F(0) = 2\int_{0}^{\infty} S(\tau)\,E_2(\tau)\,d\tau ,$$

where $E_2(\tau)$ is the second exponential integral.