ASTR 606 Stellar Structure and Evolution Spring 2008

Problem Set No. 2 Due Feb 19, 2008

This exercise is intended to give you a quantitative impression of how important stellar parameters such as the central pressure and the gravitational potential energy depend upon the density distribution within the star. We may also use these results later to make some simple analytic models of stars.

Consider stars with the same total mass M and radius R. The mean density of the star, $\bar{\rho}$, is given by

$$\bar{\rho} = \frac{M}{\frac{4}{3}\pi R^3} \ .$$

We will *assume* three forms for the density distribution within the star:

(a) $\rho(r) = \bar{\rho} =$ constant,
(b) $\rho(r) = \rho_c\,(1-x)$, where $x = (r/R)$, and
(c) $\rho(r) = \rho_c\, e^{-sx}$, where $x = (r/R)$ and $s =$ is a constant.

1. Find M_r for (a), (b), and (c). Write the results in the form $M_r = Mf(x)$, where, for (a) and (b), $f(x)$ will be a polynomial with integer coefficients.
2. What is the ratio of central density to mean density, $\rho_c/\ \bar{\rho}$, in models (b) and (c)? For case (c), evaluate the ratio for $s = 1.5$, 2 and 3.
3. Plot $\rho(r)$ and M_r/M as a function of $x = (r/R)$ for (a), (b), and (c). (Again, for (c), consider $s = 1.5$, 2 and 3.)
4. Find the central pressure, P_c, for (a) and (b) from the integral of the Eulerian equation of hydrostatic equilibrium (H,K&T, eqn. 1.6):

   $$P_c = \int_0^R \frac{G\,M_r}{r^2}\,\rho(r)\,dr \ .$$

   Your result for P_c will have the form $P_c = (n/8\pi)GM^2/R^4$, where n is an integer.
5. Find the gravitational potential energy, Ω , for (a), (b), and (c) from

   $$\Omega = -\int_0^M \frac{G\,M_r}{r}\,dM_r = -\frac{GM^2}{R}\int_0^1 \frac{(M_r/M)}{(r/R)}d(M_r/M) \ .$$

   Your answers for (a) and (b) will have the form $\Omega = -(n/m)GM^2/R$, where n & m are integers. Compare the values of these coefficients and comment on the variation of potential energy with the degree of central condensation. For case (c), you will have to do the integral numerically. (Again, consider $s = 1.5$, 2 and 3. Remember that stars of the same mass but different s will differ in ρ_c.)
6. Look again at case (b). Find the pressure, $P(x)$, not just at the center, but also for all radial points x. If the equation of state were that of an ideal gas, with a constant mean molecular weight, find the corresponding temperature variation, $T(x)$, that would follow from $P(x)$ and $\rho(x)$. Plot $P(r)$ and $T(r)$ for this case. Do you think such a temperature gradient would be likely (or possible)?