ASTR 606 Stellar Structure and Evolution Spring 2008

Problem Set No. 5 Due May 13, 2008

1. In this exercise you will use the program ZAMS to construct a series of hydrogen burning stellar models. (You can find this program on the disk at the back of the text, and also on the course web page. It should compile on Unix boxes with "f77 ZAMS.FOR", etc.)

   If (contrary to what we believe is the case!) stars were completely mixed during their evolution, they would still burn their hydrogen to helium before any reactions to produce heavy elements began. Such a sequence would be represented by stars like the homogeneous zero-age models, but with decreasing X and increasing Y, while the metalicity Z remains constant.

   (a) Begin by making a ZAMS model of a 2 $M_\odot$ star with a composition of X=0.74, Y=0.24 (so that Z=0.02). This is one of the models found in Tables 2.5 and 2.6 of Hansen, Kawaler & Trimble (pp 121-122). Suppose that this star is always well-mixed as it evolves. To represent this, make a series of 2 $M_\odot$ models with more and more helium: (X=0.72, Y=0.26), (X=0.70, Y=0.28), (X=0.68, Y=0.30), (X=0.66, Y=0.32), (X=0.64, Y=0.34), (X=0.62, Y=0.36), (X=0.60, Y=0.38), (X=0.58, Y=0.40), (X=0.55, Y=0.43), (X=0.50, Y=0.48), (X=0.45, Y=0.53), (X=0.40, Y=0.58), (X=0.35, Y=0.63), (X=0.30, Y=0.68) (X=0.25, Y=0.73) and (X=0.20, Y=0.78). How do the high-helium models differ from the initial (X=0.74, Y=0.24) model? What do you think is the basic physical reason for the differences?

   Try to push as close to (X=0, Y=1) as you can before the program crashes or the results become non-physical. What physical processes are neglected in the program that may become important in this limit?

   (b) Plot a theorist's H-R diagram (i.e., $\log_{10}(L/L_\odot) vs \log_{10}(T_{eff})$) showing the (middle of the) normal ZAMS using the values given in Tables 2.5 and 2.6. Then, on the same figure, plot your well-mixed evolutionary track for the one solar mass star. How does it differ from normal stellar evolution (e.g., Fig. 2.5, p 53 of the text)?

2. For this problem you are to integrate numerically the Oppenheimer-Volkoff (OV) equation (sometimes: Tolman-Oppenheimer-Volkoff), which is appropriate for the structure of neutron stars. This equation is [Shapiro & Teukolsky, "Black Holes, White Dwarfs and Neutron Stars", p 125 – but given there in units of c=G=1]:

$$\frac{dP}{dr} = -\frac{GM_r}{r^2}\rho\left\{1 + \frac{P}{\rho c^2}\right\}\left\{1 + \frac{3P}{<\rho> c^2}\right\}\left\{\frac{1}{1 - r_S/r}\right\}$$

   where $<\rho>$ is the mean density interior to r,

$$<\rho> = \frac{M_r}{\frac{4}{3}\pi r^3} \qquad \text{and} \qquad r_S = \frac{2GM_r}{c^2}$$

   is the Schwarzschild radius for a mass M_r. We also have the additional equation

$$\frac{dM_r}{dr} = 4\pi r^2 \rho \quad .$$

Assume a gas of non-interacting, degenerate, non-relativistic neutrons:

$$P = K\,\rho^{5/3} \qquad \text{where} \qquad K = 5.3802 \times 10^9 \ \text{(cgs)}$$

[Shapiro & Teukolsky, pp 27-28].

Find a function f so that you can write the equations in the form

$$\frac{dM_r}{dr} = 4\pi r^2 \rho \qquad \text{and} \qquad \frac{d\rho}{dr} = f(\rho,\ M_r)\ .$$

(a) Integrate from a point Δr very near the center, where you can assume $\rho = \rho_c$ and $M_r = (4\pi/3)(\Delta r)^3 \rho_c$, out to the point where ρ goes to zero. This gives you M and R as a function of ρ_c.

Explore the region from $\rho_c = 10^{15}$ to $\rho_c = 10^{16}$. You will find that $M(\rho_c)$ reaches a **maximum** in this range. At the point where M begins to decrease, your neutron star models are unstable (why ?), so that the value M_{max} (and its corresponding ρ_c) is the mass limit for a neutron star (assuming your adopted equation of state!).

(b) Replace the space curvature term $(1 - r_S/r)^{-1}$ by unity for a $\rho_c = 4 \times 10^{15}$ model and see how much M, R, and the structure changes.

(c) The stiffest possible equation of state is $P = \rho c^2$ (any higher pressure would make the sound speed exceed the speed of light). Make a series of models using $P = K\,\rho^{5/3}$ for $\rho \leq \rho_0$ and $P = P_0 + (\rho - \rho_0)c^2$ for $\rho > \rho_0$, where $\rho_0 = 4 \times 10^{14}$ and $P_0 = K\,\rho_0^{5/3}$. Determine the maximum mass in this case.