Problem I.1

(a)

$d = \ell\theta$ $\qquad$ $\ell = .6$ meters

$\theta = 3° = \left(\frac{2\pi}{360} \times 3°\right)$ rad $= 5.25 \times 10^{-2}$ rad.

$d = .6 \times (5.25 \times 10^{-2})$ m $= 3.14 \times 10^{-2}$ m $= 3.14$ cm

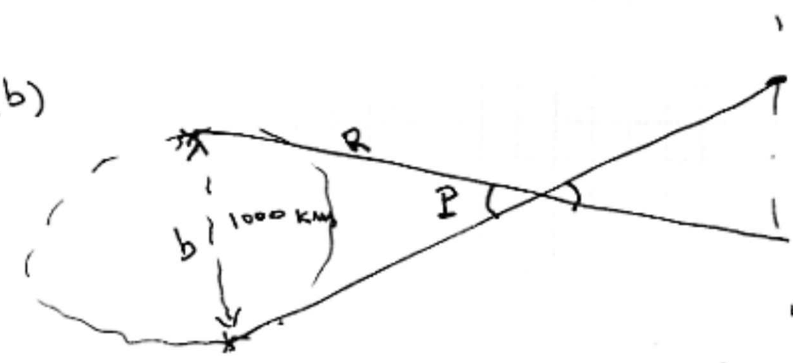
$\boxed{d = 3.14 \text{ cm}}$

OR USING TRIGONOMETRY.

$\frac{d}{2} = \ell \times \tan\frac{3°}{2} = .6 \times \tan 1.5° = 1.57 \times 10^{-2}$ m

$\boxed{d = 3.14 \text{ cm}}$

(b)

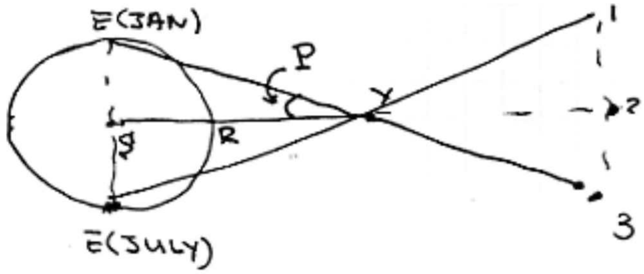


$R \times P = b$ $\qquad$ $R = \frac{b}{P} = \frac{10^3}{P}$ km

$$\boxed{P = \frac{10^3}{\frac{1.5}{2} \times 10^8} = 1.3 \times 10^{-5} \text{ rad} = .046 \text{ arcmin}}$$

36) ~~60 arcmin~~.

(c)



DEFINITION OF PARALLAX ANGLE P IS SHOW ABOVE.

TRIANGLE ESX HAS A RIGHT ANGLE AT S.

SO

$$\tan p = \frac{ES}{R} = \frac{1\,AU}{R}$$

SMALL ANGLE p $\tan p \approx p$.

$$\rightsquigarrow \quad p = \frac{1\,AU}{R}$$

$$R = \frac{1\,AU}{p}$$

THERE ARE 2.06 ARCSEC IN 1 RADIAN SO

$$\boxed{R = \frac{2.06 \times 10^5}{p''} \cdot AU} \cancel{= 2.75\,AU} =$$

MEANS ARCSEC

[NOTE THAT 1 PARSEC CORRESPONDS TO $p''=1$, OR 2.06×10^5 AU]

$$\boxed{R = \frac{2.06 \times 10^5}{.75''} \, AU = 2.75\,AU = 1.75\,LY}$$

NOTICE THAT THE REASON FOR THE USE OF PARSEC IS THAT 1'' CORRESPONDS TO 1 parsec.

NOTICE THAT

$$R(pc) = \frac{1}{p''}$$

I.E. 1 pc HAS $p = 1''$; 10 pc IS $p = .1''$; AND SO ON.

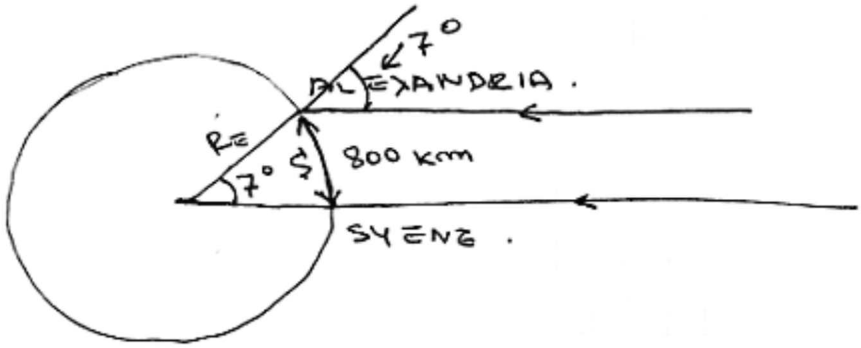
(d)

$$\boxed{1\,pc = \frac{1}{p''} = 2.06 \times 10^5\,AU = 3.26\,LY = 3 \times 10^{13}\,km}$$

PROBLEM I.34:

PROBLEM I.2

(a)



FROM THE FIGURE

$$S = R_E\,\theta \quad \text{IN RAD}$$

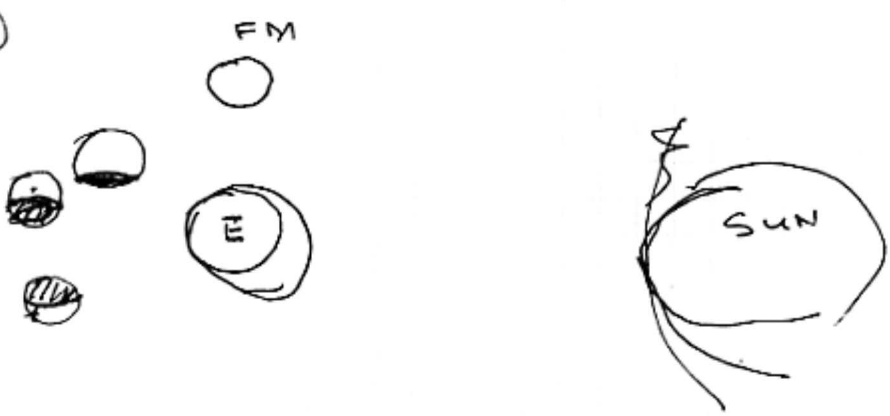
$$\theta = 7^\circ = 7^\circ \frac{2\pi}{360}$$

$$R_E = \frac{S}{7}\,\frac{360}{2\pi} = \frac{800 \times 360}{7 \times 2\pi}\ \text{km}$$

$$\boxed{R_E = 6400\ \text{km}}$$

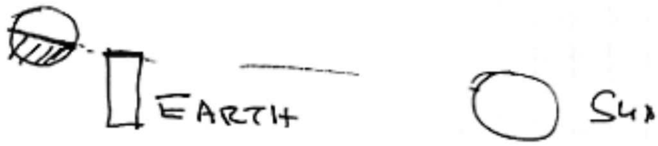
IT COMPARES WELL.

(b)



- THE EARTH'S SHADOW ON THE MOON DURING ECLIPSES OF THE MOON IS <u>ALWAYS</u> <u>ROUND</u> → EARTH SPHERICAL
- ~~IF EARTH A~~.

IF EARTH ~~SLAB~~ WAS A FLAT PLATE THE SHADOW ELONGATED AND STRAIGHT OR ELLIPTICAL.



(C)

$$D_M = \frac{1}{3}(2R_E) = \frac{2}{3}\,6400\,\text{km} = 4300\,\text{km}.$$

$$R_M = \frac{D_M}{\theta_M} = \frac{4300}{.5\times\frac{2\pi}{360}} \simeq 4.9\times10^5\,\text{km}$$

$$\boxed{R_M = 4.9\times10^5\,\text{km}}$$

[ATTACHED FIND AN AMUSING BUT EDUCATIONAL COMMENTARY ON THE ISSUE]

31. How Big Is the Earth's Diameter?

IMAGINE YOU LIVE on a tiny planet, perhaps 50 meters in radius. At some point on the planet, its sun will be directly overhead, and the length of your shadow will be zero. As you leave that spot, the sun will no

1. Only at the time of the spring and fall equinoxes does the Earth's north-south axis make a 90-degree angle with the line joining Earth and Sun, and we then have equal-length days and nights everywhere on Earth as a result. Given the 26,000-year precession of the Earth's axis, the time of the equinoxes must change by 1/26,000 of a year, or about 20 minutes, for each revolution about the Sun. You might think that this precession would cause the seasons to shift slowly over the years. But the length of the year is *defined* by the time from one vernal equinox to the next rather than the time Earth takes to make one 360-degree revolution around the Sun. As explained in essay 125, a gradual shift in the seasons does occur, but it is connected to the incommensurability between the length of the day and the year.

longer be directly overhead, and the length of your shadow will gradually grow. For example, if you walk 5 meters away (10 percent of the planet's radius), your shadow's length will be 10 percent of your height. Exactly the same idea would apply if the planet were larger in size, except that you would need to walk a greater distance to travel 10 percent of a radius. But you could still find the radius of your planet by seeing how far you needed to travel for the length of your shadow, initially zero, to become 10 percent of your height.

Using a method very similar to this, Eratosthenes, who lived in Greece around 200 B.C., first estimated the size of the Earth by measuring the lengths of the shadows of two vertical poles that were hundreds of miles apart along a north-south line. Let us assume that one vertical pole was at the equator, and the comparison of shadow lengths was made at noon on the date of an equinox. In that case, the Sun would be directly overhead at the equator, and the length of one pole's shadow would be zero. From the distance to the second pole, and the length of its shadow at noon, Eratosthenes could find the radius of the Earth. For example, if a 1-meter vertical pole located 640 kilometers north of the equator casts a $\frac{1}{10}$-meter shadow at noon, the Earth's radius would be 10×640, or 6,400 kilometers (4,000 miles).

One tricky point that Eratosthenes faced in making his observation was that he and his assistant could not be sure they were measuring the shadow lengths at the same time—remember that no mechanical clocks existed until many centuries later. Basically, the two observers made their measurements at local noon, when the Sun was most nearly overhead. Essentially, each person measured the length of the *shortest* pole shadow found at his location. Eratosthenes's method for measuring the Earth's radius does not require the two observers be on a north-south line, or even that the measurements be simultaneous—just so long as each one measures the length of the shortest pole shadow during the day.

Nowadays, in the era of space travel, when we can see the Earth whole from space, we can easily measure its diameter from a photograph. For example, suppose we had a photograph of the Earth with a 1-meter ruler in the foreground the same size as the Earth's diameter. In that case the diameter of the Earth in meters would just equal the distance to the Earth divided by the distance to the meter stick at the time the photograph was taken. You can get the idea using a

FIGURE 41. *In the days before pi was known, scientists had to find the Earth's diameter the hard way.*

corresponding earthly example: measuring the height of a light pole with a ruler. Walk a distance away from the light pole, so that it appears as tall as a 1-foot ruler held at arm's length. The height of the light pole (in feet) will, by similar triangles, have to equal its distance from you divided by the length of your arm. For example, if the pole matched the length of a 1-foot ruler when you were 10 arm lengths from the pole, it would have to be 10 feet high.

Still another way to get the diameter of the Earth relies on a few simple facts about geography. Each time zone is roughly 1,000 miles wide at the equator (which you can easily remember, since the east and west coasts of the United States are roughly 3,000 miles and three time zones apart). Knowing that there must be 24 time zones around the Earth—one for each hour of the 24-hour rotation—we see that the Earth's circumference must be in the neighborhood of 24,000 miles. Dividing this number by π gives a rough figure of 7,600 miles for the Earth's diameter, which is close enough for our purposes to the correct figure of 7,926 miles, or 8,000 miles in round numbers. This last method for finding the diameter of the Earth could be considered a measurement only if you were actually to find the average width of a time zone at the equator by observation. Basically, you would have to see how far you needed to travel east or west for there to be a 1-hour time difference between sunrises. If you didn't feel like making the trip, you could just phone a good friend as the Sun was rising at your location. If your friend were to live a distance corresponding to 15 degrees east of you in longitude, she should find that at her location the Sun had already risen 1 hour earlier. (Don't try this with someone west of you unless that person enjoys predawn phone calls.)

3. (d) Kepler.

4. (c) From study of Tycho Brahe's observational data.

5. First Law: The orbit of each planet lies in a plane passing through the Sun and is an ellipse with one focus at the Sun.

Second Law: The radial line segment from the Sun to a planet sweeps out equal areas per unit time as the planet moves along its elliptical orbit.

Third Law: The square of the orbital period of revolution of a planet is proportional to the cube of the semimajor axis of its elliptical orbit.