ASTR 340
FALL 2006
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HOMEWORK #2
SOLUTIONS

Problem #1

a. (2.6) Refer to Figure 2.2 from the book. It takes 20.985 days from 1st to 3rd quarter. 20.985/2=10.49 days halfway -> 360/42 days =8.57°/day. 8.57°/day x 10.49 days= 89.9° . cosα=(Distance zorlo to moon)/(Distance Zorlo to Sun)=.001. Therefore (Distance Zorlo to Sun)=1000 (Distance Zorlo to moon)
b. Tycho made the following experiments to discriminate between the Ptolemaic and Copernican models:
    a. Measured the distance to Mars at its closest approach. He calculated that the diurnal parallax – see figure 2.9 – should be measurable if the solar system was consistent with the Copernican system. He failed to measure it due to inaccurate knowledge of the earth's orbit by a factor of 20.
    b. He failed to detect stellar parallax – a necessity if the earth moved. He again failed due to poor instruments

c. (2.13) Kepler's 3rd law

$$P / years = \sqrt{(R / AU)^3}$$
$$R = 40AU$$
$$P = 253 \text{ days}$$

It does not depend on how elliptical the orbit is.

Problem #2

In all three portions of this question, I will refer to motion towards the right as "positive" velocity, and motion to the left as "negative." In addition, the notation for velocities will list the object being measured first, and the frame in which that measurement is made second (in parenthesis).

a. First, calculate the initial momentum of the system in the spring's frame (since the spring never changes velocity, it is convenient to refer to its intertial frame). The momentum of a body is simply its mass times its velocity, or $m_{body}v_{body}$. The total momentum of a system is the sum of the momentums of all bodies in the system, so the initial momentum of the ball-spring-ball system is:

$$m_A v_A(spring) + m_B v_B(spring) = 0\,(initial\ conditions) \tag{1}$$

where subscripts A and B refer to balls A and B. The spring contributes nothing to the total momentum, since it has no mass. The total momentum is zero because neither ball is in motion. Note that "(spring)" indicates that the velocities are measured with respect to the (stationary) spring.

The final momentum must equal the initial momentum, which in this case is zero. Since we know the final velocity of ball B, and the masses are constant, we must simply solve for the velocity of ball A. Begin with this equation:

$$m_A v_A(spring) + m_B v_B(spring) = 0\,(final\ conditions) \tag{2}$$

And rearrange into this:

$$-\frac{m_B v_B(spring)}{m_A} = v_A(spring) \tag{3}$$

Plugging in values:

$$-\frac{(4\,kg)(5\,m/s)}{(2\,kg)} = -10\,m/s \tag{4}$$

which means that the ball will be moving to the left at 10 m/s (relative to the spring).

b. This may seem like a totally different problem than part 2a., but it is in fact the same, with one difference. The question asks for the measurements of velocities in a frame in which the observer sees the ball-spring-ball system initially moving to the right at $3\,m/s$. I will denote this frame as the "lab" frame, and continue to refer to the frame in which the spring has zero velocity as the "spring" frame (this is the zero total momentum frame).

So $v_{spring}(lab) = 3\,m/s$. But from the spring's point of view, this means that the *observer* is moving at $-3m/s$, so $v_{lab}(spring) = -3\,m/s$. That is, to transform from the lab to the spring frame, subtract 3 m/s from any velocity. To transform from the spring frame to the lab frame, add 3 m/s to any velocity. Thus we can simply transform to the spring frame, calculate the velocities after the spring is released, and transform back to the lab frame.

We know from 2a that the final velocities in the spring frame are $v_A(spring) = -10\,m/s$ and $v_B(spring) = 5\,m/s$. (Note that the signs of the velocities become very important soon.) The relationship between the velocities in the observer's (lab) frame and the spring's frame is:

$$v_{body}(spring) + v_{spring}(lab) = v_{body}(lab) \tag{5}$$

Be sure to understand how the signs work out in this equation. For example, if the spring is observed to be moving to the right (positive velocity) in the observer's frame, then any velocity measured in the spring frame will be larger (more positive) if measured in the lab frame.

Note that it is equally simple to transform from the lab frame into the spring frame by:

$$v_{body}(lab) - v_{spring}(lab) = v_{body}(spring) \tag{6}$$

(One can easily see that eqn. 5 and 6 are the same equation!)

In any case, the machinery is in place to finish the problem. Just plug in values into eqn. 5:

$$-10\,m/s + 3\,m/s = -7\,m/s \;(ball A) \tag{7}$$

$$5\,m/s + 3\,m/s = 8\,m/s \;(ball B) \tag{8}$$

Thus the observer measures ball A to be moving to the left at 7 m/s, and ball B moving to the right at 8 m/s. (Note that each velocity is more positive than if it were measured in the spring's frame.)

c. We can apply the knowledge from parts 2a and 2b to this problem to solve it quickly. Again, picture initially a frame in which the balls and spring are initially stationary (spring frame). We know (from 2a) that the final velocity of ball A in this frame is $-10\,m/s$. What, then, does the relative velocity between the spring frame and the lab frame have to be for the observer in the lab to measure ball A's velocity as zero?

We already have eqn. 5, allowing us to transform from the spring to the lab frame. We require that $v_A(lab) = 0\,m/s$, and we've calculated that $v_A(spring) = -10\,m/s$, so all that remains is to solve for $v_{spring}(lab)$ and plug in:

$$v_A(lab) - v_A(spring) = v_{spring}(lab) = (0\,m/s) - (-10\,m/s) = 10\,m/s \tag{9}$$

Thus the spring system must be moving at 10 m/s towards the right (as measured in the lab frame), in order for ball A's final velocity to be measured as $0 m/s$ in the lab frame.

Problem #3

Note: be careful to use the correct 'r' in this problem wherever needed, and be sure to keep track of (and label!) your *units* at all times.

a. There are 1000 meters in 1 kilometer, so:

$$r_m = 384,400\,km \times \frac{1000\,m}{1\,km} = 3.844 \times 10^8\,m \tag{10}$$

$$R_E = 6378\,km \times \frac{1000\,m}{1\,km} = 6.378 \times 10^6\,m \tag{11}$$

b. 24 hours in 1 day, and 3600 seconds in an hour:

$$P_{moon} = 27.3\,days \times \frac{24\,hours}{1\,day} \times \frac{3600\,seconds}{1\,hour} = 2.36 \times 10^6\,s \tag{12}$$

c. $speed = \frac{distance}{time}$, and the distance traveled by the moon during its orbit is the circumference of its orbit. Use results from equations 10 and 12 and plug into the given equation:

$$\frac{2\pi r_m}{P_{moon}} = v_m = \frac{2\pi(3.844 \times 10^8\,m)}{(2.36 \times 10^6\,s)} = 1024\,m/s \tag{13}$$

d. Centripetal acceleration is $a = \frac{v^2}{r}$, with v as the orbital velocity (from eqn 13) and r the distance to the central body which is being orbited (in this case, we use the result of eqn 10). Plug in:

$$a_m = \frac{v_m^2}{r_m} = \frac{(1024\,m/s)^2}{(3.844 \times 10^8\,m)} = 2.728 \times 10^{-3}\,m/s^2 \tag{14}$$

Note that this is the acceleration due to the gravity of the Earth on *any* body that happens to be at the distance from the Earth that the Moon is - we haven't used the mass or size of the Moon anywhere!

e. Simply divide the result from 3d by the given acceleration due to gravity on the surface of the Earth (given as a_a, for "apple"):

$$\frac{a_m}{a_a} = \frac{2.728 \times 10^{-3}\,m/s^2}{9.8\,m/s^2} = 2.8 \times 10^{-4} \tag{15}$$

The distance from the center of the Earth to the apple is the radius of the Earth, or R_E. The distance from the center of the Earth to the Moon is the orbital radius of the Moon, or r_m.

We can use these values to verify Newton's equation of acceleration due to gravity:

$$a_{body(Grav)} = \frac{GM_{central}}{R^2_{central}} \tag{16}$$

with $M_{central}$ being the mass of the central body, and G Newton's gravitational constant. As we will soon see, these two values are irrelevant in this problem, and only $R_{central}$, the distance between the small object (apple or Moon) and the central body (Earth) is important.

Set the ratio of the accelerations due to gravity for the Moon and apple:

$$\frac{a_{M(Grav)}}{a_{a(Grav)}} = \frac{\frac{GM_{Earth}}{r_m^2}}{\frac{GM_{Earth}}{R_E^2}} \tag{17}$$

It is clear that G and M_{Earth} can be canceled, leaving us with:

$$\frac{a_{M(Grav)}}{a_{a(Grav)}} = \frac{\frac{1}{r_m^2}}{\frac{1}{R_E^2}} = \frac{R_E^2}{r_m^2} = \frac{(6.378 \times 10^6\,m)^2}{(3.844 \times 10^8\,m)^2} = 2.8 \times 10^{-4} \tag{18}$$

which is exactly the result we got in 3e, so gravity must follow a $1/R^2$ behavior.