ASTR 340
Fall 2006
Dennis Papadopoulos

HOMEWORK #3
Solutions

Problem 1

a) Mass of an object is the quantity of matter therein, a scalar having magnitude only.
b) Weight is the vector force on an object within the gravitational field of other objects. Its magnitude is proportional to the mass of the object.
c) Velocity of an object is the instantaneous rate of change of its position. It is a vector quantity, having magnitude, direction, and sense.
d) Speed, a scalar, is the magnitude of a velocity vector.
e) Acceleration is the instantaneous rate of change of the velocity vector of an object. It is also a vector quantity, having magnitude, direction, and sense.

Problem 1b: One can calculate the average distance from the earth to the moon only. Namely

$$T = 2\pi R_{EM} / V_\theta$$
$$M_M g(R_E / R_{EM})^2 = M_M (V_\theta / R_{EM})$$
$$R_{EM} = (g \frac{R_E^2 T^2}{4\pi^2})^{1/3}$$

Problem 1c: The earth' orbit would not change since its mass enters on both sides of the balance equation and cancels out. That is

$$G\frac{M_S \cancel{M_E}}{R_{ES}^2} = \cancel{M_E} \frac{V_\theta}{R_{ES}}$$

In fact a small change due to change in the system's center of mass of the order (M_E/M_S) will occur (Extra credit 3 points for noting this)

Problem 2:

$$\cancel{G}\frac{M_S}{R_{SX}^2}\cancel{m} = \cancel{G}\frac{M_J}{R_{JX}^2}\cancel{m}$$

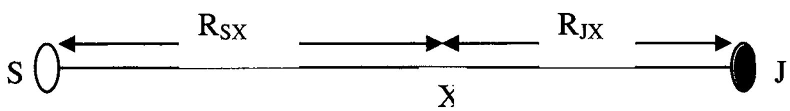
$$R_{JX} = R_{SX}\sqrt{M_J / M_S} = 3\times10^{-2} R_{SX}$$

$$3\times10^{-2} R_{SX} = 3\times10^{-2}(5.2AU - R_{JX})$$

$$R_{JX} = 3\times10^{-2}(5.2AU - R_{JX})$$

$$1.03R_{JX} = .156AU$$

$$R_{JX} = .153AU$$



Problem 3

In all three portions of this question, I will refer to positive motion as the direction in which the skateboarder is moving, measured with respect to the road.

The notation for velocities will list the object being measured first, and the frame in which that measurement is made second (in parenthesis). The three frames introduced here will be measured with respect to the skateboard (board), the antagonists (bad) and the road on which they are traveling (road).

a. In the *road* frame, the velocity of the frisbee is:

$$\frac{distance}{time} = velocity = \frac{10\,m}{0.5\,s} = 20\,m/s \tag{7}$$

So $v_{frisbee}(road) = 20\,m/s$. You are moving at +3 m/s relative to the road, or $v_{board}(road) = 3\,m/s$, so we can calculate the velocity of the frisbee in your frame:

$$v_{frisbee}(board) + v_{board}(road) = v_{frisbee}(road) \tag{8}$$

Or, rewriting:

$$v_{frisbee}(road) - v_{board}(road) = v_{frisbee}(board) = 20\,m/s - 3\,m/s = +17\,m/s \tag{9}$$

So you threw the frisbee ***forward*** at a speed of 17 m/s, as measured in your (board) frame.

b. First, we calculate the velocity of the tomato in the 'road' frame. We know the velocity of the board frame measured in the road frame already, and the problem says you threw the tomato with the 'same force' as the frisbee, so that means $v_{tomato}(board) = -17\,m/s$. Thus:

$$v_{tomato}(board) + v_{board}(road) = v_{tomato}(road) = (-17\,m/s) + (3\,m/s) = -14\,m/s \tag{10}$$

which is similar to eqn. 8. So the antagonists (who aren't moving relative to the road) observe the tomato coming at them at 14 m/s. The distance covered by the tomato is 20 meters, so:

$$\frac{distance}{speed} = time = \frac{20\,m}{14\,m/s} = 1.4s \tag{11}$$

Thus the tomato hits the pursuers 1.4 seconds after you throw it.

c. This problem involves all 3 reference frames, since the pursuers are now in motion, so we will calculate things relative to the road in the end.

First, in 0.9 seconds, you cover extra distance in the road frame, so:

$$speed \times time = distance = 3\,m/s \times 0.9\,s = 2.7\,m \tag{12}$$

So the egg must actually travel $15\,m + 2.7\,m = 17.7\,m$ in the road frame in order to hit you. Using this distance, the egg's velocity in the road frame is:

$$\frac{distance}{time}(road) = v_{egg}(road) = \frac{17.7\,m}{0.9\,s} = 19.7\,m/s \qquad (13)$$

We know how fast the antagonists threw the egg, so $v_{egg}(bad) = 16\,m/s$. Thus, using a rearranged version of eqn. 8:

$$v_{egg}(road) - v_{egg}(bad) = v_{bad}(road) = 19.7\,m/s - 16\,m/s = 3.7\,m/s \qquad (14)$$

So the pursuers have a road velocity of 3.7 m/s. That means that they are gaining on you at 0.7 meters per second! ($v_{bad}(road) - v_{board}(road) = v_{bad}(board)$) You should hurry!

Problem 6

There are various ways of approaching this problem – here's a couple of them.

Suppose we ignored gravity. Then, when the Monkey let go of the branch, he would not fall. Also, the dart would fly in a straight line from the gun to the Monkey, and (since the Monkey was not moving) the Monkey would get hit. Now imaging "turning on" gravity. The Monkey would start falling once he let go of the branch, and the dart would also "fall" thereby making it follow a curved trajectory. But, by the weak equivalence principle, the Monkey and the dart fall at exactly the same rate. So, by the time the dart has reached the Monkey, both the Monkey and the dart have fallen the same distance, and the dart still hits the Monkey!

Another way of looking at it – because of the weak equivalence principle, free-falling objects (including the dart) appear to travel in straight lines when observed from a frame of reference that is itself free-falling. This is exactly the basis behind the phenomenon of "weightlessness" experienced by astronauts. So, once the Monkey lets go of the branch, he will observe the dart moving in a straight line towards him and, eventually, get hit by the dart.