# Quasi-Periodic Brightness Oscillations from Accreting Neutron Stars and Black Holes

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**Abstract**

Observations with the Rossi X-ray Timing Explorer have revealed a number of characteristic frequencies of variation in the X-ray emission from accreting neutron stars and black holes. These frequencies are large enough that they must be affected by the strongly curved spacetime near these compact objects. As a result, study of them has the potential to probe strong gravity, and may also constrain the state of dense matter in neutron stars. I discuss the phenomenology of these oscillations as well as current ideas about their causes and implications.

# 1 Introduction

Neutron stars and black holes represent extreme physics and states of matter in many ways. For example, the most strongly curved spacetime in the universe is near black holes and neutron stars, and the densest non-singular matter and strongest magnetic fields in the universe are found in neutron stars. Therefore, study of these ultracompact objects can teach us about fundamental physics as well as astrophysics.

In this review, I discuss the phenomenon of quasiperiodic brightness oscillations (QPOs) from black holes and weakly magnetic neutron stars, and what we can learn from them about strong gravity and dense matter. After a brief discussion in § 2 of the information that we have available, in § 3 I give highlights of the observations. In § 4 I give some basic considerations for models, and in § 5 I discuss in depth the principles behind beat frequency models of neutron star QPOs. This is done for pedagogical purposes, to explore how an astrophysical model can be constructed, but is not meant to imply that there is complete agreement in the community that this is the correct class of models. In § 6 I give an overview of model ideas for black hole QPOs, and in § 7 I indicate how neutron star QPOs can be used to probe strong gravity and dense matter, independent of detailed models. In § 8 I discuss what qualitative advances would be possible with a future high-area X-ray timing mission, and I conclude in § 9. In the appendices I give some basic derivations of length and time scales, and some sample exercises with hints.

# 2 Extreme Physics and Timing Phenomena

We are hampered in our study of astrophysics because of the reality that astronomy is a "look but don't touch" science; we can observe objects from a distance, but not experiment with them. This is rather a fortunate situation for life, which would otherwise be substantially challenged by, e.g., neutron star surface gravitational fields a hundred billion times that of Earth. This does mean, though, that astrophysicists are required to make indirect inferences.

We can therefore ask what information is in principle available. With perfect instruments, we could have imaging, spectroscopy, and timing information.

## 2.1 Imaging

For stellar-mass compact objects, imaging is out of the question in the near term. Consider a 10 km radius neutron star that is at a distance of 100 pc, which is fairly close. The angular size is $3 \times 10^{-13}$ radian, or about $6 \times 10^{-8}$ arcseconds. A diffraction-limited telescope at $0.5\mu$m would need to have an effective aperture of roughly 2000 km to achieve this resolution. Obviously this is impossible for a single telescope, but interferometric arrays might be constructed in the distant future at this level (in space, to avoid atmospheric interference). The supermassive black hole in the center of our Galaxy, with a mass of $\approx 4 \times 10^6\,M_\odot$ and a distance of $\approx 8$ kpc (e.g., [1]), would have an angular size of $5 \times 10^{-11}$ radians or $10^{-5}$ arcseconds. There is currently discussion of X-ray interferometric experiments that would achieve such a resolution, and would thus be able to image the event horizon for the Galactic center black hole and the $\approx 3 \times 10^9\,M_\odot$ black hole in the center of the galaxy M87, but not many others. Imaging, therefore, is unlikely in the near future to contribute significantly to our understanding of the regions close to neutron stars and black holes.

## 2.2 Spectroscopy

The next obvious possibility is spectroscopy, which has contributed overwhelmingly to every field of astronomy. This is a field that actually has benefited significantly from laboratory experiments. Measurements of molecular, atomic, and nuclear energy levels, and the strengths of transitions between states, have historically allowed inferences of compositions, temperatures, densities, magnetic field strengths, rotation rates, and bulk velocities from many astronomical sources. Can spectroscopy make a similar contribution to the study of compact objects?

The answer is yes, but a qualified yes. As shown in Appendix 1, near a compact object the temperature of an accretion disk is millions of degrees Kelvin. This implies that all the hydrogen and helium will be fully ionized, and that even heavier elements such as iron will be almost completely ionized. This reduces the number of lines or edges that can be observed, but does not eliminate them completely. In addition, the rapid motions deep in the gravitational well tend to smear out lines. As an example, the speed of a particle at the innermost stable circular orbit (ISCO; see Appendix 1) in a Schwarzschild (non-rotating) spacetime is half the speed of light, as measured by an observer in a local static frame. This causes a spectral line to broaden by some tens of percent of its rest frame energy, reducing contrast and making the line tougher to detect. Nonetheless, these effects have been seen. Iron K$\alpha$ line fluorescence, broadened by Doppler shifts and reddened by gravitational redshifts, has been seen from both stellar-mass and supermassive black holes (for a review, see [2]). However, most spectral lines are smeared out or ionized away, making spectroscopy much more difficult to apply than it is for planets, stars, and galaxies.

## 2.3 Timing

Our remaining possibility lies in the temporal variations in the flux from accreting neutron stars and black holes. As demonstrated above, the temperatures are high enough that most of the emission will be in X-rays. Modern satellites can detect the time of arrival of each X-ray photon that hits the detector (current detectors have an area of up to a few thousand square centimeters), with an accuracy of microseconds. This produces a countrate time series $c(t)$, which can be Fourier transformed and squared to give a power density. The normalization is not unique, but it is particularly convenient to use the Leahy normalization (see [3] for a detailed discussion of timing techniques):

$$P(\omega) = (2/N)\left|\int_0^T c(t)\exp(-i\omega t)dt\right|^2 , \tag{1}$$

where $N$ is the total number of counts and $T$ is the duration of the observation. This has the property that pure Poisson noise has a chi squared distribution, with the probability of a power greater than $P$ in a given bin being $\exp(-P/2)$. This makes it straightforward to estimate the chance probability of a particular peak in a power density spectrum. In addition, the fractional root mean squared variability at frequency $\omega$ is $[P(\omega)/N]^{1/2}$.

The power density spectra of accreting black holes or neutron stars have many features analogous to an energy spectrum. Long stretches are often well-described by power laws (similar to a continuum spectrum), with occasional changes in the slope (analogous to edges) and sharper features (corresponding to spectral lines). An exactly periodic variation in the brightness (e.g., from a pulsar) would show up as a delta function for a long observation. In addition, there are a number of broader peaked features seen in the power density spectra. These are the quasi-periodic brightness oscillations, and are the subject of this review.

# 3 QPOs from Neutron Stars and Black Holes

We will focus on neutron stars or black holes accreting from low-mass stars, which have masses $\lesssim M_\odot$. Typical power density spectra are shown in Figure 1 for neutron stars and black holes. Only the high frequency portions are shown, because these have attracted the most theoretical interest. At first glance the power density spectra look similar, because both have a pair of high-frequency QPOs. However, there are important differences.

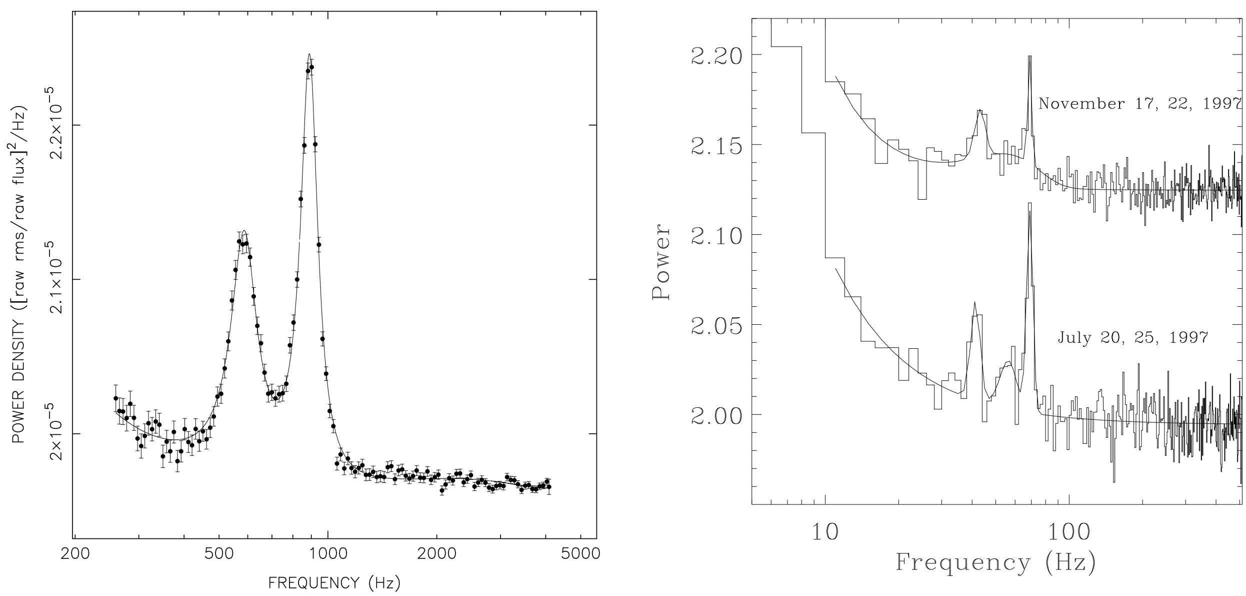


Figure 1: Sample high-frequency quasi-periodic oscillations from a neutron star (Sco X-1, left, from [5]) and a black hole (GRS 1915+105, right, summarized in [6]). Note that the neutron star power density has an arbitrary normalization. The figure for the black hole shows power density spectra from two different states, demonstrating the constancy of the QPO frequency. Neutron star QPOs are higher frequency, stronger, and sharper than black hole QPOs, and neutron star QPOs change significantly as the flux changes.

Neutron star high frequency QPOs:

- Have frequencies that change by up to a factor $\sim 2$. The frequencies tend to increase with increasing flux, although the short-term behavior can be complicated.

- Have a frequency separation $\Delta\nu$ between the pair that, in many sources, appears to be related to the spin frequency $\nu_{\rm spin}$ by $\Delta\nu \approx (\frac{1}{2} \text{ or } 1)\nu_{\rm spin}$. The frequency separation changes in several sources, hence this relation is not exact, but it is close enough in many sources to be striking (see Table 1).

- Are strong, with fractional rms amplitudes up to $\sim 15\%$ in the 2–60 keV energy band. The fractional amplitude tends to increase with increasing photon energy.

- Are sharp, with quality factors $Q \equiv \nu/FWHM$ up to $\sim 200$ in some sources, where $FWHM$ is the full width at half maximum for the QPO.

- Have no preferred frequency ratio (see Figure 2, for the ratio of QPO frequencies in the source Sco X-1).

- Are very high frequency, ranging between $\sim 500 - 1330$ Hz.

For a more detailed discussion of these and other properties, see [4].

Table 1: Spin frequency and frequency separation for neutron star low-mass X-ray binaries. 4U 1702, SAX J1808, and KS 1731 have single measurements of $\Delta\nu$, with uncertainties indicated.

| Source | $\nu_{\rm spin}$ (Hz) | $\Delta\nu$ (Hz) |
|---|---|---|
| 4U 1916–053 | 270 | 290–348 |
| 4U 1702–429 | 329 | 333±5 |
| 4U 1728–34 | 363 | 342–363 |
| SAX J1808–3658 | 401 | 195±6 |
| KS 1731–260 | 524 | 260±10 |
| 4U 1636–536 | 581 | 250–323 |
| 4U 1608–52 | 620 | 225–313 |

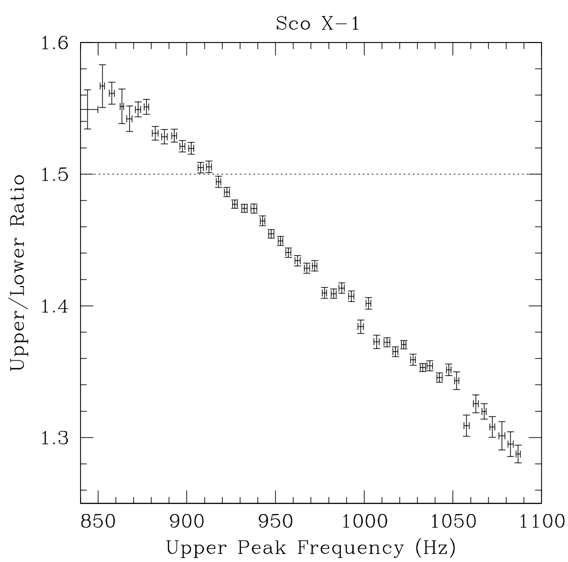


Figure 2: Ratio of the upper to lower QPO frequencies in Sco X-1, versus the frequency of the upper peak QPO. The dotted line marks a 3:2 ratio. Unlike in black hole sources, neutron star QPOs do not cluster around a 3:2 ratio. Data kindly provided by Mariano Méndez.

Black hole high frequency QPOs:

- Have essentially constant frequencies, changing by no more than a few percent even as the flux changes by almost an order of magnitude.
- Are weak, with fractional rms amplitudes typically $< 2\%$ in the 2–60 keV energy band. Like neutron star QPOs, the fractional amplitude increases with increasing energy.
- Are broad, with typical quality factors of only a few (the source GRS 1915+105 is the record holder, with $Q \approx 20$ in some observations).
- Have a frequency ratio that is close to 3:2 in several sources. However, the weakness and breadth of the QPOs means that the centroid frequency of a QPO has significant uncertainty, hence the ratio of the two QPO frequencies has large error bars (see Figure 3).
- Are lower frequency, ranging from $\sim 40 - 450$ Hz.

For more details on spectral and timing properties of black hole binaries, see [6]

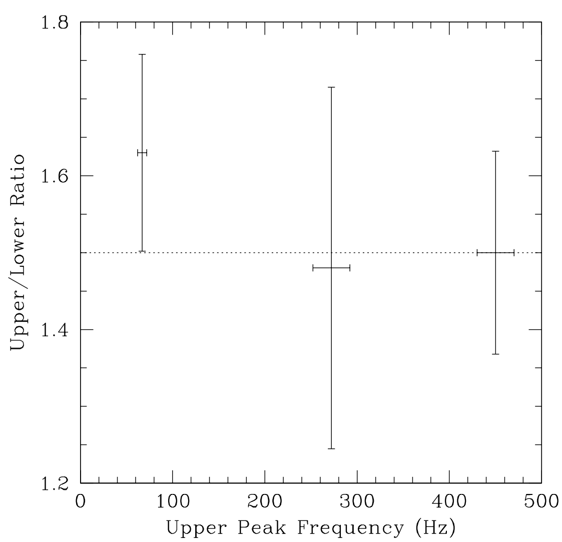


Figure 3: QPO frequency ratios and uncertainties for three black hole sources: GRS 1915+105 (left), XTE J1550–564 (middle), and GRO J1655–40 (right). In all cases the ratio is consistent with 3:2 (indicated by the dotted line), although the uncertainties are substantial and other ratios are also allowed. Note that, unlike for neutron star QPOs, the frequencies of black hole high-frequency QPOs do not vary significantly.

In addition to the QPOs seen during accretion-powered emission, many neutron stars in low-mass X-ray binaries also display nearly coherent oscillations (i.e., very sharp QPOs, with quality factors on the order of 1000 or more) during thermonuclear X-ray bursts (see Figure 4). Thermonuclear X-ray bursts, sometimes called type I bursts in the literature, occur because the hydrogen or helium (or sometimes, carbon) that has piled up on the surface because of accretion can become unstable to nuclear burning (see, e.g., [7] for a review of the nuclear physics of bursts). Briefly, when matter is at very high densities, fusion can happen due to quantum tunneling even if the temperature is low (this is called "pycnonuclear", or "cold", fusion to contrast it with thermonuclear fusion). The fusion releases energy, increasing the local temperature, which increases the fusion rate. If the rate of energy release increases faster than the cooling rate, there is a runaway and all of the fusable material fuses in a short time. If the material is primarily helium, the fusion happens so rapidly that the limiting timescale of the observed burst is the time needed for photons to diffuse from the burning layer (typically at a column depth of $\sim 10^8$ g cm$^{-2}$) to the surface, which is a few seconds. If the material is primarily hydrogen, then weak decay timescales are the limiting factor and the burst lasts longer.

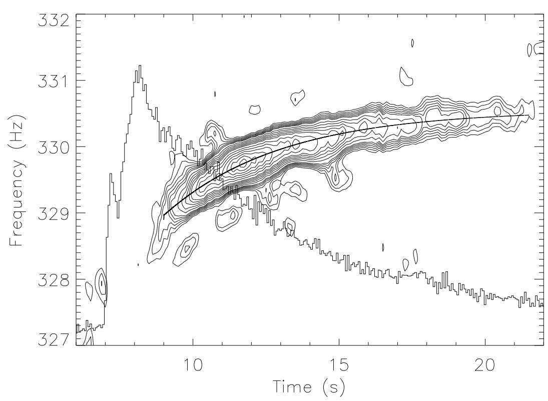


Figure 4: Thermonuclear burst from 4U 1702–429 with nearly coherent oscillations (from [8]). Shown are both the relative flux and the burst oscillation frequency as a function of time. This is a typical helium-dominated burst, lasting a few seconds, and it shows that although the frequency is close to constant throughout, there is a steady rise of a few Hertz.

Early analyses of this process assumed spherically symmetric burning for simplicity. However, it was pointed out that accretion onto the surface need not be spherically symmetric (e.g., if a magnetic field channels matter), hence the burning is at least a two-dimensional phenomenon. Indeed, when nearly coherent oscillations were discovered from many sources (see Table 2), they were interpreted almost immediately as "hot spots", whose observed flux was modulated at the stellar rotational frequency. This general idea is now universally believed, because bursts with oscillations were observed from the millisecond X-ray pulsars SAX J1808–3658 ([9],[10]) and XTE J1814–338 ([11]), and the burst oscillations are at the same frequency as the rotational frequency inferred from the persistent oscillations.

There are, however, some puzzles remaining. The frequency during a burst is not exactly constant, typically rising by 1–3 Hz to an asymptotic frequency (and occasionally reaching a maximum and then dropping; see [12]). In addition, it is not clear why the spot would remain hot and confined for the entire burst, rather than spreading rapidly over the star and reducing the amplitude of oscillations. For that matter, given that the inferred magnetic dipole moments of neutron stars in low-mass X-ray binaries are rather weak (equating to $\sim 10^{7-10}$ G at the stellar surface; see Appendix 1), it is not obvious that the accreting matter could ever be confined in a small region. Without confinement, the matter should spread over the surface within tenths of a second. Therefore, there are many details about the observed oscillations that are not understood, but there is at least a clear identification of the burst oscillation frequency with the stellar spin frequency.

Table 2: Spin frequencies of neutron stars in low-mass X-ray binaries. Most spin frequencies are inferred from thermonuclear burst brightness oscillations; the five persistent oscillators are marked.

| Source | $\nu_{\rm spin}$(Hz) | Persistent? |
|---|---|---|
| EXO 0748-676 | 45 | |
| XTE J0929-314 | 185 | Y |
| XTE J1807-294 | 191 | Y |
| 4U 1916-053 | 270 | |
| XTE J1814-338 | 314 | Y |
| 4U 1702-429 | 329 | |
| 4U 1728-34 | 363 | |
| SAX J1808-3658 | 401 | Y |
| SAX J1748.9-2021 | 410 | |
| XTE J1751-305 | 435 | Y |
| KS 1731-260 | 524 | |
| Aql X-1 | 549 | |
| MXB 1659-298 | 567 | |
| 4U 1636-536 | 581 | |
| MXB 1743-29 | 589 | |
| 4U 1608-52 | 620 | |

# 4 Models of QPOs

A natural starting point for QPO models is to determine what types of processes might produce frequencies in the observed range. We find that the fundamental source of the QPOs must be very close to the compact objects, hence the QPOs are informative about strong gravity and dense matter. This conclusion follows because only near a neutron star or black hole can any fundamental time scales (e.g., orbital, vibrational, free fall, etc.) be as short as milliseconds.

More specifically, one can consider the fundamental frequencies of a test particle in geodesic motion, i.e., under the influence of only gravity. If we consider such a particle in initially circular motion in the equatorial plane of a spinning compact object (but, of course, outside the object), then if the particle is perturbed one has the orbital frequency, the radial epicyclic frequency (i.e., the frequency to undergo a full cycle in radial motion), and the vertical epicyclic frequency (i.e., the frequency to undergo a full cycle in latitudinal motion). In Newtonian gravity, all these frequencies are degenerate, but deviations from a $1/r^2$ acceleration law and dragging of inertial frames cause them to separate when close to a rotating object. If the compact object is a black hole, then the external spacetime geometry is the Kerr spacetime, and the frequencies are

Orbital frequency:

$$2\pi\nu_K = (GM/r^3)^{1/2}\left[(1+j\hat{r}^{-3/2})\right]^{-1} \; . \tag{2}$$

Radial epicyclic frequency:

$$\nu_r = |\nu_K|\left(1-\hat{r}^{-1}+8j\hat{r}^{-3/2}-3j^2\hat{r}^{-2}\right)^{1/2} \; . \tag{3}$$

Vertical epicyclic frequency:

$$\nu_\theta = |\nu_K|\left(1-4j\hat{r}^{-3/2}+3j^2\hat{r}^{-2}\right)^{1/2} \; . \tag{4}$$

In these expressions, $\hat{r} \equiv rc^2/GM$ for radius $r$ and gravitational mass $M$ of the central object, and $j \equiv cJ/GM^2$ is the dimensionless angular momentum (where $J$ is the angular momentum itself). Black holes have $-1 \leq j \leq 1$; negative values indicate particles on retrograde orbits, and $j = 0$ is a nonrotating black hole, which is described with the Schwarzschild spacetime. Note that all frequencies are proportional to $1/M$ at fixed $\hat{r}$. Therefore, the low frequencies of black hole QPOs compared to neutron star QPOs could be a consequence of the higher masses of black holes. At the ISCO, $\nu_K \approx 2200$ Hz$(M_\odot/M)$ for $j \ll 1$.

These expressions are exact for black holes, but at order $j^2$ the spacetime geometry around a rotating neutron star deviates from these formulae (at lower orders, the spacetimes are identical; see [13]). In practice, however, known neutron stars in low-mass X-ray binaries are likely to have $j < 0.2$, so the Kerr expressions are probably sufficiently accurate for most purposes. See Figure 5 for a plot of these frequencies versus radius for $M = 10\,M_\odot$ and $j = 0.5$. Note that the orbital frequency is always the largest of the three, and that the radial epicyclic frequency vanishes at the ISCO, as it must because at that radius there is no restoring acceleration if the particle moves inwards.

Although these geodesic frequencies are interesting starting points for models, a more detailed picture requires some underlying physics that would select a particular frequency or radius. This is especially restrictive for neutron stars, where the QPO can be very narrow indeed. We now consider one particular class of NS QPOs, the beat frequency models. We do not intend to imply that this is the only model under consideration, but we want to give an example of how one constructs a detailed astrophysical model.

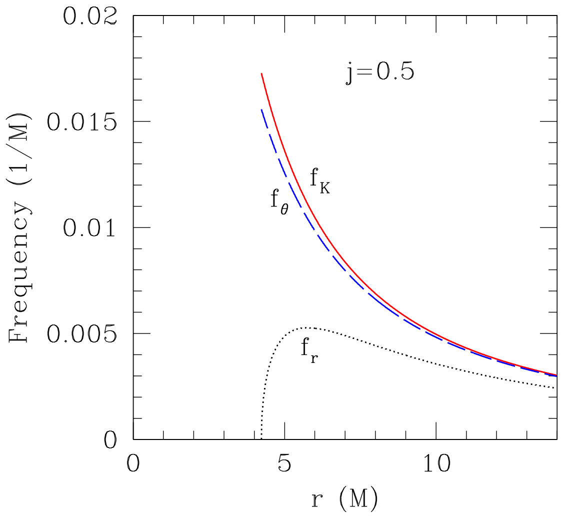


Figure 5: Orbital (solid line), vertical epicyclic (dashed line), and radial epicyclic (dotted line) frequencies versus radius for prograde orbits around a black hole with spin parameter $j = 0.5$. The radial epicyclic frequency becomes zero at the ISCO, which is $4.23\,GM/c^2$ for $j = 0.5$.

# 5 Beat Frequency Models for NS QPOs

Observations of neutron star QPOs are now rather detailed, thanks to the timing capability, collecting area, and flexible scheduling of the *Rossi* X-ray Timing Explorer (RXTE). More than twenty systems display a pair of high-frequency QPOs (called kilohertz QPOs in this context, thanks to their $\sim 500-1300$ Hz range). Many trends and exceptions to those trends have been discovered. In fact, there is so much information on these systems that no simple model can hope to fit every single bump and wiggle in every power density spectrum. It is therefore common to focus on especially important trends and model those.

One striking aspect of neutron star kHz QPOs is that the frequency separation $\Delta\nu$ between a pair of QPOs in a given source is often very close to either $\nu_{\rm spin}$ or $\nu_{\rm spin}/2$, where the spin frequency $\nu_{\rm spin}$ is inferred from either persistent pulsations during accretion-powered emission, or from the frequency of brightness oscillations during thermonuclear X-ray bursts. See Table 1 for several cases. On the other hand, $\Delta\nu$ is not exactly constant (see Table 1); it tends to decrease with increasing frequency, but by at most a few tens of Hertz, which is only a few percent of the QPO frequencies themselves.

How should this be interpreted? This is an example of how one must decide which aspects of the data are fundamental, and which are secondary. In my opinion, the near harmonic relation between $\Delta\nu$ and $\nu_{\rm spin}$ is a clear indication that the spin frequency is somehow communicated to the accretion flow. Pursuing this line of thought leads to beat frequency models.

To understand the principle behind beat frequency models, consider the hands of a clock. Suppose that the minute hand and the hour hand are wired in such a way that every time they are perfectly lined up, a bell sounds. How frequently will the bell sound? If the frequency of the minute hand is $\nu_{\rm minute}$, and of the hour hand is $\nu_{\rm hour}$, then the bell will sound with a frequency $\nu_{\rm bell} = \nu_{\rm minute} - \nu_{\rm hour}$. The negative sign arises because the minute and hour hands are turning in the same direction. In a mutant clock whose minute and hour hands turn in opposite directions, the frequencies would be added. In accreting neutron star systems, the torque applied by accretion is expected to at least roughly align the rotational axes of the star and disk, hence all such spins should be in approximately the same direction.

A key aspect of beat frequency models is that they generate only a single sideband frequency, rather than two frequencies. For example, consider an alternate model in which you have frequencies $\nu_1$ and $\nu_2$, producing an amplitude of the form

$$A = \cos(2\pi\nu_1)\cos(2\pi\nu_2)\ . \tag{5}$$

What frequencies does this generate? Given that

$$\cos(2\pi\nu_1)\cos(2\pi\nu_2) = \frac{1}{2}\cos[2\pi(\nu_1+\nu_2)] + \frac{1}{2}\cos[2\pi(\nu_1-\nu_2)]\ , \tag{6}$$

this modulation mechanism generates equally strong oscillations at $\nu_1 + \nu_2$ and $\nu_1 - \nu_2$. However, the two QPOs seen from neutron star sources are *not* equivalent. The often differ strongly in strength and width (see [4]). Therefore, it is thought that one frequency (the upper frequency, usually) is in some sense a fundamental frequency, and the other is produced by modulation of that fundamental frequency. Returning to the clock analogy, it would be as if a bell sounds every time the minute and hour hands line up, *and* when the minute hand hits 12.

To make this a more physically based model, it is necessary to answer at least two important questions. First, what sets the particular frequency of the upper QPO? Note that the mechanism has to be able to allow large changes in frequency in different source states, yet at any given time the QPO that is produced has to be sharp. Second, how can $\nu_{\rm spin}/2$ appear in the problem? There are many straightforward ways to involve $\nu_{\rm spin}$, but half the spin frequency is not so clear. We will now address these issues in order.

## 5.1 Candidates for sharp frequencies

Our first task is to establish a frequency that is very sharp, yet can change substantially depending on the properties of the accretion. The requirement of change means that the frequency cannot be associated solely with the star or with the properties of the external spacetime, because these change on timescales of millions of years, much longer than the hour timescales on which the frequencies are observed to change. Therefore, we have to focus on the gas in the accretion disk. In addition to the three epicyclic frequencies discussed above, we could also imagine global oscillation modes of the disk, or possibly local oscillation modes of some narrow annulus. As we will explore more thoroughly when we discuss black hole QPOs, global modes tend to be relatively independent of disk properties, hence are not ideal for our current purposes. The other possibilities (epicyclic frequencies or local modes) depend on the radius of the disk at which they are generated, therefore our search for a sharp frequency reduces to a search for a sharp radius.

What are candidates for a sharp radius? We need to think of places where the properties of the accretion disk undergo dramatic changes over a fractionally small range in radii. Examples are:

- The boundary layer where the disk plasma meets the star.
- The innermost stable circular orbit.
- The transition region where the flow becomes strongly affected by the stellar magnetic field.

The third option is a promising explanation for some of the lower-frequency QPOs seen in neutron star low-mass X-ray binaries. However, it is not sharp enough to explain the kilohertz QPOs. The ISCO is at a fixed radius, hence cannot produce changes in frequency. The boundary layer is also at a fixed radius. In principle one could imagine that as the vertical or radial thickness of the layer changes in response to accretion rate, the frequency could change as well, but it is difficult to believe that a narrow QPO could emerge from the boundary layer given that it should be rife with strong shocks and turbulence.

We are therefore driven to look for another radius. Miller, Lamb, & Psaltis [14] propose that a good candidate is the region in the flow where the inward radial velocity increases sharply with decreasing radius. Because this region usually corresponds to where the radial velocity goes from subsonic to supersonic, we adopted the term "sonic point" as a convenient descriptor. Note, however, that it is the rapid increase in radial speed, rather than whether the flow is subsonic or supersonic, that is the key here.

The sharp transition in radial speed occurs because of the effects of radiation drag. Consider an element of gas in the midplane of a geometrically thin but optically thick disk. No radiation from the star affects this gas, hence it spirals in slowly under the influence of magnetic transport (see [15] for a discussion). Close enough to the star, however, the radial optical depth from the surface of the star becomes small enough (an approximate value is $\tau \approx 5$) that some of the radiation from the star can reach the gas element without having scattered and isotropized in the disk. Observed neutron stars rotate more slowly (a maximum of $\sim 600$ Hz) than the orbital frequency near the star (typically $\sim 1000 - 1500$ Hz), and of course the gas is at a larger distance than the stellar radius. Therefore, the specific angular momentum of the radiation is less than that of the gas element. As a result, on average, when the radiation scatters off the gas element, the gas element loses angular momentum.

What happens then? A particle in a nearly-circular orbit that loses angular momentum will fall towards the star. This means that the radial velocity increases. By continuity, the increase in radial velocity causes a decrease in the density. The decrease in the density lowers the optical depth to the surface of the star, letting more low angular momentum radiation scatter and increasing the angular momentum loss rate of the matter. This speeds up the inward radial velocity of the matter, and the whole process is one of positive feedback. Miller et al. [14] find that quantities such as the radial speed and optical depth undergo large changes on length scales that can be $< 1\%$ of the radius, which is the sharpness required by the observed QPO widths.

Note that this mechanism provides a sharply defined radius that could be used as an input in more general classes of models, not just beat frequency models. For example, Stella & Vietri ([16]) have proposed that the QPOs are in fact direct reflections of epicyclic frequencies of test particles on geodesics. D. Psaltis and C. Norman (unpublished) suggest that these might really be oscillation modes of a thin annulus in a disk, and that the radius of the annulus could be the sonic point radius. In these models the relation $\Delta\nu \approx \nu_{\rm spin}$ or $\nu_{\rm spin}/2$ is not expected, hence we will not discuss them further here, but it is important to emphasize that the sharp transition expected at the sonic point is a general feature.

In beat frequency models, the final link is a proposed mechanism by which the orbital frequency at the sonic point is transformed into the upper QPO. Miller et al. [14] suggest that density enhancements ("clumps") in the flow are constantly being created and destroyed, but that a clump near the sonic point can have its gas spiral onto the stellar surface in a stream, which produces a bright spot on the surface. As this impact point moves around the star, its brightness modulations are seen by a distant observer. Miller et al. [14] show that the observed frequency of the spot is the orbital frequency at the sonic point, and *not* the stellar rotation frequency. Perhaps the easiest way to see this is to consider a nonrotating star, and imagine a satellite orbiting above it with someone pouring paint from the satellite. As the satellite orbits, the paint will not hit just a single point on the star, but will rather make a band around the entire star, at the orbital frequency of the satellite. Similarly, the gas from the clump will hit the star as the clump orbits, and the rotation of the star is essentially irrelevant for the generation of the upper QPO.

Production of the lower QPO in beat frequency models requires interaction with a second frequency. If the separation is $\Delta\nu \approx \nu_{\rm spin}$, several possibilities exist. In the original sonic point model paper, Miller et al. [14] suggested that interaction with the stellar magnetic field (which, of course, rotates at $\nu_{\rm spin}$) would do the trick. For example, when the clump sweeps past a magnetic maximum, it might lose gas at an enhanced rate. This would mean that the stream of gas from the clump has density maxima and minima, leading to modulation of the accretion rate and hence of the flux. This modulation would occur at roughly $\nu_{\rm sonic} - \nu_{\rm spin}$. The clump itself will drift in slowly as it orbits, and as [17] show this will produce some frequency changes that could explain why $\Delta\nu$ is not exactly constant.

However, detailed observations of the transient low-mass X-ray binary SAX J1808–3658, which has a known spin frequency of 401 Hz, show that the frequency separation is actually 200 Hz ([18],[19]). Therefore, new ideas are needed, which we now discuss.

## 5.2 Spin frequency resonance

At first glance, one might think it straightforward to involve half the spin frequency in the flow. A standard guess involves something related to geometry. For example, is it possible to have some two-fold symmetry on the star that leads to $\nu_{\rm spin}/2$?

No, it isn't. To see this, return to the clock analogy. Suppose that there are now two hour hands, one fixed at 12 and the other fixed at 6. The minute hand moves as normal, and a bell sounds when it aligns with either hour hand. The frequency is now $2\nu_{\rm minute}$. More generally, if there are two hour hands 180° apart that move at an angular frequency $\nu_{\rm hour}$, the observed frequency is simply $2(\nu_{\rm minute} - \nu_{\rm hour})$. $N$-fold symmetry produces $N$ times the fundamental beat frequency, so this is not a way to have $\nu_{\rm spin}/2$ appear.

There is, however, an elegant way that $\nu_{\rm spin}/2$ can affect the flow, and it involves a resonance. Consider an element of gas in the disk, and assume that this gas is essentially orbiting in a circle at frequency $\nu_{\rm orb}$. Suppose that, *as measured in the rest frame of the element*, the element is subjected to a vertical forcing term with frequency $\nu_{\rm force,rest}$. At most forcing frequencies, over many orbits the phases of the forcing will add incoherently, and the vertical motion will not be large. However, if the vertical motion has a natural frequency, then forcing at that frequency will lead to resonance and will drive substantial motion. Indeed, there is such a natural frequency. Again, as measured in the rest frame of the gas element (which is the orbital frame at that radius), we can consider the observed frequency of free vertical motion. This is just the vertical epicyclic frequency that we discussed in § 4. Thus, if the forcing frequency as measured in the orbital frame equals the vertical epicyclic frequency, a resonance occurs, so

$$\nu_{\rm vert} = \nu_{\rm force,rest}\ . \tag{7}$$

Let us now say that the forcing frequency is $\nu_{\rm force}$ *as measured in a global, nonrotating* *frame*. The frequency observed in the orbiting frame is then

$$\nu_{\rm force,rest} = \nu_{\rm force} - \nu_{\rm orb} \ . \tag{8}$$

Combining these two equations we have

$$\begin{aligned} \nu_{\rm vert} &= \nu_{\rm force} - \nu_{\rm orb} \\ \nu_{\rm vert} + \nu_{\rm orb} &= \nu_{\rm force} \ . \end{aligned} \tag{9}$$

From Figure 5 we see that the vertical epicyclic frequency is very close to the orbital frequency for all radii of interest. In addition, we will assume that the forcing frequency in the global frame is just the stellar spin frequency. Therefore, $\nu_{\rm vert} \approx \nu_{\rm orb}$, $\nu_{\rm force} \approx \nu_{\rm spin}$, and

$$\begin{aligned} 2\nu_{\rm orb} &\approx \nu_{\rm spin} \\ \nu_{\rm orb} &\approx \nu_{\rm spin}/2 \ . \end{aligned} \tag{10}$$

This effect is displayed in Figure 6 (from [20]), which shows a simple model of the oscillation. As predicted, the vertical excursions of gas in the disk occur most strongly at the radius giving $\nu_{\rm orb} = \nu_{\rm spin}/2$.

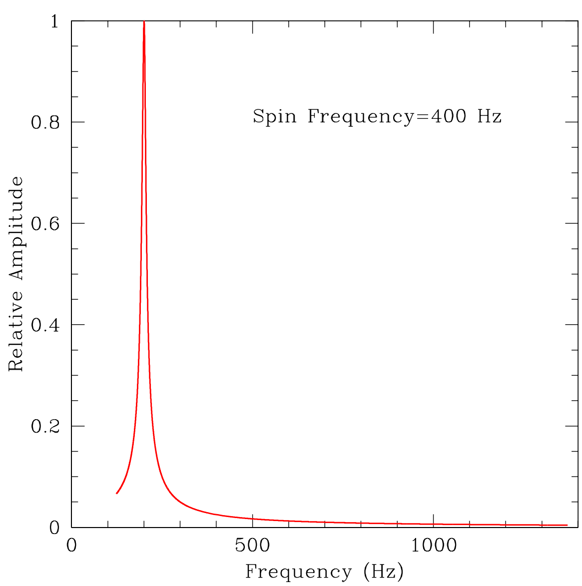


Figure 6: Simulated response of accretion disk to driving force at $\nu_{\rm spin}$ = 400 Hz. The resonance at $\nu_{\rm spin}/2$ produces a sharp response far stronger than at any other radius in the disk. Original figure and caption from [20].

Generically, then, one could imagine a flux modulation at $\nu_{\rm sonic}$ that, via a beat frequency mechanism, also produces a flux modulation at $\nu_{\rm sonic} - \nu_{\rm spin}/2$. For example, if the disk (or a particular gas element) has large vertical excursions at this particular radius, then reflection or obscuration of the flux at $\nu_{\rm sonic}$ by the raised portion of the disk could produce the required modulation at the beat frequency (see [20]).

There is, however, a complication. It is *not* always the case that $\Delta\nu \approx \nu_{\rm spin}/2$. There are several well-observed sources for which $\Delta\nu \approx \nu_{\rm spin}$. Perhaps in such cases some of the previously considered mechanisms operate, but it turns out that the spin-resonance picture described above can also explain separation by the spin frequency.

To see this, let us contrast a situation in which there are very few clumps at the spin resonance radius (i.e., where $\nu_{\rm orb} \approx \nu_{\rm spin}/2$), so this is rough flow, with a case in which there are many clumps at the spin resonance radius, i.e., comparatively smooth flow.

First, consider a single clump, which orbits at $\approx \nu_{\rm spin}/2$. For specificity, let us assume that the driving force is magnetic in origin. That is, we suppose that when the magnetic maximum is aligned with the orbital phase of the clump, there is a downward push. When a system is driven by a periodic force, it rapidly becomes in phase with the driving force. Therefore, the clump will be at its vertical maximum when its orbital phase is aligned with that of the magnetic maximum. However, since there is only a single clump, the main modulation relevant to the beat frequency mechanism is simply its orbital motion, hence the dominant difference frequency is $\nu_{\rm spin}/2$.

Now imagine a very large number of clumps. Each of them orbits at $\approx \nu_{\rm spin}/2$. In addition, each of them is phased with the driving force, therefore each of them reaches its vertical maximum just when its orbital phase is aligned with the magnetic maximum. As a result, at any given instant, the greatest vertical extent at that radius is at the location of the magnetic maximum. Therefore, the vertical maximum tracks the magnetic maximum, which is simply another way of saying that the vertical maximum has a frequency equal to $\nu_{\rm spin}$. Note that this is so even though no *single* clump moves at $\nu_{\rm spin}$. The *pattern* speed moves at $\nu_{\rm spin}$, twice as fast as any individual element of gas at that radius. Similarly, in a water wave, individual fluid elements undergo circular motion even as the wave itself sweeps into shore.

If the number of clumps is large (i.e., if the flow is comparatively smooth), then it is this pattern speed that is most important in the beat frequency mechanism (just as one perceives a water wave rather than the motions of individual fluid elements). Figure 7, from [20], shows the results of a simple computation of this effect. With a small number of clumps the individual motion dominates, whereas with a large number the pattern dominates. In this way, one can explain observations of $\Delta\nu \approx \nu_{\rm spin}/2$ and $\Delta\nu \approx \nu_{\rm spin}$ in a single model.

In summary, it should be emphasized again that this is a detailed description of a single class of models. There is no universal agreement about the cause of the kilohertz QPOs in neutron stars. Various people have suggested ideas about geodesic frequencies ([16]), disk resonances ([21]), disk modes ([22]) and photon bubble interactions at the stellar surface ([23]). However, in my opinion, the link between $\Delta\nu$ and $\nu_{\rm spin}$ makes beat frequency models the most promising at this point, which is why I have focused on them.

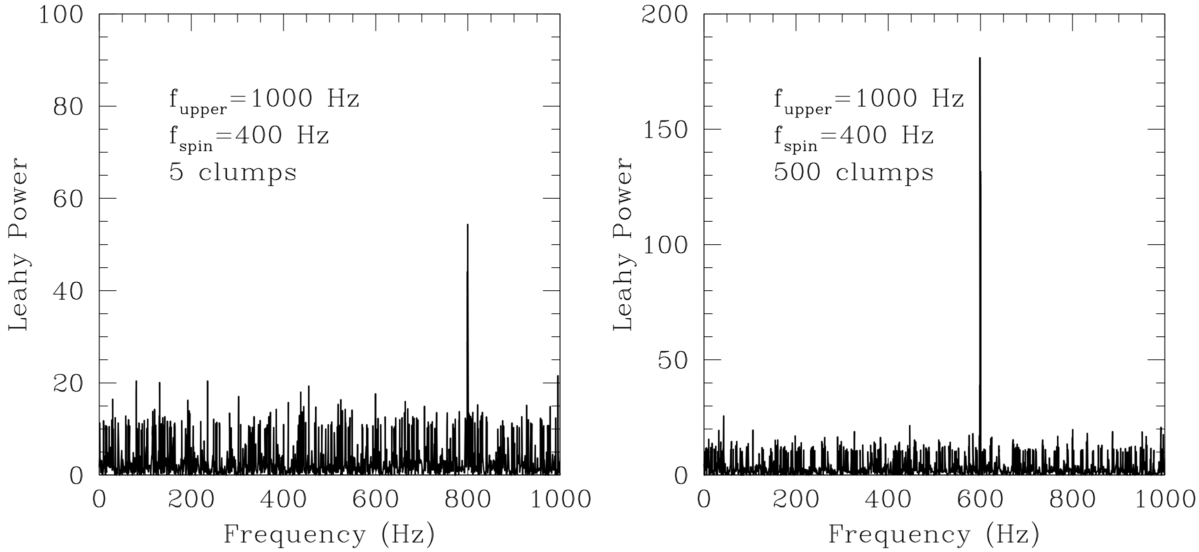


Figure 7: Simulated power spectra for disks containing a small number of clumps at the resonance radius (left panel) and a large number of clumps at the resonance radius (right panel). Here $\nu_{\rm sonic} = 1000$ Hz is the orbital frequency at the sonic radius, and $\nu_{\rm spin} = 400$ Hz. These power spectra demonstrate that when the flow is clumpy, the most prominent beat frequency is the one with the motion of individual clumps, at $\nu_{\rm spin}/2$. When the flow is smoother, however, it is the beat with the pattern frequency $\nu_{\rm spin}$ that is the strongest. This simulation does not include any signal at $\nu_{\rm sonic}$. Original figure and caption are from [20].

# 6 Models of Black Hole QPOs

There is even less agreement about the physical cause of high-frequency black hole QPOs than there is for neutron star QPOs. This is true for two reasons. First, the observations are not so easy, because the QPOs are weaker and broader than they are in the kilohertz QPOs, hence fine details of their phenomenology are tougher to establish. Second, unlike in the neutron star case, there does not exist a single frequency (such as the spin frequency) whose origin we know, thus there is no fixed point from which models can spin off. Nonetheless, theory and observations have suggested certain classes of models.

The stability of the high-frequency QPOs has led to two main categories of models:

- Global modes in the disk.
- Resonant interactions between characteristic frequencies.

*Global oscillation modes.*— Global modes have been analyzed on a hydrodynamic background (no magnetic fields) in spacetimes that are pseudo-Newtonian [24]-[28], Schwarzschild

[29], and Kerr [30]-[36]. The basic idea is that a geometrically thin disk has certain characteristic oscillation modes, and that these could affect the overall energy release and/or the properties of the corona in a way consistent with the observations of QPOs. The modes are divided into three types. The g-modes have gravity as their primary restoring force. The p-modes have pressure gradients as their main restoring force. The c-modes, a new type of mode compared to more familiar modes in spherically symmetric systems such as stars, are nonaxisymmetric twisting or corrugation modes of geometrically thin disks.

The appeal of these ideas is that they provide a natural physical mechanism by which a particular frequency is selected. As pointed out in the references above, the maximum in the radial epicyclic frequency curve implies that radial modes can be confined in a cavity, with a given frequency. The mode frequencies depend only very weakly on the accretion rate, through pressure gradient effects (e.g., [37]). This may be consistent with the slight variations in frequency seen in, e.g., GRS 1915+105 [6].

These models, however, have important observational and theoretical question marks. In the standard disk mode picture, it is expected that there will be multiple modes with different frequencies, but, at least in the linear treatment, the 3:2 relation is not recovered naturally. Therefore, observation of such relations in several sources would have to be seen as coincidental in this picture, although it is possible that non-linear interactions naturally couple the mode frequencies into a harmonic relationship (see [38] for a recent idea). In addition, there is the theoretical concern that the perturbation analyses used to derive the mode properties assume a smooth hydrodynamical background without magnetic fields. It has been understood for well over a decade that magnetic fields play an essential role in transporting angular momentum in ionized accretion disks, through the magnetorotational instability (see, e.g., [15] for a review). A consequence of this is that the disk is highly turbulent. It may therefore be that the hydrodynamical modes do not exist at all in a magnetohydrodynamic disk.

*Resonance ideas.*— A different approach has been to take the 3:2 frequency relations as fundamental. One suggestion is that the fundamental epicyclic frequencies interact with each other in a way similar to a parametric resonance [21]. If the frequencies are coupled properly, this would naturally produce high amplitudes in epicyclic motion when the frequencies were in a 3:2 relation. However, there is at present no consensus on a physical model that produces this behavior. That is, although one can set up a mathematical model that generates a parametric resonance [21], there is not yet a clear candidate for the physical coupling (see [38] for ideas related to warps). Such a candidate is needed because true geodesic orbits (i.e., orbits under only the influence of gravity) do not experience any resonance; the epicyclic motions are essentially independent of each other. The most straightforward candidates for coupling do not work as needed. For example, magnetic coupling between fluid elements leads to the magnetorotational instability rather than a parametric resonance, and pressure gradient forces tend to lead to shocks and damping rather than amplification of motion. In particular, [39] show that substantial radial epicyclic motion is not expected to last for many cycles, due to dissipation in shocks. In no way does this rule out the general idea, but it does show that at a minimum there is a major piece of the puzzle missing.

An alternate idea is to consider global modes in a setting that will naturally produce small integer ratios of frequencies. Rezzolla and colleagues have investigated modes in a torus with a nearly constant angular momentum profile ([40],[41]). As expected, the natural mode frequencies are integer multiples of each other. It is, however, necessary for this model that the modes be confined to a region with well-defined boundaries. It is not clear whether such a situation can be set up in reality (one possibility is a region near the ISCO that is a fattened disk), and especially whether such modes could survive in the presence of turbulence. In particular, the tori considered by Rezzolla et al. have constant or very nearly constant angular momentum profiles, and hence are close to instability on a dynamical timescale by the powerful Papaloizou-Pringle instability [42].

In summary, there are a number of analytic and semianalytic ideas for the causes of black hole QPOs, but there is no consensus and every proposed idea has potentially major hurdles to overcome before it can be considered physically realistic.

As discussed above, the orbital frequency at the ISCO, which is $220(10\,M_\odot/M)$ Hz for a nonrotating black hole, is the theoretical maximum for a reasonably sharp fundamental QPO (although it is possible for the higher harmonics to have frequencies significantly greater than $\nu_{\rm K}$). Interestingly, the higher-frequency of the two QPOs in GRO J1655-40 has a frequency, 450 Hz, that is larger than the maximum of ≈360 Hz allowed for a nonrotating black hole of the measured mass ($6.0 - 6.6\,M_\odot$; see [43] for a catalog of black hole masses). It has therefore been argued that the very existence of this QPO argues strongly in favor of frame-dragging effects [44]. However, there is also a 300 Hz oscillation from this source, leading to speculation that the fundamental frequency is actually 150 Hz (see [6] for a summary). In addition, numerical simulations display significant variability even well above the orbital frequency at the ISCO. The claim of evidence for frame-dragging therefore must wait for the understanding of these systems that will come from numerical simulations.

# 7 Neutron Star QPOs as Probes

I have given a summary of one particular model of the kilohertz QPOs found in many accreting neutron star systems, in which the upper QPO frequency is close to an orbital frequency. There are certainly others, the most prominent of which have assumed that the upper QPO is a vertical epicyclic frequency and the lower QPO is a radial epicyclic frequency at the same radius, with the radius determined by unknown factors or possibly by a resonance. In either case, however, it turns out that one can use these QPOs to probe dense matter and strong gravity.

The key is that, as seen in Figure 5, the orbital frequency and the vertical epicyclic frequency are very close to each other for realistic stellar angular momenta. For both, it is clear that the radius at which the QPOs are generated must be (1) outside the star, and (2) outside the innermost stable circular orbit, because otherwise the oscillations would last at most a few cycles and would thus not have the observed high quality factor. To first order in the dimensionless angular momentum $j = cJ/GM^2$, these two constraints turn out to yield upper limits on both the stellar gravitational mass and the stellar radius [14]:

$$\begin{aligned} M_{\rm max} &= 2.2\ M_\odot(1000\ {\rm Hz}/\nu_{\rm max})(1+0.75j) \\ R_{\rm max} &= 19.5\ {\rm km}(1000\ {\rm Hz}/\nu_{\rm max})(1+0.2j)\ . \end{aligned} \tag{11}$$

Here $\nu_{\rm max}$ is the highest frequency QPO observed from a particular source. More generally, the $r > R$ and $r > R_{\rm ISCO}$ limits yield a maximum $M$ and a maximum radius as a function of $M$. The highest frequency conclusively observed from any source is $\nu_{\rm QPO} = 1330$ Hz, from 4U 0614+091 [45]. Assuming that the equation of state of neutron star matter is the same for all neutron stars, the highest-frequency source therefore provides the most stringent constraints on the masses and radii, and one can in principle eliminate some equations of state from consideration if they predict too large a radius for a given mass. See Figure 8 for the current constraints based on the 1330 Hz QPO, compared with the mass-radius relations that follow from several candidate equations of state.

If one could be confident that a particular QPO is generated very near to the ISCO, then one would immediately have a good estimate of the mass of the star. More importantly, one would have evidence for a prediction of general relativity (the existence of unstable orbits) that has no parallel in Newtonian gravity. Such a discovery would therefore have tremendous importance, hence it would need to be subjected to a rigorous critical examination.

What might constitute evidence for the ISCO? We know that the frequencies of the QPOs tend to increase with increasing X-ray countrate. However, we also know that the maximum QPO frequency is roughly the orbital frequency at the ISCO. Therefore, a frequency versus countrate behavior that has the usual steep increase followed by a flattening at some frequency, seen reproducibly at a particular frequency for a given source, could be a signature of the ISCO. Similarly, a sharp amplitude or coherence drop at a fixed frequency for a given source might be caused by the ISCO.

Indeed, frequency flattening was reported for the source 4U 1820–30 at 1060 Hz [50]. If

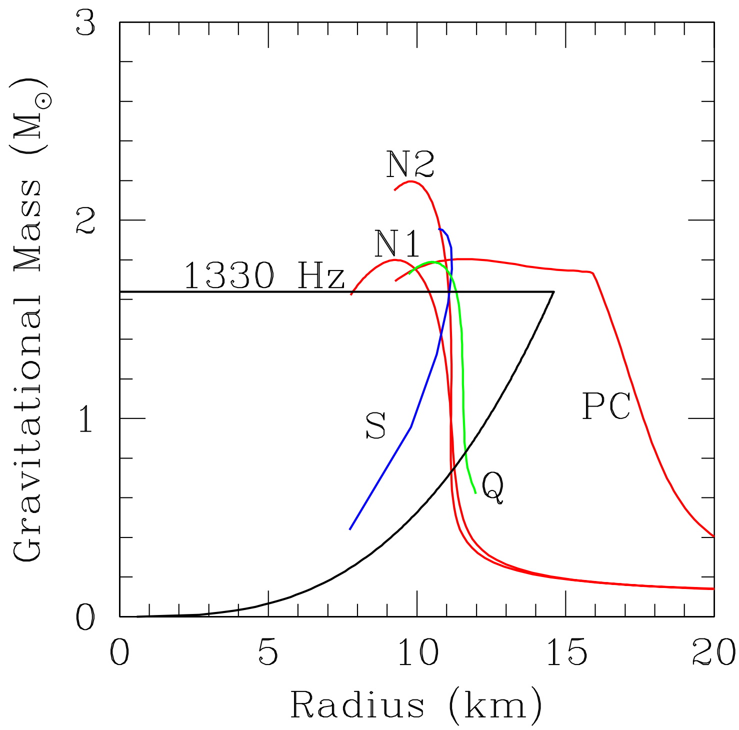


Figure 8: Constraints from orbital frequencies. The 1330 Hz curve is for the highest kilohertz quasi-periodic oscillation frequency yet measured (for 4U 0614+091 [45]). This curve is for a nonrotating star; the constraint wedge would be enlarged slightly for a rotating star (see [14]). The solid lines are mass-radius curves for different representative high-density equations of state. The mass-radius curves are all for equilibrium nonrotating stars; note that rotation only affects these curves to second order and higher. Curves N1 and N2 are for nucleonic equations of state; N1 is relatively soft [46], whereas N2 includes significant three-body repulsion [47]. PC has a sharp change to a Bose-Einstein condensate of pions in the core when the mass reaches $\approx 1.8\,M_\odot$ [48]. Equations of state N1, N2, and PC are not modern (i.e., not fitted to the most current nuclear scattering data), but are included for easy comparison to previous work on equation of state constraints. Curve S is for a strange star equation of state [49]. Curve Q is for a quark matter equation of state with a Gaussian form factor and a diquark condensate (kindly provided by David Blaschke and Hovik Gregorian). Original figure and caption from my contribution to the proceedings of the X-ray Timing 2003 meeting in Cambridge, MA, USA.

this was indeed caused by the ISCO, it would imply a mass of $2.2\,M_\odot$, which would have major implications for the properties of high density matter. In particular, it would suggest that such matter is restricted to nucleonic degrees of freedom (rather than quark or hyperonic degrees of freedom), and that three-nucleon repulsion is significant. However, further examination showed that at the point of flattening the source underwent a state change that could mimic this behavior, hence there is insufficient evidence that this was caused by an approach to the ISCO. As we discuss in the next section, we anticipate that higher-area timing instruments will be sensitive to higher-frequency QPOs, allowing an excellent chance of catching systems with QPOs near their innermost stable circular orbits.

# 8 Prospects With a Higher-Area Instrument

All of the preceding discoveries were made using the *Rossi* X-ray Timing Explorer, which has a maximum collecting area of 6000 $\mathrm{cm}^2$. What advances could be expected with a follow-up instrument with significantly higher area, hence greater sensitivity?

Phenomenologically, we know that for neutron star low-mass X-ray binary sources, $\nu_{\rm QPO}$ increases but the amplitude decreases as the countrate goes up. Eventually, therefore, the QPO simply sinks into the noise. However, since the countrate is still measurable and still is seen to increase, we can project that either the frequency will continue to go up or there will be a rollover due to, e.g., effects of the ISCO. Michiel van der Klis (Astronomical Institute "Anton Pannekoek", Amsterdam) has estimated based on the frequency and amplitude trends that a 10 $\mathrm{m}^2$ instrument would therefore see QPOs with frequencies $\gtrsim 1500$ Hz in sources such as 4U 0614+091. This is important, because $\nu_{\rm ISCO} < 1500$ Hz for $M > 1.6\,M_\odot$; note that neutron stars in low-mass X-ray binaries are expected to have accreted a few tenths of a solar mass, therefore this is in the right range to detect signatures of the ISCO. Such high frequencies would also strengthen constraints on the masses and radii of neutron stars (see Figure 9 for the mass-radius constraint wedge that would follow from $\nu_{\rm QPO} = 1500$ Hz).

As pointed out by van der Klis (personal communication), a higher area instrument would allow another qualitative advance. Currently, even for the relatively strong neutron star QPOs, it is impossible to detect a QPO within a coherence time (that is, within $\nu/FWHM$ cycles, which is the time over which an oscillation maintains approximate phase coherence). Instead, it has to be averaged over many coherence times. It is therefore uncertain whether, for example, the QPO is present at all times or whether it is a superposition of more coherent oscillations. With a 10 $\mathrm{m}^2$ instrument, it would be possible to detect a bright source such as Sco X-1 within 0.004 s in some states, easily resolving it in a coherence time.

The key advance facilitated by such detections is that they would allow the analysis of phase-resolved continuum spectra. If, for example, the QPOs are produced by discrete "clumps", one would therefore be able to detect Doppler shifts as the clumps orbit. This

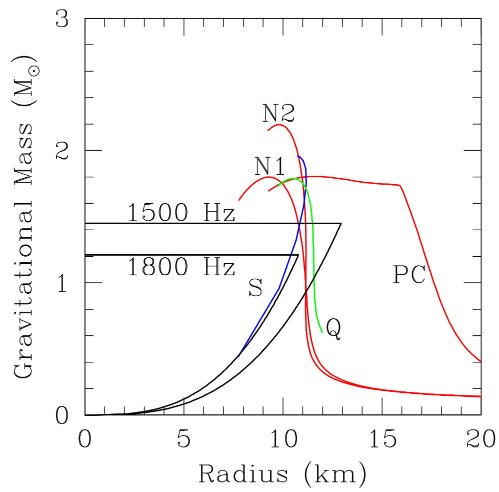


Figure 9: Constraints on mass and radius for hypothetical detections of a 1500 Hz QPO and a 1800 Hz QPO, if they are identified with an orbital frequency. At 1500 Hz one expects signatures of the ISCO to be present; a detection of 1800 Hz would present strong difficulties for standard nucleonic equations of state. The equation of state curves are as in Figure 8. Rotational effects are not included in this figure. Original figure and caption are from my contribution to the proceedings of the X-ray Timing 2003 meeting in Cambridge, MA, USA.

would provide a critical piece of information that is currently impossible to obtain. That is, the observed frequencies are basically $\sim (GM/r^3)^{1/2}$, but $M$ and $r$ are not known independently. Doppler shifts would provide information about the speed, which is $\sim (M/r)^{1/2}$, so that, modulo an uncertain line of sight inclination, one could determine $M$ and $r$ directly. Combined with inclination estimates from optical observations of the binary system, this would produce the very first reliable measurements of the masses of neutron stars in accreting low-mass X-ray binaries, which would then allow much more precise constraints on radius and hence on the equation of state of neutron star matter.

# 9 Conclusions

In the last decade there have been enormous advances in the study of QPOs from neutron stars and black holes. The high frequency QPOs were unknown and unsuspected prior to the launch of RXTE, but now there are many sources and multiple well-established phenomenological trends. With current data, we are just short of the expected signatures of the innermost stable orbit, but it is expected that a future high-area timing mission will see this and allow qualitatively new explorations of strong gravity and dense matter.

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**Appendix 1: Basic Compact Object Astrophysics**

A neutron star or black hole is most easily observed when it is accreting matter. For stellar-mass compact objects, this occurs at a high rate only when the object is in a close binary system and matter flows from the companion star in either a wind or as part of Roche-lobe overflow. The luminosity produced by an accretion rate $\dot{M}$ depends on the accretion efficiency $\epsilon$ via $L = \epsilon \dot{M} c^2$. For matter falling onto a neutron star surface, $\epsilon \approx 0.2 - 0.3$. For matter spiraling into a black hole, a usual assumption is $\epsilon \approx 0.1$. The basis for this is that in general relativity, circular test particle orbits are unstable inside of the so-called innermost stable circular orbit. Heuristically, one can understand this based on the idea that gravity becomes stronger more rapidly with decreasing radius in general relativity than it does in Newtonian theory. A particle in a circular orbit at some radius therefore has to have a larger angular momentum than it would in Newtonian gravity, and in fact the specific angular momentum reaches a minimum, then increases again at smaller radii. This implies instability, as one can see by considering a small perturbation of such an orbit; a perturbation inwards leaves the particle with less angular momentum than it needs for a circular orbit, hence the particle simply spirals inwards rather than undergoing an elliptical orbit.

For a nonrotating black hole, the radius of the ISCO is $6GM/c^2 \approx 9$ km$(M/M_\odot)$, where $M$ is the mass of the object and $M_\odot = 1.989 \times 10^{33}$ g is the mass of the Sun. It is commonly assumed that when gas spirals inside the ISCO, it goes into the event horizon without further loss of energy (this need not be true if magnetic effects are important; see [51]). For a nonrotating black hole, the specific binding energy (i.e., per unit mass) at the ISCO is $0.057c^2$, and for a rotating black hole, the specific binding energy is higher for a prograde orbit, hence the approximation $\epsilon \approx 0.1$.

Accretion-powered sources are limited in their luminosity, because beyond some critical luminosity the outward radial force of radiation balances gravity and shuts off the accretion. For quasi-spherical emission and accretion of fully ionized hydrogen, this critical luminosity is the *Eddington luminosity*:

$$L_E = 4\pi G M m_p c/\sigma_T \approx 1.3 \times 10^{38} (M/M_\odot) \text{ erg s}^{-1} . \quad (12)$$

Here $m_p$ is the mass of a proton and $\sigma_T$ is the Thomson scattering cross section (which is relevant if electron scattering dominates the opacity, as it typically does for accreting stellar-mass compact objects).

The matter flowing onto the compact object from the companion has non-negligible angular momentum (from the binary orbit, if from nothing else), hence it tends to spiral in as a relatively flat "accretion disk". One well-used model assumes that each annulus of the disk radiates as a blackbody with some radius-dependent temperature $T(r)$. To get an order of magnitude estimate of the temperature near the inner portion of the disk, note that because the energy comes from the gravitational potential, most of the energy is released fairly close to the central object. Suppose that the luminosity is a fraction $\eta$ of the Eddington luminosity, and that the radiating region has a radius equal to that of the ISCO (in reality most of the emission will be *outside* the ISCO, but this works for a quick estimate). Then in cgs units the blackbody equation is

$$\begin{aligned} L &= \sigma_B(\pi R^2)T^4 \\ 1.3\times 10^{38}\eta(M/M_\odot) &\approx 1.3\times 10^{8}(M/M_\odot)^2 T^4 \\ T &\approx 3\times 10^{7}\eta^{1/4}(M/M_\odot)^{-1/4}\ \mathrm{K}\ . \end{aligned} \tag{13}$$

Therefore, objects of a few solar masses accreting near the Eddington rate have temperatures in the millions or tens of millions of Kelvin.

The high temperature means that matter will be substantially ionized, and capable of coupling to any magnetic fields present. Black holes have no intrinsic magnetic fields, hence the only relevent fields are those in the accretion disk itself. In fact, the interaction of disk fields with the accreting matter is of fundamental importance, and it turns out that it is the means by which angular momentum is transported (see [15]), and it also generates turbulence.

Neutron stars do have an intrinsic magnetic field, and this field can couple strongly enough to the disk plasma to force the plasma to follow field lines near the star. In fact, generally there is a region very far from the star where the plasma stresses dominate, a region close to the star where the magnetic stresses dominate, and a transition region in between. This is seen simply by considering the radial dependences of the stresses. For a dipolar magnetic field, $B \sim r^{-3}$ and the magnetic stress scales as $B^2 \sim r^{-6}$. Material stresses scale as $\rho v^2$, where $\rho$ is the density and $v$ is the speed of the flow (the nonrelativistic approximation is good even fairly near the neutron star). The speed scales as $v \sim r^{-1/2}$, where we assume that the motion is mainly orbital. The density depends on the disk half-thickness profile as well as the radial speed. If we assume that the disk half-thickness $h$ is proportional to $r$ and that the radial speed is a fixed fraction of the orbital speed, then the equation of continuity

$$\rho A v_r = \rho 4\pi h r v_r \propto \rho r^2 v_r = \mathrm{const} \tag{14}$$

implies $\rho \sim r^{-3/2}$, so that the plasma stresses scale as $r^{-5/2}$. This is a much weaker dependence on radius than the dependence of the magnetic stresses, hence magnetic stresses become more important near the star.

Balancing these stresses leads to an "Alfvén radius" of

$$r_A \approx 3\times 10^{6}\ \mathrm{cm}(\dot{M}/10^{16}\ \mathrm{g\ s^{-1}})^{-2/7}(B/10^{8}\ \mathrm{G})^{4/7}\ , \tag{15}$$

although careful consideration of the $r\phi$ components of the flow suggests that the effective radius at which torque is applied could be a factor of $\sim 2-3$ less than this [52]. Here $B$ is the surface dipole magnetic field strength; multipolar components, or squeezing of the field if the torque radius is small enough, will modify this expression. Given enough time, an accreting neutron star will come to a torque balance, at a spin frequency equal to the orbital frequency at the torque radius. For the neutron stars in low-mass X-ray binaries, the spin frequencies (see Table 2) and other observations suggest $B \sim 10^{7-10}$ G, although direct measures do not exist. Neutron stars in high-mass X-ray binaries (i.e., with companions of at least several solar masses) appear to have fields $\sim 10^{12}$ G, similar to those of isolated pulsars. Although many ideas have been suggested for why the field strengths are so different, at this point there is no consensus.

### Appendix 2: Sample Exercises

1. Derive the expression for the Eddington critical luminosity far from a compact object of mass $M$. Assume spherical radiation. Remember that the Eddington luminosity is the luminosity at which the outward radiation force exactly balance gravity, hence you should simple compute the radiative and Newtonian gravitational accelerations and balance them. Take the standard assumption that one has fully ionized hydrogen, so that radiation interacts with the electron via Thomson scattering (cross section $\sigma_T = 6.65 \times 10^{-25}$ cm$^2$), but that the effective mass is the mass of the proton ($m_p = 1.67 \times 10^{-24}$ g). **For extra thought:** the gravitational acceleration near a compact object differs from the Newtonian value. Does this change the maximum luminosity observable at a very large distance? Think carefully; this is tricky.

2. This problem will reproduce the classic Landau derivation of the Chandrasekhar critical mass for compact objects.

a. Using the uncertainty principle, estimate the Fermi momentum of particles of number density $n$. The important thing here is dependences, not constant factors in front.

b. From part a, what are the approximate Fermi energies in the nonrelativistic and relativistic limits?

c. Consider a star of mass $M$ and radius $R$ that is composed primarily of baryons of mass $m_B$. Within factors of order unity, compute the total energy per baryon in the star (i.e., the gravitational potential energy plus the Fermi energy) in the nonrelativistic and relativistic limits.

d. Based on energy minimization considerations, show that in the nonrelativistic limit there will always be an equilibrium radius for the star, whereas in the extreme relativistic limit there is a maximum mass beyond which the star is unstable. Calculate this maximum mass, as a function of Planck's constant, G, c, and $m_B$. What is the value in solar masses if $m_B = 1.7 \times 10^{-24}$ g, the mass of a nucleon?

**For extra thought:** what are the implied radii if the degeneracy pressure is provided by electrons (as in a white dwarf), versus neutrons (as in a neutron star)? You can also do the problem more carefully, to find the dependence of the maximum mass on the number of baryons per degenerate particle; you should find that the maximum mass you get for a neutron star is much greater than the actual maximum mass. What is the main physical effect that we've left out, that explains the difference?

3. For a nonrotating star ($j = 0$), show how the conditions that the orbital radius be greater than the stellar radius and the radius of the ISCO results in the mass and radius constraints quoted in § 7.

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