

# Testing the rotating hotspot model using X-ray burst oscillations from 4U 1636–536

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## ABSTRACT

Precise and accurate measurements of neutron star masses and radii would provide valuable information about the still uncertain properties of cold matter at supranuclear densities. One promising approach to making such measurements involves an analysis of the X-ray flux oscillations often seen during thermonuclear (type 1) X-ray bursts. These oscillations are almost certainly produced by emission from hotter regions on the stellar surface modulated by the rotation of the star. One consequence of the rotation is that the oscillation should appear earlier at higher photon energies than at lower energies. Ford found compelling evidence for such a hard lead in the tail oscillations of one type 1 burst from Aql X-1. Subsequently, Muno, Özel & Chakrabarty analysed oscillations in the tails of type 1 bursts observed using the *Rossini X-ray Timing Explorer*. They found significant evidence for variation of the oscillation phase with energy in 13 of the 51 oscillation trains they analysed and an apparent linear trend of the phase with energy in six of nine average oscillation profiles produced by folding the energy-resolved oscillation waveforms from five stars and then averaging them in groups. In four of these nine averaged energy-resolved profiles, the oscillation appeared to arrive earlier at lower energies than at higher energies. Such a trend is inconsistent with a simple rotating hotspot model of the burst oscillations and, if confirmed, would mean that this model cannot be used to constrain the masses and radii of these stars and would raise questions about its applicability to other stars. We have therefore re-analysed individually the oscillations observed in the tails of the four type 1 bursts from 4U 1636–536 that, when averaged, provided the strongest evidence for a soft lead in the analysis by Muno et al. We have also analysed the oscillation observed during the superburst from this star. We find that the data from these five bursts, treated both individually and jointly, are fully consistent with a rotating hotspot model. Unfortunately, the uncertainties in these data are too large to provide interesting constraints on the mass and radius of this star.

**Key words:** equation of state – relativistic processes – stars: neutron – X-rays: binaries – X-rays: bursts.

## 1 INTRODUCTION

The central regions of neutron stars contain highly degenerate matter with densities up to several times nuclear saturation density. The properties of this matter cannot be studied in terrestrial lab-

oratories but can be probed by determining the masses and radii of neutron stars (see e.g. Lattimer & Prakash 2007). One method proposed to obtain this information involves fitting detailed models to the waveforms of the oscillations often seen during type 1 (thermonuclear) X-ray bursts (see e.g. Strohmayer 1992, 2004; Miller & Lamb 1998; Braje, Romani & Rauch 2000; Weinberg, Miller & Lamb 2001; Nath, Strohmayer & Swank 2002; Bhattacharyya et al. 2005; Cadeau, Leahy & Morsink 2005; Cadeau et al. 2007;

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Bogdanov 2013; see Watts 2012 for a recent overview of the properties of these bursts). These oscillations are thought to be produced by rotational modulation of the emission from a hotspot that rotates at or close to the stellar spin frequency (Strohmayer et al. 1996). The amplitude of the oscillation and its harmonic content are affected by special relativistic Doppler boosts and aberration and gravitational light deflection, which depend on the mass and radius of the star. The mass and radius of the star can therefore be determined by fitting sufficiently accurate waveform models to waveform data of sufficient quality. It is likely that an analysis of data from oscillations during the tails of bursts will provide the best constraints on the mass and radius of the star (see Lo et al. 2013).

A strong prediction of the rotating hotspot model is that the oscillation should appear earlier at higher photon energies than at lower energies (e.g. Strohmayer 2000). This is most easily seen in an extreme case. The spectral shape of the emission from burst atmospheres typically has a shape close to the shape of a Bose–Einstein spectrum (see e.g. Boutloukos, Miller & Lamb 2010). At energies well above the peak of the spectrum, the observed maximum of the burst oscillation will occur at the stellar rotational phase that maximizes the blueshift of the hotspot relative to the observer, which happens when the spot is near the approaching limb of the star. In contrast, at energies near the peak of the spectrum the maximum of the oscillation will occur later, when the projected area of the spot is close to its maximum. Ford (1999) found just such a hard lead in *Rossi X-ray Timing Explorer* (*RXTE*) observations of the oscillation during the tail of a type 1 burst from Aql X-1.

Muno, Özel & Chakrabarty (2003) subsequently analysed 51 oscillation trains in *RXTE* data from the tails of type 1 bursts from six neutron stars. They folded each of these oscillation trains to produce oscillation profiles and then measured the phases of these profiles using linear least-squares fits of sinusoids to the oscillation profiles. Although the uncertainties of the oscillation phases were relatively large, they found significant evidence for phase variations with photon energy in 13 of the 51 oscillation trains. Of these 13 trains, 7 showed random phase variations while the remaining 6 showed either no significant lags or a roughly linear phase trend corresponding to a hard lag.

Muno et al. (2003) then grouped all the energy-resolved oscillation profiles from a given star for each *RXTE* Proportional Counter Array (PCA) gain epoch and averaged the profiles within each of the resulting nine groups. They found that five of the nine averaged energy-resolved oscillation profiles showed significant phase variations with energy. The phase variations in one of these five profiles appeared to be random while the phase variations in the other four were consistent with a linear increase in phase with energy. The apparent hard lags in these profiles are inconsistent with production of these oscillations by a rotating heated area that emits thermal radiation. Muno et al. speculated that these hard lags might be caused by Comptonization. If confirmed, statistically significant hard lags would mean that a simple rotating spot model of burst oscillations cannot be used to constrain the masses and radii of these stars and would raise questions about the applicability of this model to other stars.

Here we re-analyse individually the oscillations observed in the tails of four type 1 bursts from 4U 1636–536. The average oscillation profiles (Muno et al. 2003) constructed by summing the individual profiles from these bursts provided the strongest evidence for a soft lead in their burst oscillation analysis. We also analyse the oscillation observed during the superburst from this star. In contrast to Muno et al., we find that the data from these bursts, treated both individually and jointly, are fully consistent with a rotating hotspot

model. The uncertainties in the *RXTE* data are too large to provide interesting constraints on the mass and radius of this star.

The number of counts in these burst oscillation profiles is so small that the consistency of these profiles with the rotating spot model cannot be taken as confirmation of this model. However, this consistency does mean that these data do not challenge the rotating spot model and that fitting this model to burst oscillation data remains a viable way to constrain neutron star masses and radii using higher quality data from future large-area timing missions such as the *Large Observatory for X-ray Timing* (*LOFT*; Del Monte, Donnarumma & Consortium 2012; Feroci et al. 2012; Mignani et al. 2012) and the *Advanced X-ray Timing Array* (*AXTAR*; Chakrabarty, Ray & Strohmayer 2008; Ray et al. 2011).

In Section 2, we describe the X-ray data we use in this study and the results of our timing analysis. In Section 3, we discuss our results and conclusions.

## 2 DATA, ANALYSIS AND RESULTS

Our analysis is based on event mode data from the *RXTE* PCA. This mode has a time resolution of 1/8192 s and is therefore well suited for studying burst oscillations with frequencies of hundreds of hertz. We focused on the 4U 1636–536 type 1 bursts observed during PCA gain Epoch 4 (the boundaries of the PCA energy channels are slightly different in different epochs), because the averaged energy-resolved folded profile of the oscillations during these bursts that was constructed by Muno et al. (2003) appeared to show a systematic soft lead (see the top-middle panel of their fig. 2), contrary to what is predicted by a simple rotating hotspot model. We also analysed data from *RXTE* observations of the hours-long superburst from 4U 1636–536 (Strohmayer & Markwardt 2002), which occurred during PCA gain Epoch 5, to determine whether we could obtain an acceptable joint fit to the data on all these bursts.

There were four type 1 bursts from 4U 1636–536 during PCA gain Epoch 4 that had detectable oscillations; their ObsIDs are listed in the first four rows of Table 1. Epoch 4 ObsIDs 40028-01-08-00 and 40031-01-01-06 each had one additional oscillation train some seconds after the segments we analysed, but these trains were too weak to satisfy our significance criterion and hence we have not included them in our analysis.

We followed Muno et al. (2003) by using five energy bins spanning photon energies from 2 to 23 keV. We also followed Muno et al. by analysing the oscillations during the tails of the bursts rather than during their rising portions, where the changing frequencies and rapidly changing amplitudes of the oscillation trains could introduce additional complications.

For each burst, we first searched for the starting time and duration of the single segment of the burst that maximized the significance of the oscillating signal. Searches using the Leahy power (Leahy et al. 1983) or, equivalently, the  $Z_1^2$  statistic (Buccheri et al. 1983; Strohmayer & Markwardt 1999), and the deviation of  $\chi^2$  for the best folded profile from its value assuming no oscillation all yielded

**Table 1.** Properties of the 4U 1636–536 burst segments analysed.

ObsID	Start time	Duration (s)	Mean frequency (Hz)
40028-01-06-00	171 611 735.014	3.260	580.3985
40028-01-08-00	172 366 987.063	1.301	580.5576
40030-03-04-00	172 431 061.302	1.926	580.4122
40031-01-01-06	172 609 556.020	2.645	581.1129
50030-02-08-01	225 479 593.957	149.251	581.9692

equivalent results. This approach implicitly assumes that the oscillation frequency is constant. For the four type 1 bursts, we explored durations ranging from 0.5 to 6 s in steps of 1/16 of a second, with starting times 1/16 of a second apart throughout the burst tail. For the superburst, we tried durations between 64 and 1024 s in steps of 8 s throughout the portion of the burst where X-ray flux oscillations were observed. The superburst has two distinct trains of detectable oscillations, separated by a 200 s gap (see fig. 6 of Strohmayer & Markwardt 2002). In principle, the single segment of data in which the oscillation was most significant could have spanned the two trains or been confined to one train or the other. We found that the segment in which the oscillation was most significant lasted  $\sim 150$  s and was entirely within the second train. This is the segment that we used in our analysis. For the first type 1 burst, we also tried fits using a frequency that varies linearly or exponentially with time. For the superburst, we tried these two frequency models and the frequency model of Strohmayer & Markwardt (2002), which assumes a constant intrinsic frequency but includes the Doppler shift produced by their orbital solution. The measured phase lags and associated error bars using all these different frequency models were fully consistent with those derived using a constant frequency model. Because there is a possibility that the intrinsic frequency is not constant and because the constant observed frequency model fits the data very well, we preferred to use this simpler model in our analysis. This also made our analysis of the superburst exactly the same as our analyses of the shorter bursts. We then used epoch folding (see e.g. Davies 1990) to determine the frequency of the oscillation during each burst segment and to construct a folded oscillation profile for each set of energy channels. The start time, duration and oscillation frequency for each of the five burst segments we analysed are listed in Table 1.

We determined the zero phase (maximum flux) of the bolometric oscillation profile and the oscillation profile in each set of energy channels by fitting a sinusoidal model to each of these profiles. The parameters in this model were the phase, amplitude and DC level. We estimated the  $1\sigma$  uncertainties in these parameters using the standard approach. Namely, we began with the overall best-fitting values of the three parameters. We then varied one parameter from its best-fitting value while minimizing  $\chi^2$  with respect to the other two parameters, until we found the value of the first parameter that gave a  $\Delta\chi^2$  of 1 relative to the  $\chi^2$  for the best-fitting parameter values. We used this value of the first parameter to define its  $1\sigma$  uncertainty region. We then repeated this procedure for the other two parameters.

Following Munro et al. (2003), we used the parameter values in our fitted sinusoidal models to determine the phase of the oscillation profile in each set of energy channels relative to the phase of the bolometric oscillation profile. We also computed the relative phases by cross-correlating the profiles. The results were consistent with the results from the sinusoidal fits. Both methods confirmed the hard lead found by Ford (1999) for the oscillation in a segment of the tail of a burst from Aql X-1. We also studied earlier portions of that burst but found large data gaps that render any conclusions concerning these portions untrustworthy.

Fig. 1 shows our results for the phases of the oscillation profiles as a function of photon energy. These results are insensitive to small changes in the start times and durations of the data segment and the oscillation frequencies. In this figure, a negative lag implies that the maximum of the oscillation in that energy range arrived, on average, before the maximum of the oscillation in the bolometric profile. A positive lag means the opposite. As explained earlier, models in which the oscillation is produced by thermal emission from a

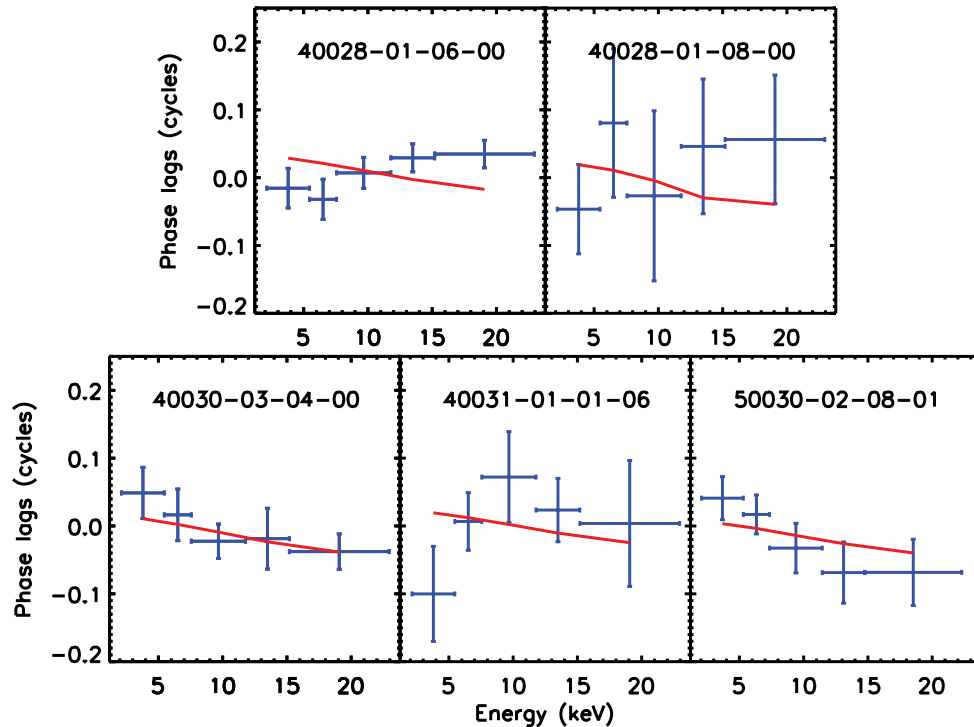
rotating spot predict an increasingly negative lag with increasing energy.

Fig. 1 also shows the results of a fit to these data of the model waveform produced by a uniform circular hotspot fixed to the rotating star with a spectrum having the shape of a Planck spectrum, computed using the algorithms described briefly in Lamb et al. (2009a,b) and more extensively in Lo et al. (2013). In fitting this model, the mass, radius, observer latitude and distance to the star were kept fixed for all five bursts, but the latitude of the spot centre, the spot angular radius and the spot colour temperature were allowed to be different for each burst. In order to extract maximal information from the data, we used standard Bayesian techniques to compare our model predictions with the data, which were broken into 16 phase bins and 64 energy channels, as provided by the PCA event modes. We used data from energy channels 3 through 26. The highest energy channel corresponds roughly to 23 keV, above which the background tends to dominate, for soft sources such as bursts. We folded the energy-resolved oscillation profile model through the appropriate response matrix for each of the data segments we analysed. The fitting process generates a best-fitting model of the background, as described in Lo et al. (2013). The error bars on the fitted parameters take into account the modelled background. The fit shown is for a star with a mass of  $1.5 M_{\odot}$  and a radius of 8.8 km, an observer inclination relative to the stellar spin axis of  $56^{\circ}$  and a distance of 6.5 kpc [consistent with the distance of  $(6.0 \pm 0.5)(M/1.4 M_{\odot})^{1/2}$  kpc estimated by Galloway et al. (2006)]. A wide range of other stellar masses and radii give comparably good fits to these data, so we are unable to derive interesting constraints on the properties of this neutron star using these data.

A simple rotating hotspot model is fully consistent with the 4U 1636–536 data that we analysed. The fits of this model go through 76 per cent (19 of 25) of the one-sigma error regions. The Cash statistic (Cash 1979, note that this statistic is an analogue of  $\chi^2$  for small numbers of counts and asymptotes to  $\chi^2$  for large numbers of counts) for the joint fit is 1859 for 1850 degrees of freedom. If we treat this as a  $\chi^2$  distribution, then if the model is correct we will find  $\chi^2$  of 1859 or larger for 1850 degrees of freedom in approximately 44 per cent of realizations. The Cash statistic when only the data in the top-left panel of the five panels of Fig. 1 are compared to the model that best fits the data in all five panels is 401.8 for 372 degrees of freedom. A  $\chi^2$  this large or larger is expected 14 per cent of the time if this model is correct. The simple rotating hotspot model is therefore fully consistent with these data, especially because the data shown for each burst were obtained by folding the oscillation waveform over several seconds of a burst that itself lasted only a few seconds, a procedure that undoubtedly smeared the oscillation profile. The uncertainties in the lag estimates are sufficiently large that occasional apparent soft leads (such as the one in the leftmost panel) are consistent with being statistical fluctuations. The superburst, which has far more counts than any of the type 1 bursts, does appear to show a hard lead, in accordance with the prediction of a simple rotating spot model.

### 3 DISCUSSION

We have individually analysed the X-ray flux oscillations during four type 1 X-ray bursts from 4U 1636–536 observed using *RXTE* during its PCA gain Epoch 4 and the superburst from this star. We find that the variations of the phases of these oscillations with photon energy are fully consistent with a rotating hotspot model, whether the data from each burst are fitted individually or jointly.



**Figure 1.** Phase lags for oscillations in the tails of five X-ray bursts from 4U 1636–536 (data points) and a joint fit of a simple rotating circular hotspot model to all the data (solid lines). This model is consistent with the data, as shown by the fact that model curves go through 19 of 25 (i.e. 76 per cent) of the 68 per cent confidence intervals and that the deviations of the data points from the model as large or larger than those shown would be expected in approximately 44 per cent of realizations if the model is correct. Although the deviations from the model curve of the data in the top left of the five panels may at first sight appear significant, deviations this large or larger would be expected by chance about 1/7 of the time if the model is correct. See the text for further details. The phases shown for the data points and the model points are both relative to the zero phase of the observed bolometric oscillation profile, which is defined by the maximum flux of the profile. Each panel shows the data and ObsID for one of the five bursts. The horizontal error bars indicate the approximate energy range covered by each channel while the vertical error bars show the  $1\sigma$  uncertainties determined by fitting a sinusoid to the data. In fitting the rotating spot model to these data, the mass, radius, observer latitude and distance to the star were kept fixed for all five bursts, but the spot latitude, spot radius and spot colour temperature were allowed to vary from burst to burst. See the text for further details.

Unfortunately, the uncertainties in these data are too large to provide interesting constraints on the mass and radius of this star.

Our results for the variations of the phases of the oscillation profiles with photon energy differ from the results reported by Muno et al. (2003), which appeared to be inconsistent with a simple rotating hotspot model. The analysis by Muno et al. differed from ours in several respects. In particular, Muno et al. averaged the energy-resolved profiles of the oscillations observed during the type 1 X-ray bursts from 4U 1636–536 during *RXTE* PCA gain Epoch 4 and then analysed them, whereas we analysed the energy-resolved profiles of the oscillations observed during each burst individually and then jointly.

Averaging the oscillation profiles before analysing them is not the optimal procedure, because the variations of the phases of the oscillation profiles with energy differ from burst to burst in this data set, as shown in Fig. 1. This is to be expected, because the properties of the hotspot (such as its radius or inclination from the spin axis) can change from burst to burst. Indeed, Muno et al. found that the variations with energy of the phases of the oscillation profiles in the 4U 1636–536 bursts changed with time (the dependence during PCA gain Epochs 3, 4 and 5 differed from one another). This suggests that the oscillation profile does vary from burst to burst.

Our most important result is that the X-ray flux oscillations during the four type 1 X-ray bursts from 4U 1636–536 that we analysed and the superburst from this star are fully consistent with a simple

rotating hotspot model, whether the data from each burst are fitted individually or jointly. Thus, these data do not call into question the use of such a model to fit X-ray burst oscillation data and thereby constrain the masses and radii of individual neutron stars. At the same time, it is clear that more and better data will be required to obtain tight constraints. A similar conclusion was reached by Strohmayer (2000).

Proposed space missions that will obtain much better timing data include *NICER*, which will focus on deep observations of X-ray emitting rotation-powered millisecond pulsars (Gendreau, Arzoumanian & Okajima 2012; see Bogdanov 2013 for constraints recently obtained using *XMM-Newton* data), *LOFT*, which will have a collecting area more than an order of magnitude larger than the *RXTE* PCA (see Del Monte et al. 2012; Feroci et al. 2012; Mignani et al. 2012) and *AXTAR*, which would also have a collecting area much larger than the *RXTE* PCA (Chakrabarty et al. 2008; Ray et al. 2011). Our simulations of the precision of the phase lag measurements that could be achieved using *LOFT* show that it could achieve a precision of  $\sim 0.01$  cycles, compared to the  $\sim 0.05$  precision achieved using *RXTE*. A full Bayesian analysis of the constraints on the masses and radii of neutron stars that could be achieved by fitting waveform models to *LOFT* observations of burst oscillations (Lo et al. 2013) indicates that tight constraints can be achieved for systems that have a favourable geometry. Thus, there is a good prospect that these missions will provide neutron star mass and radius estimates

precise enough to tightly constrain the properties of cold supranuclear matter.

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