

SUPERMASSIVE BLACK HOLE FORMATION VIA GAS ACCRETION IN NUCLEAR STELLAR CLUSTERS

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ABSTRACT

Black holes exceeding a billion solar masses have been detected at redshifts greater than six. The rapid formation of these objects may suggest a massive early seed or a period of growth faster than Eddington. Here we suggest a new mechanism along these lines. We propose that in the process of hierarchical structure assembly, dense star clusters can be contracted on dynamical timescales due to the nearly free-fall inflow of self-gravitating gas with a mass comparable to or larger than that of the clusters. This process increases the velocity dispersion to the point where the few remaining hard binaries can no longer effectively heat the cluster, and the cluster goes into a period of homologous core collapse. The cluster core can then reach a central density high enough for fast mergers of stellar-mass black holes and hence the rapid production of a black hole seed that could be $10^5 M_{\odot}$ or larger.

Key words: black hole physics – galaxies: evolution – galaxies: formation – galaxies: nuclei

1. INTRODUCTION

Evidence from observations of high redshift active galactic nuclei shows that massive black holes ($m_{\text{bh}} \sim 10^9 M_{\odot}$) have already formed at redshifts $z > 6$ (Willott et al. 2003; Barth et al. 2003). It is challenging to produce such masses so early via Eddington-limited gas accretion onto initially stellar-mass black holes, hence there have been various explorations of the possibility of higher-mass seeds, e.g., Population III stars (Madau & Rees 2001; Volonteri et al. 2003), massive stars formed through runaway collisions (Devecchi & Volonteri 2009), or quasi-stars (Begelman et al. 2006; Lodato & Natarajan 2006). Here we renew an earlier suggestion: that a sufficiently massive and dense star cluster with a central black hole sub-cluster could undergo collapse, including mergers due to gravitational radiation, building up a massive black hole much faster than would be possible by Eddington-limited gas accretion onto a stellar-mass seed.

Supermassive black hole (SMBH) formation within stellar clusters in galactic nuclei has been explored in a series of papers by Quinlan & Shapiro (1987, 1989, 1990). They considered core collapse in a cluster of compact objects where the cluster core contracts driven by two-body relaxation to the point where the central potential well is so deep that a relativistic instability sets in and a single black hole is formed via the collapse of the central core of compact objects. However, this instability requires a central redshift $z \sim 0.5$, meaning that the core would have to be extremely massive and dense.

In this Letter, we consider a variation on the cluster-collapse model. We begin with cluster conditions similar to those seen in nuclear stellar clusters, which have masses and radii roughly comparable to globular clusters (e.g., Seth et al. 2008). In these clusters, the more-massive stellar-mass black holes are likely to segregate to the cluster center forming their own dark core. Binaries within the core will provide the cluster with energy via binary–single encounters. These binaries are relatively wide (~ 0.1 – 1 AU), hence the timescale for them to merge via the effects of gravitational radiation is extremely long. Binary heating will thus provide a fuel to support the cluster core from complete collapse.

Infall of gas into such a nuclear stellar cluster may have profound consequences for its evolution. Infall is likely to occur

during a merger between two galaxies (Mayer et al. 2010), which will be common at high redshifts (Bellovary et al. 2011). The addition of significant mass will cause the cluster to shrink after virialization, and in some circumstances significant accretion onto cluster stars may occur. The cluster potential well will be deepened, increasing velocity dispersions and decreasing the number and semi-major axes of the binaries that heat the cluster. Because the inspiral timescale due to gravitational radiation depends sensitively on the initial separation, these smaller binaries will have significantly decreased merger timescales. Mergers remove binaries and thus remove the energy source for the cluster. Without an energy source, the cluster core will undergo deep core collapse, potentially causing a runaway merger of the black holes residing there. For this collapse to occur we require not only that the merger timescale of binaries due to the emission of gravitational radiation is small but also that the merger products (which will receive kicks due to the asymmetric emission of gravitational waves) are retained within the cluster core.

In Section 2, we set out the conditions for a nuclear stellar cluster such that black hole binaries within the core are likely to merge (and be retained) on shorter timescales than cluster heating via binary–single encounters. In Section 3, we consider the immediate effects of infall gas into a nuclear stellar cluster. In Section 4, we consider the subsequent evolution of a nuclear stellar cluster after an episode of gas infall, possibly leading to a chain of events producing an SMBH.

2. DRY NUCLEAR STELLAR CLUSTERS

We begin by considering a cluster free of gas and assume that the cluster contains only two species: stellar-mass black holes and less-massive main-sequence stars. The heavier black holes will sink to the core by the effects of mass segregation. The cluster will likely be vulnerable to the *Spitzer* instability, whereby the black holes will form their own central sub-cluster within the cluster core. Although some black-hole–black-hole interactions can lead to ejections, a significant fraction of the initial black hole population remains even after several Gyr at standard cluster densities (Mackey et al. 2007, 2008).

Black hole binaries within the dark core that are soft (i.e., their binding energy is less than the typical kinetic energy of a single black hole) will be split rapidly by binary–single interactions. In contrast, hard binaries tend to harden further after binary–single interactions (and heavier black holes tend to swap into the binaries), meaning that both the single and binary get a kick after the interaction and the cluster is heated as a result (Heggie 1975). The timescale for a given object to have a gravitationally focused encounter with another object within the cluster is given by

$$\tau_{\text{enc}} = 7 \times 10^{10} n_5^{-1} v_{\infty,10} r_{\text{min}}^{-1} m^{-1} \text{ yr}, \quad (1)$$

where n_5 is the number density of stars/black holes in units of 10^5 objects/pc³, r_{min} is the minimum distance during the encounter in solar radii, m is the sum of the masses of the objects involved in the encounter in solar masses, and $v_{\infty,10}$ is the relative speed at infinity in units of 10 km s^{-1} (Binney & Tremaine 2008). When considering encounters between binaries and single stars, it is reasonable to set $r_{\text{min}} = a_{\text{bin}}$, the semi-major axis of the binary. If we consider encounters between binary and single black holes, all of equal mass, then $m = 3m_{\text{bh}}$. The number density of objects in the core is $n \simeq 3N_{\text{bh}}/4\pi r_c^3$, where N_{bh} is the total number of black holes and r_c is the core radius. Assuming the cluster is in virial equilibrium, one may estimate that $v_{\infty} \simeq \sqrt{0.4GM_c/r_h}$ (Binney & Tremaine 2008), where M_c is the total cluster mass. This implies $v_{\infty,10} \simeq 4.36\sqrt{M_{c,6}/r_h}$ where $M_{c,6}$ is the total cluster mass in units of $10^6 M_{\odot}$ and r_h is the cluster half-mass radius in pc.

The semi-major axis of a binary is given by

$$a_{\text{bin}} \simeq \frac{1000 R_{\odot}}{x} \left(\frac{m_{\text{bh}}}{v_{\infty,10}^2} \right), \quad (2)$$

where x is the ratio of the binary binding energy to the kinetic energy of the stars/black holes and m_{bh} is given in solar masses. One can show that the timescale for an encounter between a given black hole binary and a single black hole is given by

$$\tau_{2+1} \simeq 5 \times 10^{14} \frac{r_c^3}{N_{\text{bh}} m_{\text{bh}}^2} \left(\frac{M_{c,6}}{r_h} \right)^{3/2} \text{ yr}, \quad (3)$$

where r_c is the cluster radius in pc. Setting $r_c = \beta r_h$, $m_{\text{bh}} = 10 M_{\odot}$, $m_{\star} = 1 M_{\odot}$, and $N_{\text{bh}} = \alpha N_{\star}$, where N_{\star} is the total number of stars in the cluster, we obtain

$$\tau_{2+1} \simeq 10^9 \frac{\beta^3 x}{\alpha} \frac{M_{c,6}^2}{v_{\infty,10}^3} \text{ yr}. \quad (4)$$

Assuming a Salpeter initial mass function (IMF), with minimum stellar mass of $0.2 M_{\odot}$ and that all stars more massive than $25 M_{\odot}$ produce black holes, we have $\alpha = 0.0015$. Setting $\beta = 0.1$ gives

$$\tau_{2+1} \simeq 6 \times 10^8 x \frac{M_{c,6}^2}{v_{\infty,10}^3} \text{ yr}. \quad (5)$$

Tight binaries will lose energy via the emission of gravitational radiation. The timescale for two black holes, in a binary of initial separation a_{bin} and eccentricity e , to spiral together is given by

$$\tau_{\text{gr}} \simeq 10^{10} \left(\frac{a_{\text{bin}}}{3.3 R_{\odot}} \right)^4 \left(\frac{1}{2m_{\text{bh}}^3} \right) (1 - e^2)^{7/2} \text{ yr (Peters 1964)}. \quad (6)$$

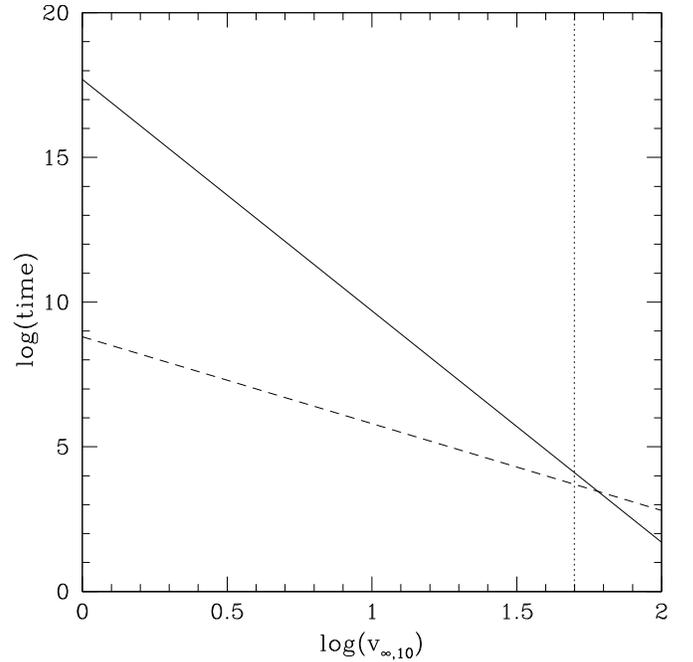


Figure 1. log of timescales (in years) for binary–single encounters (dashed line) and gravitational radiation inspiral (solid line) in binaries as a function of log of velocity dispersion within clusters, $v_{\infty,10}$, in units of 10 km s^{-1} , assuming binaries with hardness parameter $x = 3$ and eccentricity $e = 1/\sqrt{2}$. The vertical dotted line represents the value of $v_{\infty,10}$ when merging black holes are likely to be retained.

Binary–single scattering will leave the binaries with a thermal distribution of eccentricities where the distribution of orbital eccentricities is given by $dn/de = 2e$. The median eccentricity is thus $e_{\text{med}} = 1/\sqrt{2}$, hence a typical binary merger time is reduced by a factor ~ 10 and could be reduced by much more for higher eccentricities. Using the expression for a_{bin} given above, we obtain

$$\tau_{\text{gr}} \simeq 5 \times 10^{19} \frac{m_{\text{bh}}}{v_{\infty,10}^8} x^{-4} (1 - e^2)^{7/2} \text{ yr}. \quad (7)$$

We plot τ_{gr} and τ_{2+1} as a function of $v_{\infty,10}$ in Figure 1, taking $x = 3$, and assuming the binaries initially have the median eccentricity of a thermal distribution ($e = e_{\text{med}} = 1/\sqrt{2}$). Thus, for moderately hard ($x \sim 3$) binaries in a typical nuclear stellar cluster ($v_{\infty,10} \sim 3$), t_{gr} is extremely large: black hole binaries are unlikely to merge via the effects of gravitational radiation. If $\tau_{\text{gr}} < \tau_{2+1}$, binaries will merge before they have the chance to heat the cluster. The exact value of $v_{\infty,10}$ when this is the case will depend on the hardness and eccentricity of the binary, but for moderately hard, and eccentric, binaries, this is likely to be true for $v_{\infty,10} \gtrsim 50$.

When two inspiraling black holes merge, they receive a kick due to the asymmetric emission of gravitational radiation. This kick depends on the mass ratio of the two black holes and their spins, but can be as large as $\sim 4000 \text{ km s}^{-1}$ for an optimal configuration (e.g., Baker et al. 2008). In order to retain merger products within the stellar cluster, we require the cluster escape speed to exceed 1000 km s^{-1} (or $v_{\infty,10} \sim 50$). This is equivalent to requiring that

$$\left(\frac{M_{c,6}}{r_h} \right)^{1/2} \geq 10. \quad (8)$$

Any merger products ejected *extremely close* to the escape speed may be left on very wide orbits outside of the cluster, however,

objects ejected at less than about 80% of the escape speed will remain in the cluster. We thus see that the conditions required for the retention of merger products are rather similar to those which give us $\tau_{\text{gr}} < \tau_{2+1}$. If the merger products are retained within the cluster, we expect the dark core to continue to contract, having the potential to lead to a runaway merger of black holes. The outcome of such a process could be the production of an intermediate-mass black hole which would act as the seed for an SMBH.

However, most observed nuclear stellar clusters will *not* satisfy Equation (8). We therefore require some mechanism to cause the cluster to shrink and/or increase its mass in order to reach the conditions required for runaway black hole mergers to occur within the dark core.

3. THE EFFECT OF GAS INFALL INTO THE CLUSTER

Suppose that gas with mass comparable to or larger than the stellar mass of the cluster falls quickly to the center of the cluster. Just as in the inverse problem (e.g., where mass is expelled from a binary in a supernova), we assume that the angular momentum of individual stellar orbits is conserved, meaning that the characteristic size of the orbits is proportional to $1/M$, where M is the mass interior to the orbit. Thus, for example, adding a gas mass equal to the cluster mass halves the sizes of the orbits exterior to the gas and increases the stellar density by a factor of eight. As shown below, at redshifts $z > 10$ one can indeed have gas inflows with total masses up to ~ 10 times the mass of the stellar cluster. The key question is whether much of the gas flows to the inner $\sim \text{pc}$ or closer, rather than stalling and forming stars much farther away, which would not contract the central cluster.

One might imagine that a cooling flow could form that would bring the gas in. Indeed, such a scenario has been suggested by Vesperini et al. (2010) to grow intermediate-mass black holes in early globular clusters. However, as demonstrated by D’Ercole et al. (2008), the distributed injection of even a comparatively small amount of luminosity ($Q_{\text{cr}} \sim 6 \times 10^{37} \text{ erg s}^{-1}$ in their treatment) is sufficient to hold off the cooling flow. This is considerably less than the Eddington luminosity from even a single $10 M_{\odot}$ black hole, and given that this luminosity is typically distributed over a large spread in photon energies that is therefore absorbed over a wide range in gas column depths, it appears unlikely that a cooling flow will develop in the situation we consider here.

We instead turn to the scenario proposed by Mayer et al. (2010) in which the self-gravitating gas is subject to instabilities that funnel much of the gas to the center in a low angular momentum flow that, in their calculations, gets to 0.2 pc or closer to the center. Gravitationally driven turbulence prevents the gas from fragmenting into stars. Such a flow may lead to the formation of a massive black hole, as Mayer et al. (2010) suggest, but the large range of scales between the final ~ 0.1 pc simulated by Mayer et al. 2010 and the ~ 0.01 AU scale of the black hole means that the final collapse is still unresolved and might be hampered by a variety of effects. Nonetheless, the inflow will also very effectively contract an existing stellar cluster, leading to high central densities, rapid mass segregation, and fast interactions among the existing stellar-mass black holes that could lead to quick coalescence and the formation of a massive black hole seed. This scenario is particularly probable given that such processes likely occurred in miniature in the smaller clusters and gas flows that accumulated to form the central cluster. Thus, the initial black hole masses would have

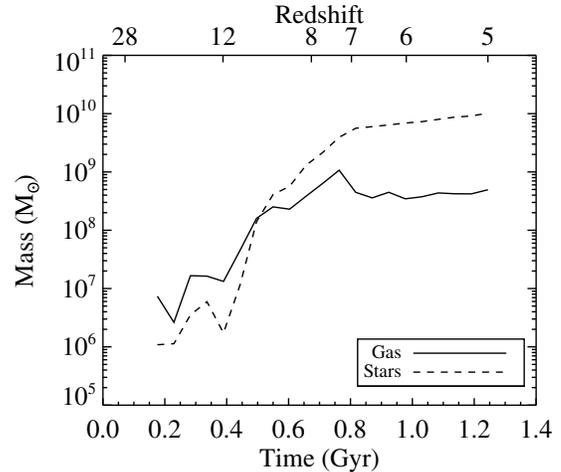


Figure 2. Amount of mass as a function of time for gas (black line) and stars (dashed line) in the cosmological simulation. Masses are the total mass enclosed within 520 pc (two softening lengths) from the center of the primary galaxy in the simulation.

had a wide range; this is known to speed up mass segregation and coalescence (Quinlan & Shapiro 1987).

As an illustration of the gas inflow into nuclear regions which can occur, we turn to high-resolution cosmological simulations of galaxy formation, considering the evolution of a massive galaxy destined to become a massive elliptical at low redshift (Bellovary et al. 2011). In Figure 2, we show the mass enclosed within 520 pc of the galaxy center versus time for the gas (black line) and stars (dashed line). The gaseous and stellar inflow are due to a combination of direct accretion of matter from the cosmic Web/filaments as well as galaxy mergers. From this plot we see that significant inflow of both gas and stars occurs at high redshift. Repeating this measurement for similar simulations reveals that high redshift ($z > 10$) massive inflow on this scale is a common occurrence for galaxies in this mass range (a few $\times 10^{11} M_{\odot}$ at $z = 5$).

One might wonder whether the rapid contraction of a cluster by a factor of a few would lead to a high rate of interactions of single objects with hard binaries and hence to re-expansion of the cluster due to energy input. The degree to which this can happen clearly depends on the fraction of stars or black holes in hard binaries that are not so hard that they merge or collide rapidly. This fraction, in turn, depends on the velocity dispersion; higher velocity dispersion means fewer hard binaries and less binary binding energy that can potentially be tapped to hold off core collapse. We also note that close three-body interactions between objects of comparable mass yield a thermal distribution of eccentricities. The eccentricities that give pericenter distances low enough for collisions (~ 0.01 AU for solar-type stars) or fast merger by gravitational radiation (also ~ 0.01 AU for $\sim 10 M_{\odot}$ black holes to merge in a few million years or less) will destroy the binary. If the hard-soft boundary is $a < 1$ AU (corresponding to a velocity dispersion $\sigma \sim 30 \text{ km s}^{-1}$ for solar-type stars or $\sigma \sim 100 \text{ km s}^{-1}$ for $\sim 10 M_{\odot}$ black holes), the available binding energy in binaries is less than the binding energy in the singles, hence binary-single interactions are inefficient at holding off core collapse.

4. THE FATE OF THE MASS-LOADED CLUSTER

We now consider the subsequent evolution of the stellar cluster, assuming it has received a significant influx of gas, and

perhaps stars. Beginning with a nuclear stellar cluster of mass $M_{c,6} \sim 1$, and half-mass radius $r_h \sim 1$, the infall of $10^7 M_\odot$ of gas is likely to shrink the cluster by a factor of about 10 leaving initial core densities $n \sim 10^8$ stars pc^{-3} and velocity dispersions $v_{\infty,10} \sim 30\text{--}50$. Several things will happen to this cluster. Wider binaries (which previously heated the cluster and prevented core collapse) will now be soft, as the velocity dispersion has increased, and will be broken up. Any remaining binaries will quickly merge via gravitational radiation inspiral or collisions. Merger products are likely to be retained within the cluster owing to the increased escape speed which resulted from the gas infall. It is possible that some merger products will in turn exchange into other binaries which then merge. This process may even be repeated.

After a short time, the cluster core will contain black holes having a broader range of masses than previously. In addition, all binaries will either have been broken up or have merged. Thus, the cluster core will lose its source of energy, and will begin to contract approaching core collapse as it transfers energy to the cluster halo via two-body scattering. As the core density increases, runaway mergers between any non-compact stars within the cluster core become possible (Quinlan & Shapiro 1990). Such collisions may produce more massive stars which in turn will add to the population of black holes and neutron stars.

At high densities, extremely close encounters occur between two compact objects where binary formation via gravitational wave emission is possible. The timescale for capture is given by (Quinlan & Shapiro 1989)

$$\tau_{\text{cap}} \approx 7 \times 10^{12} n_5^{-1} \mu_{10}^{-2/7} M_{100}^{-12/7} v_{\infty,500}^{-11/7} \text{yr}, \quad (9)$$

where μ_{10} is the reduced mass of the two compact objects capturing each other (in units of $10 M_\odot$) and M_{100} is their total mass (in units of $100 M_\odot$). We see that when the core reaches densities $n \sim 10^{12}$ stars pc^{-3} , which at a velocity dispersion $v \sim 300 \text{ km s}^{-1}$ contains few $\times 10^4 M_\odot$, the formation of black-hole–black-hole binaries via gravitational wave emission becomes possible on a timescale $\lesssim \text{Myr}$. These binaries will have very short inspiral timescales. We will thus have a second phase of black hole binary mergers. The black holes produced via such mergers have greater mass, and given that the timescale for mergers scales as $M^{-12/7}$, higher-mass black holes have a greater probability of merger and the process will run away. Such a phase may well collect most of the mass of the stellar black holes into a single black hole, which would have several percent of the original stellar mass of the system for typical IMFs. Thus, the mass of the single black hole could be $\sim 10^5 M_\odot$ for situations similar to that shown in Figure 2 (assuming most infalling gas reaches the center). Accretion of neutron stars and white dwarfs, both of which will be swallowed whole for $M \gtrsim 10^5 M_\odot$, could boost the mass by a factor of a few, depending on the mass at which stellar interactions in the radius

of influence of the black hole become an effective heating source for the cluster (e.g., Gill et al. 2008). Overall, the time elapsed between mass infall to the production of the seed SMBH will be roughly 100 Myr (e.g., Quinlan & Shapiro 1990), hence producing a $\sim 10^5 M_\odot$ black hole within $\sim 300\text{--}400$ Myr after the beginning of the universe. Growth to $\sim 10^9 M_\odot$ via gas accretion could then occur by a redshift $z \sim 6$ via standard Eddington-limited accretion onto moderately spinning black holes.

In summary, we have proposed that clusters with initial structural parameters similar to current-day nuclear clusters may (1) form as part of early hierarchical merging, (2) accrete gas with a total mass comparable to or greater than that of the cluster at redshifts $z > 10$, (3) contract as a result so that binary heating is ineffective, (4) undergo core collapse to a density high enough that stellar-mass black holes merge, and thus (5) have most of the mass originally in stellar-mass black holes collect into a single black hole that could be $\sim 10^5 M_\odot$ or larger. This black hole would therefore be a high-mass seed that could comfortably grow to supermassive size by the observed redshifts $z \sim 6$.

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