

PROBING GENERAL RELATIVITY WITH MERGERS OF SUPERMASSIVE AND INTERMEDIATE-MASS BLACK HOLES

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ABSTRACT

Recent observations and stellar dynamics simulations suggest that $\sim 10^3 M_\odot$ black holes can form in compact massive young star clusters. Any such cluster in the bulge of its host galaxy will, within a few hundred million years, spiral to the center, where its intermediate-mass black hole is likely to eventually merge with the galaxy’s supermassive black hole. If such mergers are common, then future space-based gravitational wave detectors such as the *Laser Interferometer Space Antenna* will detect them with such a high signal-to-noise ratio that toward the end of the inspiral the orbits will be visible in a simple power density spectrum, without the need for matched filtering. We discuss the astrophysics of the inspiral of clusters in the nuclear region of a galaxy and the subsequent merger of intermediate-mass with supermassive black holes. We also examine the prospects for understanding the spacetime geometry of rotating black holes, based on phase connection of the strong signals visible near the end of these extreme mass ratio inspirals.

Subject headings: black hole physics — gravitational waves — relativity — stellar dynamics

1. INTRODUCTION

Observations of many star-forming galaxies show that a common mode of star formation involves the production of young massive star clusters (or “super star clusters”), which might have masses $\sim \text{few} \times 10^5 M_\odot$ with half-mass radii of $\sim \text{few}$ parsecs (e.g., van den Bergh 1971 and numerous subsequent papers; see Maíz-Appellániz 2001 for a review of the structural parameters of such clusters). According to recent N -body simulations (Ebisuzaki et al. 2001; Portegies Zwart & McMillan 2002; Portegies Zwart et al. 2004), Monte Carlo simulations (Gürkan et al. 2004), and semianalytical treatments (Mouri & Taniguchi 2002b), when such a cluster is compact enough it can evolve dynamically in such a way as to produce runaway collisions in its center in the few million years before the most massive stars explode. Such collisions could produce a black hole of several hundred solar masses, and subsequent dynamical processes could add additional mass to the black hole (Miller & Hamilton 2002a, 2002b; Mouri & Taniguchi 2002a; Gültekin et al. 2004). As suggested by Ebisuzaki et al. (2001), a cluster of this type that starts close enough to the center of its host galaxy will, within a few billion years, sink to the center, where it will eventually release its black hole. Rough estimates (see § 3) suggest that a few times per year, a merger of such an intermediate-mass black hole (IMBH) with the central supermassive black hole (SMBH) will be detected by space-based gravitational wave detectors such as the *Laser Interferometer Space Antenna* (*LISA*).

If these mergers occur, they will be ideal sources with which to probe the spacetime geometry around rotating black holes (see the discussion in Cutler & Thorne 2002). The mass ratio (typically 10^3 – 10^4) is large enough that the IMBH acts almost as a test particle, but the signal strength is much larger than it is for mergers of stellar-mass black holes with SMBHs (Hughes 2001; Cutler & Thorne 2002; Glampedakis et al. 2002). As a result, although there is much greater uncertainty about the event rate for IMBH-SMBH mergers than for mergers of stellar-mass and supermassive black holes (thus design considerations for *LISA* should focus on the latter), if even a single IMBH-SMBH merger is detected, then high-precision

constraints on gravitational radiation and the Kerr spacetime will be possible with greatly simplified data analysis.

Here we discuss the dynamics and implications of such IMBH-SMBH mergers. In § 2 we describe the astrophysical scenario of an influx of IMBHs into the center of a galaxy. In § 3 we make estimates of the strength of the signal and discuss data analysis in the *LISA* context. We summarize in § 4.

2. ASTROPHYSICAL SCENARIO

Throughout this paper, we consider the interactions of an SMBH of mass M with one or more IMBHs of mass $\mu \ll M$. We scale these masses by 10^6 and $10^3 M_\odot$, respectively.

If a super star cluster of mass M_{cl} is embedded in a much lower density stellar environment, it will act dynamically as a single object. Adapting equation (7-26) of Binney & Tremaine (1987) and equation (2) of Ebisuzaki et al. (2001), the dynamical friction time for such a cluster to sink from a distance r to the center of a galaxy with three-dimensional velocity dispersion σ_{gal} is

$$t_{\text{df}} = (1.65 / \ln \Lambda) (r^2 \sigma_{\text{gal}} / GM) \\ \approx 4 \times 10^8 \text{ yr} (\sigma_{\text{gal}} / 100 \text{ km s}^{-1}) (r / 100 \text{ pc})^2 (10^5 M_\odot / M_{\text{cl}}), \quad (1)$$

where $\ln \Lambda$ is the Coulomb logarithm. Therefore, a cluster will be able to sink to the center within much less than a Hubble time if it starts anywhere within the inner few hundred parsecs of its host galaxy.

From this point, we expect the following sequence: (1) the cluster sinks until it is stripped or tidally disrupted, thus releasing its IMBH; (2) the IMBH sinks rapidly until the stellar mass interior to it is less than the mass of the IMBH; (3) the orbital radius of the IMBH around the SMBH shrinks via interactions with stars, as long as the relaxation time for the surrounding stars is less than a Hubble time; and (4) the IMBH either merges with the SMBH as a result of interactions with stars followed by inspiral caused by gravitational radiation, or one or more additional IMBHs settle to the center and interact

dynamically, causing mergers. We now discuss each of these steps.

Cluster mass loss.—As the cluster sinks, it can lose stars in several ways (a similar discussion in the context of stars at the Galactic center is in Hansen & Milosavljevic 2003). The first is tidal stripping. That is, if the cluster mass and radius are M_{cl} and R_{cl} , respectively, then the outer portions of the cluster will be stripped away when the cluster is a distance $r < r_{\text{tide}}$ from the center of the galaxy, where the tidal radius r_{tide} is given by

$$r_{\text{tide}} = [3M(<r_{\text{tide}})/M_{\text{cl}}]^{1/3} R_{\text{cl}}. \quad (2)$$

Observations of the central regions of many galaxies suggest that the velocity dispersion is relatively constant (K. Gebhardt 2004, private communication). Therefore, this is consistent with an isothermal density profile in which $M(<r) = 2\sigma^2 r/G$, where σ is the three-dimensional velocity dispersion (see eq. [4-123] of Binney & Tremaine 1987). Rewriting, we find that the tidal radius is

$$r_{\text{tide}} = [6\sigma^2/(GM_{\text{cl}}/R_{\text{cl}})]^{1/2} R_{\text{cl}}, \quad (3)$$

or about $10\text{--}20 R_{\text{cl}}$ for $M_{\text{cl}} \sim \text{few} \times 10^5 M_{\odot}$ and a half-mass radius $R_{\text{cl}} \sim \text{few pc}$. This will typically allow the cluster to sink to $\sim 30\text{--}50$ pc, which it does within $\sim 10^8$ yr if it started at ~ 100 pc. If we assume that the cluster itself has mass distributed roughly as an isothermal sphere, then $M_{\text{cl}} \propto R_{\text{cl}}$; therefore, the relaxation time scales as $t_{\text{rel}} \propto r^2/M \propto R_{\text{cl}}$ because the tidal radius scales as R_{cl} .

However, the cluster itself will also evolve dynamically. From the Pryor & Meylan (1993) catalog of Galactic globular clusters, the typical half-mass relaxation time for a globular is $\sim 10^8\text{--}10^9$ yr. For a cluster with N stars and a crossing time of $t_{\text{cross}} = R_{\text{cl}}/\sigma_{\text{cl}}$ (where σ_{cl} is the three-dimensional velocity dispersion of the cluster), the cluster relaxation time is $t_{\text{rel,cl}} \approx (0.1N/\ln N)t_{\text{cross}}$ (e.g., Binney & Tremaine 1987). For an isothermal sphere, $N \propto R_{\text{cl}}$ and $t_{\text{cross}} \propto R_{\text{cl}}$. Thus, $t_{\text{rel,cl}} \propto R_{\text{cl}}^2$.

Therefore, once tidal stripping of the cluster begins, the cluster relaxation time will decrease faster than the dynamical friction time. When $t_{\text{rel,cl}} < t_{\text{df}}$, the cluster will disperse and the IMBH will be on its own. For typical masses and radii of clusters, the simplified treatment above would suggest that this will happen when $r \sim 10$ pc. Given that clusters that form IMBHs tend to have short relaxation times, there could be a concern that these clusters would disrupt earlier. However, simulations by Kim et al. (2004) and A. Gürkan & F. Rasio (2005, in preparation) support the suggestion of Hansen & Milosavljevic (2003) that the presence of an IMBH in the center of a cluster increases the velocity dispersion of the stars and hence their relaxation time. Therefore, it is found numerically that the IMBH is in fact released at $\sim \text{few parsecs}$. Thus, equation (1) suggests that the IMBH will take $\lesssim 10^8 \text{ yr} (\sigma/100 \text{ km s}^{-1})(10^3 M_{\odot}/\mu)$ to sink to the center. Clusters that start within ~ 100 pc of the center will be able to deliver their central IMBHs to the center within a few hundred million years.

For completeness, we now discuss another way in which a cluster could theoretically be dispersed. Given that the velocity dispersion of stars in a galactic bulge is much greater than the velocity dispersion of stars in a cluster, passage of bulge stars through the cluster will soften the cluster somewhat and will eventually cause it to evaporate. One can show, however, that this effect is unimportant. As in Binney & Tremaine (1987; eq. [4-6a]), the typical change in squared transverse velocity of

a particle of mass m going at speed v through a cluster of N particles of mass m within a radius R is

$$\Delta v_{\perp}^2 \approx 8N(Gm/Rv)^2 \ln \Lambda, \quad (4)$$

where $\ln \Lambda \sim 10\text{--}20$ is a Coulomb logarithm. If we use $v = \sigma_{\text{gal}}$ and assume a cluster velocity dispersion of $\sigma_{\text{cl}}^2 \approx GNm/R$, then this becomes

$$\Delta v_{\perp}^2 \approx (8 \ln \Lambda/N) \sigma_{\text{cl}}^2 (\sigma_{\text{cl}}/\sigma_{\text{gal}})^2. \quad (5)$$

Because these are softening interactions, we assume that the energy of the cluster is always *increased* by $(1/2)m\Delta v_{\perp}^2$.

As these are fast interactions, there is little gravitational focusing; hence, the mass per time interacting with the cluster is simply $\rho(\pi R_{\text{cl}}^2)\sigma_{\text{gal}}$. For an isothermal sphere, $\rho = \sigma_{\text{gal}}^2/(4\pi r^2 G)$ at a distance r from the center. The change in energy per time is then

$$\begin{aligned} dE/dt &= \frac{1}{2} \frac{\sigma_{\text{gal}}^2}{4\pi r^2 G} (\pi R_{\text{cl}}^2) \sigma_{\text{gal}} \frac{8 \ln \Lambda}{N} \sigma_{\text{cl}}^2 (\sigma_{\text{cl}}/\sigma_{\text{gal}})^2 \\ &= \frac{\sigma_{\text{gal}}}{r^2 G} R_{\text{cl}}^2 \frac{\ln \Lambda}{N} \sigma_{\text{cl}}^4. \end{aligned} \quad (6)$$

The total binding energy of a singular isothermal sphere is

$$E = \int_0^{R_{\text{cl}}} \frac{GM(<r)}{r} \rho dV = \sigma_{\text{cl}}^4 R_{\text{cl}}/G. \quad (7)$$

Therefore, the softening time is

$$t_{\text{soft}} = \frac{E}{(dE/dt)} = \frac{r^2 N}{\sigma_{\text{gal}} R_{\text{cl}} \ln \Lambda}. \quad (8)$$

The Coulomb logarithms for t_{soft} and t_{df} will generally be different but probably not by more than a factor of a few ($\ln \Lambda$ for softening is likely to be of the order of $10\text{--}15$, but for dynamical friction is probably $3\text{--}5$; see Spinnato et al. 2003). Therefore, as an approximation, we can effectively cancel the Coulomb logarithms when we take the ratio

$$t_{\text{soft}}/t_{\text{df}} \sim N(GM_{\text{cl}}/R_{\text{cl}})/\sigma_{\text{gal}}^2 \sim N(\sigma_{\text{cl}}/\sigma_{\text{gal}})^2 \gg 1. \quad (9)$$

For example, if $N = 10^6$, $\sigma_{\text{gal}} = 100 \text{ km s}^{-1}$, and $\sigma_{\text{cl}} = 10 \text{ km s}^{-1}$, then $t_{\text{soft}} \sim 10^4 t_{\text{df}}$. Softening by interactions with bulge stars can always be neglected in comparison with other effects.

Initial inspiral of the IMBH.—After the cluster disrupts, the IMBH itself will spiral in independently. As a first stage, it will spiral in to where the mass interior to it is not much less than the mass of the IMBH itself. For a stellar number density of 10^6 pc^{-3} , this implies a distance of ~ 0.05 pc, but for a higher density it will be less. For example, Hansen & Milosavljevic (2003) quote the Genzel et al. (2003) density profile of the central cusp of the Galaxy as implying $M(<r) = 1.3 \times 10^4 M_{\odot} (r/0.04 \text{ pc})^{1.63}$, where $1'' = 0.04 \text{ pc}$ at 8 kpc . This implies a higher density, so that the rapid inspiral of a $10^3 M_{\odot}$ IMBH will occur down to a separation of ~ 0.01 pc. From above, the inspiral of the IMBH will start from a few parsecs; hence, it will come in on a timescale of $\lesssim 10^8$ yr for typical densities and velocity dispersions.

Long-term inspiral of the IMBH.—Further settling of the IMBH requires that it interact with a significant mass in stars.

If the stars have fully isotropized orbits, this is easy; for a strongly gravitationally focused encounter with a binary of total mass M and semimajor axis a the cross section is $\Sigma = \pi a(2GM/\sigma^2)$ and the timescale of interaction is $\tau = 1/(n\Sigma\sigma)$, which is much less than a year for typical masses, velocities, and densities.

However, stars that interact with the IMBH-SMBH binary are eventually thrown out of the system, so the bottleneck is the time needed for other stars to diffuse into the required orbital phase space. This “loss cone” of stars could cause SMBH binaries to stall in their inspirals before they get close enough for gravitational radiation to be significant (Begelman et al. 1980; see Sigurdsson & Rees 1997, Milosavljevic & Merritt 2003, Sigurdsson 2003, and Makino & Funato 2004 for recent discussions). For IMBHs, this is not likely to be a problem. As discussed by Yu & Tremaine (2003), once the original contingent of stars is ejected from the loss cone, the system will settle into a state in which the rate of diffusion of stars into the loss cone is balanced by the rate at which they are ejected by interaction with the IMBH-SMBH binary. From equation (38) of Yu & Tremaine (2003), the hardening timescale for a black hole binary of total mass M is

$$t_h \approx 6 \times 10^9 \text{ yr} (M/3.5 \times 10^6 M_\odot) \times (1 M_\odot/m_*) (2 \times 10^{-4} \text{ yr}^{-1}/n_{\text{diff}}), \quad (10)$$

where m_* is the typical stellar mass in the central regions and $2 \times 10^{-4} \text{ yr}^{-1}$ is a characteristic value for the diffusion rate n_{diff} into the loss cone. Therefore, depending on the details of the stellar distribution, the orbital radius of the IMBH could be reduced by several e -foldings in a Hubble time, especially if the SMBH has $M \lesssim 10^6 M_\odot$. This process could be enhanced slightly because stars that interact with the IMBH typically will not be ejected entirely from the core; hence, they will return for several interactions (Milosavljevic & Merritt 2003). In addition, gas dynamical friction from molecular clouds (see, e.g., Ostriker 1999) can shrink the orbit further.

Final merger with the SMBH.—By the time $a < 10^{-3}$ pc, gravitational radiation can be important for an IMBH-SMBH binary. The timescale to merger is

$$\tau_{\text{GR}} \approx 10^{12} \text{ yr} (\mu/10^3 M_\odot)^{-1} (M/10^6 M_\odot)^{-2} \times (a/0.001 \text{ pc})^4 (1 - e^2)^{7/2} \quad (11)$$

(from, e.g., Peters 1964), where e is the orbital eccentricity. Thus, if $a < 0.0003$ pc or the eccentricity is high, the merger can happen within a Hubble time.

Therefore, in contrast to what might be the case for two SMBHs in a binary (Begelman et al. 1980), it is unlikely that there is a hang-up problem for an IMBH-SMBH binary. The difference is that an IMBH-SMBH binary at a given separation has a much smaller binding energy than a binary with two SMBHs. Hence, the stars that are ejected or displaced in the process of hardening the binary come from a relatively smaller volume in which the relaxation time is short enough to repopulate the loss cone. Given that each IMBH by assumption brings with it several hundred thousand new stars, there will always be a fresh set of stars to supply dynamical friction. It is therefore possible that tens or even hundreds of IMBHs could be brought in sequentially, each merging with the SMBH before the next IMBH arrives.

Note that this situation is dramatically different from the processes for mergers of stellar-mass black holes with SMBHs.

In that case, the dynamical friction time for stellar-mass black holes is much too long to get to the center in a Hubble time. As a result, only rare scatters of stellar-mass black holes into extremely high eccentricity orbits, followed by capture onto the SMBH by release of energy in gravitational radiation, can lead to a merger (e.g., Freitag 2003; Sigurdsson 2003). In contrast, the scenario we describe for IMBHs leads to sinking of the IMBH toward the center on a relatively short timescale. Gravitational radiation capture of black holes on hyperbolic orbits is not necessary.

If the timescale for dynamical friction and merger is longer than the timescale for the next IMBH to sink in (e.g., because the stellar number density at the center is much less than we have assumed), then a few IMBHs will interact with each other as they orbit the SMBH. This will lead to instabilities in the orbits. The exact criterion for instability depends on mass ratios and eccentricities (e.g., see Mardling & Aarseth 2001 for a comparable-mass binary orbited by a tertiary of arbitrary mass), but if orbits of particles approach each other within a few tens of percent of their orbital radii, instability usually results.

Once this occurs, the orbiting IMBHs will interact with each other until either (1) secular resonances drive the inner IMBH close enough to the SMBH that the pair merges because of gravitational radiation (a situation that preliminary simulations suggest may be surprisingly common), or (2) one or several IMBHs are ejected, implying by energy conservation that the inner one or several IMBHs are driven closer to the SMBH. In the latter case, simulations must be performed to determine the efficiency of this process, that is, the average number of IMBHs ejected for each one that merges. If simulations of stellar-mass black holes around an IMBH are a guide, then ejections may be dominant (see, e.g., Baumgardt et al. 2004). However, the dynamics of SMBH-IMBH systems could be different in several important ways. For example, if an IMBH is ejected from the core but not the entire bulge, its periape is still of the order of the IMBH-SMBH binary semimajor axis; thus, barring significant deflection during its orbit, it could interact again on the next pass. In addition, although the IMBH-SMBH mass ratio is small enough to prevent ejection of the binary, if the inner region has been evacuated of stars because of prior interactions, then the small binary kick due to IMBH ejection will cause the binary to move significantly to where it can interact with more stars and harden further. Numerical details of the interactions also need to be computed to estimate quantities such as the eccentricity in the sensitivity band of a particular gravitational radiation detector and to determine whether two IMBHs might pass close enough to each other to form bound pairs by the loss of energy to gravitational radiation, leading to IMBH-IMBH mergers (D. Hamilton 2004, private communication).

For the purposes of this paper, however, the main point is that the IMBHs are expected to eventually merge with the central SMBH, rather than stalling or being ejected. As we now discuss, this is a high mass ratio merger (and hence comparatively easy to calculate) with a large enough signal-to-noise ratio (S/N) that it will be possible to detect it near the end of inspiral in just a few cycles, requiring very few templates.

3. DETECTION OF IMBH-SMBH GRAVITATIONAL RADIATION

The information content of the signal from an IMBH-SMBH binary depends on the S/N. To compute the signal strength, we assume for simplicity that the binary is nearly circular by the

time it enters the sensitivity band of an instrument such as *LISA*; we discuss the possibility of an eccentric binary in § 4.

The rest-frame frequency of gravitational radiation from a nearly circular binary a time T_{merge} from merger is (for the basic equations, see Peters & Mathews 1963; Peters 1964)

$$f_{\text{GW, rest}} = 7 \times 10^{-4} \text{ Hz} (\mu/10^3 M_{\odot})^{-3/8} \times (M/10^6 M_{\odot})^{-1/4} (T_{\text{merge}}/1 \text{ yr})^{-3/8}, \quad (12)$$

at which point the orbital semimajor axis in units of the gravitational radius $r_g = GM/c^2$ is

$$a/r_g = 19(\mu/10^3 M_{\odot})^{1/4} (M/10^6 M_{\odot})^{-1/2} (T_{\text{merge}}/1 \text{ yr})^{1/4}. \quad (13)$$

If the source is at a redshift z , then the observed frequency is $f_{\text{obs}} = f_{\text{GW, rest}}/(1+z)$. From, for example, Schutz (1997), the dimensionless amplitude of a circular binary at a line-of-sight comoving distance D_M , averaged over all observer angles, is

$$h = 2^{2/3} (4\pi)^{1/3} G^{5/3} c^{-4} f_{\text{GW, rest}}^2 \mu M^{2/3} / D_M \\ = 1.3 \times 10^{-21} (\mu/10^3 M_{\odot})^{3/4} (M/10^6 M_{\odot})^{1/2} \\ \times (T_{\text{merge}}/1 \text{ yr})^{-1/4} (3 \text{ Gpc}/D_M). \quad (14)$$

At the innermost stable circular orbit (ISCO) for a nonrotating SMBH, $a_{\text{ISCO}} = 6GM/c^2$, the amplitude and rest-frame frequency are

$$h_{\text{ISCO}} = 1.4 \times 10^{-20} (\mu/10^3 M_{\odot}) (3 \text{ Gpc}/D_M) \\ f_{\text{ISCO}} = 4.4 \times 10^{-3} \text{ Hz} (M/10^6 M_{\odot})^{-1}. \quad (15)$$

Note that the amplitude at the ISCO is independent of M , because $h \propto f^{2/3} M^{2/3}$ and $f \propto M^{-1}$.

The effective *LISA* noise includes contributions from the instrument and unresolved binaries (see, e.g., Larson et al. 2000).¹ From $\sim 2 \times 10^{-4}$ to 10^{-3} Hz, unresolved Galactic double white dwarf binaries exceed the instrumental noise (e.g., Farmer & Phinney 2003); from $\sim 2 \times 10^{-3}$ to 10^{-2} Hz, in contrast, there will typically be one or zero double white dwarf binaries in a 10^{-8} Hz bin; hence, after several years of operation, it will be possible to model individual binaries and subtract them from the data stream. Unresolved extragalactic double white dwarf binaries will, however, continue to make a contribution. The minimum total noise is in the few mHz range, where the total one-sided spectral noise density at a $S/N = 10$ is

$$S_n(10 \sigma) \approx 1.5 \times 10^{-19} \text{ Hz}^{-1/2}, \quad 3 \times 10^{-3} \text{ Hz} < f_{\text{obs}} < 10^{-2} \text{ Hz}. \quad (16)$$

The time necessary to detect an SMBH-IMBH binary at $S/N = 10$ is $T_{\text{obs}} = [S_n(10 \sigma)/h]^2$. If $3 \times 10^{-3} \text{ Hz} < f_{\text{obs}} < 10^{-2} \text{ Hz}$, then

$$T_{\text{obs}} \approx 1.2 \times 10^4 \text{ s} (\mu/10^3 M_{\odot})^{-3/2} (M/10^6 M_{\odot})^{-1} \\ \times (T_{\text{merge}}/1 \text{ yr})^{1/2} (D_M/3 \text{ Gpc})^2. \quad (17)$$

¹ See also <http://www.srl.caltech.edu/~shane/sensitivity/makecurve.html>.

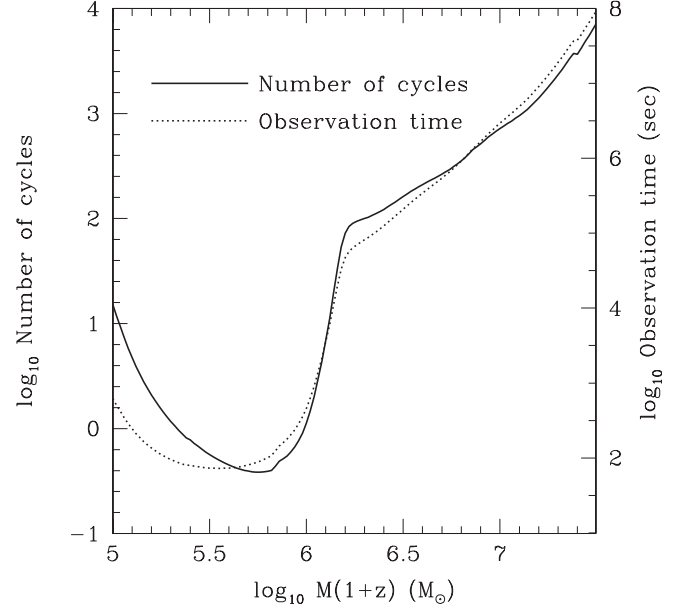


FIG. 1.—Minimum observation time (*dotted line*) and corresponding number of gravitational wave cycles (*solid line*) required to get $S/N = 10$ at the innermost stable circular orbit from a circular binary of total mass M and reduced mass $\mu = 10^3 M_{\odot}$, at a line-of-sight comoving distance of $D_M = 3 \text{ Gpc}$. This figure indicates the time and cycles needed if the IMBH were to be fixed in an orbit at the ISCO; in reality, the IMBH will typically spend several months at frequencies comparable to f_{ISCO} . Therefore, if $M(1+z) \lesssim \text{few} \times 10^6 M_{\odot}$, it is possible to achieve a high S/N in a very short time with an IMBH-SMBH binary.

Multiplying f_{obs} by T_{obs} gives the number of cycles in the time T_{obs} ,

$$N = 8(1+z)^{-1} (\mu/10^3 M_{\odot})^{-15/8} (M/10^6 M_{\odot})^{-5/4} \\ \times (T_{\text{merge}}/1 \text{ yr})^{1/8} (D_M/3 \text{ Gpc})^2. \quad (18)$$

The minimum observational time and number of cycles are obtained when the source is near the ISCO, which occurs in the most favorable frequency band $3 \times 10^{-3} \text{ Hz} < f_{\text{obs}} < 10^{-2} \text{ Hz}$ when the redshifted mass $M(1+z)$ is between 1.5×10^6 and $4.4 \times 10^5 M_{\odot}$. At this point,

$$T_{\text{obs, min}} = 1200 \text{ s} (\mu/10^3 M_{\odot})^{-2} (D_M/3 \text{ Gpc})^2 \\ N_{\text{min}} = 5(1+z)^{-1} (\mu/10^3 M_{\odot})^{-2} (M/10^6 M_{\odot})^{-1} (D_M/3 \text{ Gpc})^2. \quad (19)$$

More generally, as in Figure 1, one can compute the minimum observation time and number of cycles for $S/N = 10$, $\mu = 10^3 M_{\odot}$, and any M based on the frequency at the ISCO and the projected total noise curve. A prograde encounter with a rapidly rotating SMBH will go to higher frequencies during its inspiral than will an encounter with a nonrotating SMBH. This increases the energy released in gravitational radiation and, importantly, increases the mass threshold at which the observed signal is in the most sensitive frequency range of the *LISA* band. The numbers in Figure 1 are therefore conservative.

The expected rate of such events depends on a number of uncertain astrophysical parameters. In particular, it is clear that the low-mass end of an SMBH (such as $\lesssim 10^6 M_{\odot}$) is of great importance. Yu & Lu (2004) use the velocity dispersion data of Sheth et al. (2003) to estimate that the comoving number density of black holes in this mass range is $\sim \text{few} \times 10^{-3} \text{ Mpc}^{-3}$. Out to $\sim 3 \text{ Gpc}$ (where $z \approx 0.8$, so redshift corrections are

minor), the volume of the universe is $\approx 10^{11}$ Mpc³, implying $\sim \text{few} \times 10^8$ black holes in the required mass range. If on average N_{merge} IMBH-SMBH mergers per galaxy happen in $\sim 10^{10}$ yr, this implies an overall rate of a few percent of $N_{\text{merge}} \text{ yr}^{-1}$.

The value of N_{merge} is highly uncertain. The M - σ relation (e.g., Ferrarese & Merritt 2000; Gebhardt et al. 2000; Merritt & Ferrarese 2001a, 2001b; Tremaine et al. 2002) implies that the SMBH typically contains $\sim 10^{-3}$ of the mass of the central bulge, which means that $M_{\text{bulge}} \sim 10^9 M_{\odot}$ for $M \sim 10^6 M_{\odot}$. If $\sim 10\%$ of this mass was originally in the form of young massive clusters (that later merged with the bulge), and if a few tens of percent of such clusters form IMBHs, this suggests $N_{\text{merge}} \approx 100$ over the lifetime of the galaxy. This is consistent with observations of actively interacting galaxies such as M82, which have hundreds of super star clusters younger than 10^8 yr and presumably have had many times that number over their lifetimes. Note that the total mass added by such mergers is much less than the mass of an SMBH; hence, this number of mergers is not in conflict with limits based on the integrated light from quasars (Yu & Tremaine 2002). It is therefore reasonable that there will be several IMBH-SMBH mergers detectable with *LISA* during the few years of its lifetime.

The observable number and precision of inferences could change depending on the astrophysics involved. For example, if most massive clusters are formed at $z \sim 2$ in accordance with the peak in the star formation history of the universe (e.g., Madau et al. 1998) and their IMBHs merge with the SMBH in less than 1 Gyr, then most mergers are at a high enough redshift that the frequencies are low; hence, the S/N values are decreased. Even in this case, the signal strength could be large enough that elaborate templates are unnecessary for detection (see § 4). If, in contrast, the process of spiraling in and merging typically takes a few billion years, mergers will be distributed over time, and a significant number of them will take place at low redshift when the S/N is high in just a few cycles.

The maximum distance at which an IMBH-SMBH binary could be detected at $S/N > 10$ (with perfect signal processing) can be estimated from equation (15). The line-of-sight comoving distance saturates at high redshift (see, e.g., Peebles 1993) to ~ 10 Gpc for cosmological parameters $\Omega_M = 0.27$, $\Omega_{\Lambda} = 0.73$, and $H_0 = 72 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (e.g., Spergel et al. 2003). From equation (15), the amplitude and observed frequency near the ISCO are then $h_{\text{ISCO}} \approx 4 \times 10^{-21} (\mu/10^3 M_{\odot})$ and $f_{\text{ISCO,obs}} = 4.4 \times 10^{-3} \text{ Hz} (M/10^6 M_{\odot})$, where μ and M are measured in the rest frame. From Larson et al. (2000), $S/N = 10$ in a 1 yr *LISA* integration crosses an amplitude of 4×10^{-21} at a frequency of $\approx 2 \times 10^{-4}$ Hz, including white dwarf noise; hence, a 1000 – $10^6 M_{\odot}$ binary could be observed out to a redshift $z \approx 20$ at $S/N = 10$ in a 1 yr integration.

4. DISCUSSION

The scenario discussed in this paper relies on still-uncertain details of the production and distribution of IMBHs (see Miller & Colbert 2004 for a discussion of formation mechanisms and of issues such as wind losses in the formation of high-mass stars). Here we have focused on the particular idea that IMBHs are formed in runaway collisions in clusters. Other formation mechanisms have different implications. For example, Madau & Rees (2001) propose that IMBHs form from the evolution of solitary, nearly zero metallicity (Population III) stars in the early universe. In such a case, hierarchical merging of mini-halos could produce multiple IMBH-SMBH mergers in the

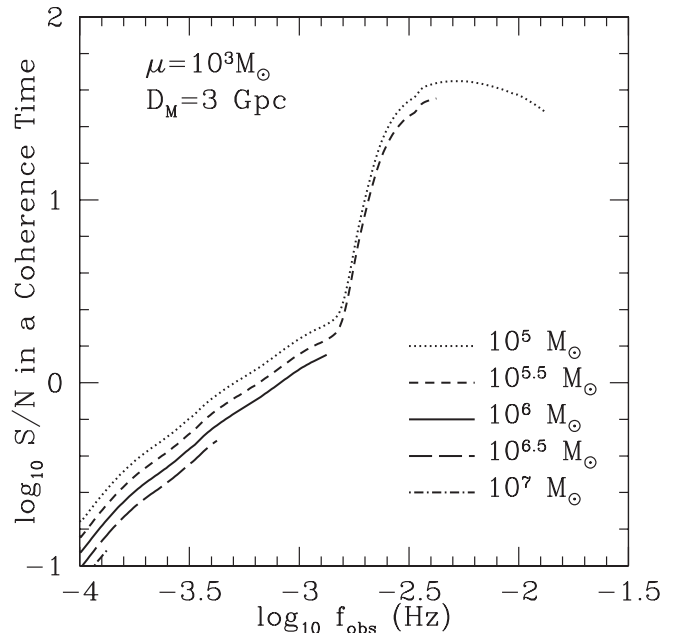


FIG. 2.—S/N in a coherence time (see text) for a binary at a line-of-sight comoving distance $D_M = 3$ Gpc that has an IMBH mass $\mu = 10^3 M_{\odot}$ and several possible total masses. Both instrumental noise and white dwarf noise are included here. We plot S/N vs. frequency from $f_{\text{obs}} = 10^{-4}$ Hz to the observed frequency at the ISCO (we assume a redshift $z = 0.8$ at 3 Gpc). This is the maximum signal obtainable in a simple power density spectrum. For $M < 10^6 M_{\odot}$, the signal will be detected strongly in a coherence time, greatly simplifying data analysis.

high-redshift universe. However, at this point too little is known about such scenarios to make informed estimates of rates. Our main point is that if even a few IMBH-SMBH mergers are detected they will be useful as uniquely precise tests of strong gravity.

To see this, consider first the inspirals of stellar-mass black holes into SMBHs. These are promising as probes of the Kerr spacetime, but a difficulty is that the waves are expected to be weak enough that thousands of orbits are required to achieve a reasonable S/N (e.g., Barack & Cutler 2004). As a result, a very large number of templates are required to detect the signal, which could make analysis difficult. In contrast, if IMBH-SMBH mergers occur a few times per year, their signal strengths will lead to detections within just a few orbits, near the end of inspiral. As a result, as we now show, only standard Fourier transforms are needed, rather than any elaborate templates.

Consider the time for a nearly circular orbit to merge (Peters & Mathews 1963; Peters 1964),

$$T_{\text{merge}} \approx 6 \times 10^{17} \text{ yr} (M_{\odot}^3 / \mu M^2) (a/1 \text{ AU})^4. \quad (20)$$

For observation times $T_{\text{obs}} \ll T_{\text{merge}}$, the change in gravitational wave frequency is $\Delta f \sim (T_{\text{obs}}/T_{\text{merge}}) f_{\text{obs}}$. The frequency resolution is $\delta f = 1/T_{\text{obs}}$, so if $\delta f > \Delta f$ the signal shows up as a single peak in a power density spectrum. Therefore, if one observes for a coherence time $T_{\text{coh}} = (T_{\text{merge}}/f_{\text{obs}})^{1/2}$ (such that $\delta f = \Delta f$), one has the maximum possible power in a single peak in a power density spectrum. In Figure 2 we show the S/N for circular orbits over a coherence time for different unredshifted SMBH masses (assuming in each case $\mu = 10^3 M_{\odot}$) for observed frequencies from 10^{-4} Hz to $f_{\text{ISCO}}/(1+z)$, where we assume $D_M = 3$ Gpc and therefore $z = 0.8$. From this figure we see that if $M < 10^6 M_{\odot}$, then a circular signal will be

detectable with $S/N > 10$ in a coherence time near the end of inspiral. If there are closer mergers, say, with $D_M = 1$ Gpc, the S/N could be as large as hundreds.

As a result, if IMBH-SMBH mergers occur, then during the end of inspiral they are detectable without modeling. At earlier times this is not the case, but it will be possible to use the late-time detections to work backward and determine the full set of orbital parameters by connecting the phases of the individual segments. It will also be possible to establish very precise initial conditions for numerical modeling of the merger phase. Some idea of the precision with which parameters will be estimated for such a merger (after fitting a yearlong wave train) can be obtained from Tables 2 and 3 of Barack & Cutler (2004). Linear scaling from these results is not appropriate, given correlations between parameters, but the much greater S/N of IMBH-SMBH mergers (thousands instead of tens) suggests that, for example, the redshifted masses and the dimensionless angular momentum of the SMBH will be estimated to fractional precisions of better than 10^{-5} .

If the orbit is eccentric, or if other effects (e.g., pericenter precession or Lense-Thirring precession) produce peaks separated in frequency by more than $1/T_{\text{coh}}$ from the main peak, then the analysis is complicated somewhat. However, these frequencies will also remain stable over T_{coh} , so with high S/N one will be able to detect each of these peaks independently and model the changes in eccentricity, orbital inclination, and so on by building up the full wave train.

As with mergers of stellar-mass with supermassive black holes, the orbits of IMBHs into SMBHs will map out the Kerr spacetime and test the no-hair theorem (e.g., Ryan 1997). In addition, we point out that the rate of inspiral (and decay of eccentricity, if this is nonnegligible) will provide a strict testbed for theoretical predictions of the flux and angular momentum functions in strong gravity. For example, for a 10^3 – $10^6 M_\odot$ binary, the total S/N is greater than 10^4 for the portion of the

orbit inside of $10 M$, so high-order contributions can be inferred empirically.

In future work we will proceed in two directions. First, we will explore the quantitative constraints on current post-Newtonian models that are possible from detection of an IMBH-SMBH merger. Second, we will investigate astrophysical scenarios in which the orbit would have significant eccentricity when the source is in the detectability band of *LISA*. Such eccentric orbits could arise from the scenario we discuss here, from plunge orbits (as in stellar-mass–supermassive mergers), or possibly from other mechanisms. If such scenarios are plausible, there is substantial extra information to be gleaned. Virtually all templates constructed thus far are specialized for ground-based detections of high-frequency waves, hence assuming that the orbits would have nearly circularized by the time the gravitational waves entered instrumental bands (see, e.g., Damour et al. 2002 for an update to 3.5PN order). The lack of analysis of post-Newtonian expansions of eccentric orbits means that observed eccentricity decay will at least provide self-consistency checks and possibly constrain additional PN parameters beyond those that are being investigated currently. Even if the orbits turn out to be mostly circular, there is a wealth of data that could be extracted from mergers of supermassive and intermediate-mass black holes.

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