

PROMPT MERGERS OF NEUTRON STARS WITH BLACK HOLES

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ABSTRACT

Mergers of neutron stars with black holes have been suggested as candidates for short gamma-ray bursts. They have also been studied for their potential as gravitational wave sources observable with ground-based detectors. For these purposes, it is important to know under what circumstances such a merger could leave an accretion disk or result in a period of stable mass transfer. We show that, consistent with recent numerical simulations, it is expected that mergers between neutron stars and black holes will be prompt, with no accretion disk and no stable mass transfer, if the black hole has a mass greater than that of the neutron star and is spinning slowly. The reason is that for comparable masses, angular momentum loss to gravitational radiation starts a plunge orbit well outside the innermost stable circular orbit, causing direct merging rather than extended mass transfer. Even when the black hole is spinning rapidly and exactly prograde with respect to the orbit, we show that it is possible within current understanding that no accretion disk will form under any circumstances, but resolution of this will require full general relativistic numerical simulations with no approximations.

Subject headings: black hole physics — gamma rays: bursts — gravitational waves — stars: neutron — stellar dynamics

1. INTRODUCTION

Compact object mergers are expected to be a prominent source for ground-based gravitational wave detectors (e.g., Nutzman et al. 2004) and have also been invoked as the central engines of short gamma-ray bursts (e.g., Narayan et al. 1992; Mészáros et al. 1999; Piran 1999; Fryer et al. 1999). There has therefore been significant effort devoted to numerical modeling of such mergers. Most of this effort has focused on mergers between two black holes (BHs) or two neutron stars (NSs), but as BH-NS coalescence could be a prominent source of high-frequency gravitational waves there have also been some studies of this type of event.

The earliest simulations used a Newtonian potential for simplicity (Kluźniak & Lee 1998; Uryu & Eriguchi 1999; Lee & Kluźniak 1999a, 1999b; Janka et al. 1999; Lee 2000, 2001; Rosswog et al. 2004). These simulations typically showed a period of stable mass transfer from the neutron star to the black hole, in which the widening of the orbit caused by the transfer is combined with loss of angular momentum to gravitational radiation (other papers discussing this idea include Clark & Eardley 1977, Jaranowski & Krolak 1992, Portegies Zwart 1998, and Davies et al. 2005). As a result, the merger is extended greatly in duration. Some of these simulations even showed a “bouncing” effect, by which the onset of mass transfer produces a significant eccentricity in the orbit and mass is donated in bursts rather than continuously. If this were a realistic description of such mergers, it would have a major effect on the gravitational radiation waveforms and electromagnetic signatures of these events.

However, as we discuss here, Newtonian simulations are *not* realistic, and in fact give a qualitatively incorrect picture of the merger. Indeed, some recent numerical simulations of BH-star mergers that include relativistic pseudopotentials (e.g., Kluźniak & Lee 2002; Rosswog 2005) or motion of a neutron star in the background spacetime of a massive black hole (Rasio et al. 2005) show that in many circumstances there is simply a direct merger, rather than stable mass transfer or the formation

of an accretion disk. This was anticipated by Kochanek (1992; see the end of his § 7) and has dramatically different implications for short gamma-ray bursts and gravitational waves.

Here we show that prompt mergers are a natural result of general relativistic effects that have no Newtonian analog. As emphasized first in this context by Rasio & Shapiro (1999, pp. 18–19), of special importance is the existence of a minimum in the specific angular momentum of circular orbits, at the innermost stable circular orbit (ISCO). Only if tidal disruption occurs outside the ISCO can there be an accretion disk or stable mass transfer; otherwise, the matter simply plunges directly into the black hole. The ratio of tidal radius to ISCO radius decreases with increasing black hole mass, hence only low-mass black holes can disrupt neutron stars outside the ISCO. In this limit, however, other effects enter to enhance the likelihood of a plunge. First, if the mass of the neutron star is nonnegligible, then the dynamical instability of the orbit occurs at a greater separation than the radius of the ISCO computed using just the mass of the black hole. Second, the flatness of the specific angular momentum profile near the ISCO means that even a small loss of angular momentum will lead to a rapid plunge. For a comparable-mass BH-NS binary, gravitational radiation is a significant sink of angular momentum. As a result, the effective radius of dynamical instability is appreciably outside the ISCO, and a one-time plunge becomes possible even for low-mass black holes, if the spin parameter of the black hole is modest.

In § 2 we expand quantitatively on this argument for non-spinning black holes, using recent semianalytic results. In § 3 we discuss the role of the spin. We show that although encounters with rapidly spinning black holes can result in accretion disks and possibly stable mass transfer, the parameter space for this is surprisingly small because for a comparable-mass binary in which one component (the neutron star) is spinning slowly, the effective spin of the system is reduced substantially and hence the radius of the ISCO is increased greatly. We discuss the implications of our findings in § 4.

2. NONROTATING BLACK HOLES

To determine whether tidal disruption occurs inside or outside the ISCO, we need to compute (1) the separation at which tidal stripping begins to occur, (2) the separation for which dynamical inspiral begins (i.e., the effective separation at the ISCO), and (3) the effects of gravitational radiation on the plunge. The first two points will require numerical simulations for full understanding, but as a guide we will use recent semi-analytic results. For the last point we will apply the lowest order quadrupolar radiation expressions derived by Peters & Mathews (1963) and Peters (1964) to get a rough idea of the magnitude of the effect.

In this section and the rest of the Letter, we assume that a neutron star of gravitational mass m_{NS} orbits a black hole of gravitational mass m_{BH} , with an orbital separation r . Note that to be strictly correct we would need to state precisely in which coordinate system we measure r , but for the estimates in this Letter we will assume that r is roughly a Boyer-Lindquist radius even though for a comparable-mass binary the spacetime will not be Kerr. With these masses we define a total mass $M \equiv m_{\text{NS}} + m_{\text{BH}}$, a reduced mass $\mu \equiv m_{\text{NS}}m_{\text{BH}}/M$, and a symmetric mass ratio $\nu \equiv \mu/M$. Note that the maximum value of ν is $\frac{1}{4}$; this makes it a good expansion parameter, a feature exploited in various post-Newtonian (e.g., Blanchet 2002) and equivalent one-body (Buonanno & Damour 1999; Damour et al. 2000; Damour 2001) treatments of strong gravity. We also assume that the dimensionless spin parameter of the black hole is $\hat{a} \equiv J/m_{\text{BH}}^2$ (in geometrized units in which $G = c = 1$, which we use henceforth; $\hat{a} = 0$ for this section only) and that the equilibrium radius of the neutron star far from tidal fields is R_0 . The lack of sufficient viscosity to enforce corotation (Kochanek 1992; Bildsten & Cutler 1992) suggests that the neutron star will be rotating slowly.

We now treat in order the tidal stripping, the separation at the ISCO, and the effects of gravitational radiation.

2.1. Radius of Tidal Stripping

Several authors have generalized the Newtonian fluid analysis of tidally locked ellipsoids to general relativistic orbits (e.g., Fishbone 1973; Mashoon 1975; Wiggins & Lai 2000). Here we follow the treatment of Wiggins & Lai (2000), who analyze the physically more realistic irrotational case as well as the corotating orbits examined in previous treatments (note that, as shown by Bildsten & Cutler 1992, the weaker tidal fields from more massive objects imply that it is even more difficult to synchronize a BH-NS system than a NS-NS system).

Wiggins & Lai (2000), like other authors, simplify by assuming that $m_{\text{BH}} \gg m_{\text{NS}}$, but as is standard we will proceed with caution by applying their formulae even to comparable-mass binaries. We note that even in the limit of equal mass objects, their results are reasonable: for example, they predict that in this case mass transfer would begin when the separation is slightly more than twice the unperturbed radius of the neutron star, which is expected because the NS will be distorted in the direction of the black hole. They also note that their treatment of the neutron star's self-gravity as Newtonian (a standard assumption in such work) actually underestimates its binding, because at $m_{\text{NS}}/R_0 \sim 0.2$, general relativistic gravity is significantly stronger than Newtonian gravity. The effective tidal radii are therefore actually smaller than we quote here, hence our conclusions are conservative (i.e. neutron stars are even more likely to merge directly without forming an accretion disk).

Wiggins & Lai (2000) define two dimensionless parameters,

$$\hat{R}_0 \equiv \frac{R_0}{m_{\text{NS}}} (m_{\text{NS}}/m_{\text{BH}})^{2/3},$$

$$\hat{r} \equiv (r/m_{\text{BH}})/\hat{R}_0, \quad (1)$$

to characterize the effect of tides. When \hat{r} is less than some critical value \hat{r}_{tide} , tidal stripping begins. For $\hat{a} = 0$ they find $\hat{r}_{\text{tide}} \approx 2-2.4$, depending on the polytropic index n and only very weakly dependent on whether the neutron star is corotating or irrotational (Wiggins & Lai 2000 show that for corotating binaries \hat{r}_{tide} is larger, but only by $\sim 3\%$, than it is for irrotational binaries). Given a value of \hat{R}_0 , one can then compute the actual separation r at which the stripping occurs. For a particular critical r/M (e.g., $r/M = 6$, the radius of the ISCO in the Schwarzschild spacetime), one can therefore define $\hat{R}_{0,\text{crit}}$ such that for $\hat{R} < \hat{R}_{0,\text{crit}}$ disruption occurs inside the critical radius. For $r_{\text{crit}}/M = 6$ and $\hat{a} = 0$, they find $\hat{R}_{0,\text{crit}} = 2.54$ for $n = 1$ and $\hat{R}_{0,\text{crit}} = 2.76$ for $n = 3/2$. By itself this would suggest that for a neutron star of mass $m_{\text{NS}} = 1.4 M_{\odot}$ and radius $R_0 = 5m_{\text{NS}}$ (≈ 10 km), a black hole of mass greater than $\approx 4.4 M_{\odot}$ would swallow the star whole. As we show in the next two sections, however, other effects lower this threshold significantly.

2.2. Radius of the ISCO

Interest in the gravitational wave signatures of merging binary black holes has led to the development of schemes to approximate the spacetime when the two objects in the binary have comparable mass (e.g., Buonanno & Damour 1999; Damour et al. 2000; Damour 2001; Blanchet 2002). These usually involve expansions to some post-Newtonian (PN) order, where an n PN expansion means including terms out to $(v^2/c^2)^n$. As tidal effects enter only at the 5PN order (e.g., Wiggins & Lai 2000 and many other references), they can largely be ignored.

Several authors have used these schemes to estimate the separation of a binary at the ISCO for arbitrary mass ratios. For zero spin and two equal masses ($\nu = \frac{1}{4}$), they find $r_{\text{ISCO}} \approx 5M$ (see Damour et al. 2000, eqs. [4.36c], [4.40c], and [4.41] for the formulae needed to compute angular momenta for circular orbits with two arbitrary masses; note that their parameter $\omega_{\text{static}} = 0$, as is determined in Damour et al. 2002). Recalling that $M = m_{\text{NS}} + m_{\text{BH}}$, this means that the separation is significantly greater than the $6m_{\text{BH}}$ separation one would obtain by just considering one black hole. The symmetric mass ratio is relatively flat for comparable masses (e.g., for $m_{\text{BH}} = 2m_{\text{NS}}$ we have $\nu = \frac{2}{9}$), so $r_{\text{ISCO}} \approx 5M$ holds for all mass ratios in this range. Therefore, let us reconsider the critical black hole mass such that a neutron star is disrupted outside the ISCO. For $m_{\text{BH}}/m_{\text{NS}} = 2$ and $R_0 = 5m_{\text{NS}}$, $\hat{R}_0 = 3.15$. At this \hat{R}_0 , Wiggins & Lai (2000) find $\hat{r}_{\text{tide}} \approx 3$ and hence $r/m_{\text{BH}} = 7.25$. Therefore, $r/M = 4.83$, and this orbit is actually inside the ISCO. Consider now a minimal mass black hole of $m_{\text{BH}} = 2.2 M_{\odot}$ in a binary with a neutron star of $m_{\text{NS}} = 1.4 M_{\odot}$ and $R_0 = 5m_{\text{NS}}$. Then $\hat{R}_0 = 3.70$, $\hat{r}_{\text{tide}} = 2.27$ according to Wiggins & Lai (2000) for $n = 1$, and thus $r/m_{\text{BH}} = 8.40$. This implies $r/M = 5.13$. This is technically just outside the ISCO, but as we will see in the next section, angular momentum loss to gravitational radiation is significant and the plunge actually starts well outside this radius. Higher mass neutron stars are more compact (with $R_0 < 5m_{\text{NS}}$) and will also not be tidally disrupted. Only low-mass stars with $R_0 >$

$5m_{\text{NS}}$ might leave accretion disks, but as we now show, even this is not certain.

2.3. Effect of Gravitational Radiation Losses

The ISCO as a sharp dividing line is only strictly useful in the test particle limit, where no other effects could cause inspiral. Any mechanism that leads to inspiral of nearly circular orbits, whether magnetic angular momentum transport in accretion disks or losses to gravitational radiation, will blur this line (see, e.g., Buonanno & Damour 2000 for a detailed discussion of this phenomenon). Near to but outside of the ISCO, the specific angular momentum of circular orbits is very flat with radius, meaning that even a small amount of angular momentum loss can have drastic consequences (see, e.g., Lombardi et al. 1997). In particular, even after the neutron star has reached the tidal radius, the matter that is stripped will take some finite time to separate significantly from the star, and during this time all the matter is still emitting gravitational radiation. We suspect that the omission of these effects by Prakash et al. (2004), who did a semianalytic treatment of BH-NS mergers using the Blanchet (2002) post-Newtonian equations, is the reason that they reached qualitatively different conclusions from ours.

Consider, for example, loss to gravitational radiation. From Peters & Mathews (1963) and Peters (1964), the lowest order (quadrupolar) loss rate of angular momentum for a nearly circular orbit is

$$dL/dt = -\frac{32}{5}(r/M)^{-7/2}\nu\mu. \quad (2)$$

Let the angular momentum at the ISCO be L_{ISCO} . If enough angular momentum is lost in a time short compared to the orbital period so that $L < L_{\text{ISCO}}$, then the black hole and neutron star will plunge together. For the hypothetical $1.4\text{--}2.2 M_{\odot}$ binary discussed above, an initial circular orbit of radius $5.13M$ has an angular momentum only $0.00078\mu M$ greater than that of the ISCO. From the formula above for angular momentum loss, this is radiated within a time $0.16M$, compared to an orbital period of $\approx 75M$ at that radius. Clearly, the binary undergoes a plunge well before the neutron star is tidally stripped.

These effects are summarized in Figure 1. Again assuming no spin of the black hole or neutron star, we show the critical radius-to-mass ratio for a neutron star, as a function of the ratio of the black hole mass to neutron star mass. The solid lines are for different criteria for the plunge, including the effects of gravitational radiation. For comparison we show a R_0/m_{NS} versus mass ratio for neutron stars with hard and soft equations of state, assuming a black hole with a mass just above the neutron star maximum in each case (to maximize the likelihood of tidal disruption outside the ISCO). We see that the neutron star is likely to be disrupted inside the radius of dynamical instability. Therefore, if the black hole is only spinning slowly, it will eat a neutron star whole rather than forming an accretion disk or producing stable mass transfer.

3. EFFECTS OF BLACK HOLE SPIN

Determination of the effective separation at the ISCO is much more challenging when the spin is nonzero and orbits are prograde. This is because the separation decreases with increasing spin, hence post-Newtonian approximations get

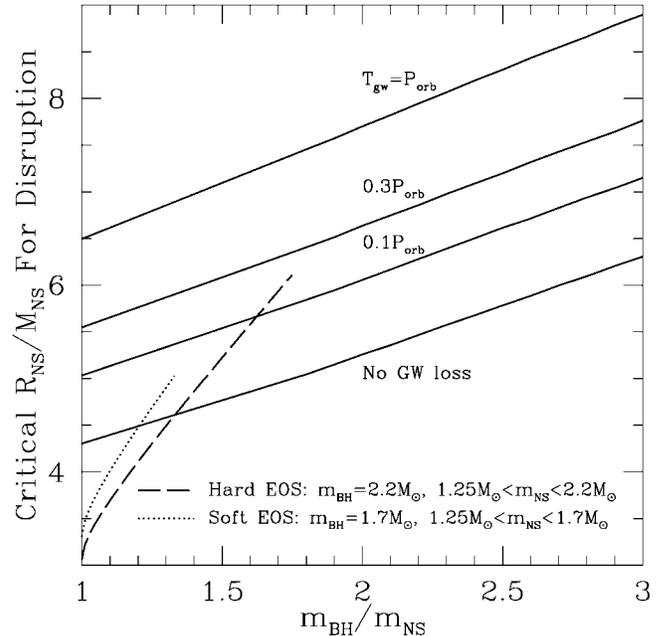


FIG. 1.—Critical radius-to-mass ratio for a neutron star, as a function of BH-to-NS mass ratio, for no rotation. Below the critical value, the neutron star is compact enough to plunge and be swallowed whole rather than be disrupted. The top solid line is constructed by assuming that the neutron star will plunge when, in one full orbit, it can reduce its angular momentum below the ISCO value via emission of gravitational radiation. The next two solid lines down reduce the allowed time to 30% and 10% of an orbit, respectively, and the bottom line ignores gravitational radiation losses entirely. For comparison, we show R_0/m_{NS} vs. $m_{\text{BH}}/m_{\text{NS}}$ for n -body equations of state (Akmal et al. 1998) that are hard (A18+UIX+dvb, in the Akmal et al. 1998 notation; dashed line) and soft (A18; dotted line). Stars constructed with relativistic mean field theory could be somewhat larger (see Lattimer & Prakash 2001 for a recent review). In both cases we assume that m_{BH} is just above the maximum mass of a neutron star, because this maximizes the likelihood of tidal disruption. We also consider m_{NS} as small as $1.25 M_{\odot}$, which is the lowest gravitational mass yet measured for a neutron star (for PSR J0737–3039B; see Lyne et al. 2004). This figure shows that direct merger, not stable mass transfer, is the likely outcome of the coalescence of a neutron star with a nonrotating black hole.

worse and full numerical relativity may be necessary. Nonetheless, we can reach some tentative conclusions following the work of Damour (2001) in constructing effective one-body spacetimes for comparable masses including spins. The critical point here is that although in principle the radius of the ISCO could be as low as M for prograde orbits in a maximally rotating Kerr spacetime, this radius plunges dramatically at high spin, hence if the effective spin is slightly less than maximal, then the ISCO radius is much larger and it becomes more difficult to disrupt neutron stars before merger. This also applies for orbits that are not exactly prograde, for which the effective radius of the ISCO is increased dramatically for rapidly spinning black holes (see Hughes & Blandford 2003).

Damour finds that for two bodies of masses m_1 and m_2 and spin angular momenta \mathbf{S}_1 and \mathbf{S}_2 , the effective angular momentum of the one-body spacetime is

$$\mathbf{S}^{\text{eff}} = \left(1 + \frac{3}{4} \frac{m_2}{m_1}\right) \mathbf{S}_1 + \left(1 + \frac{3}{4} \frac{m_1}{m_2}\right) \mathbf{S}_2. \quad (3)$$

The effective dimensionless spin parameter is then $\hat{a}_{\text{eff}} = \mathbf{S}^{\text{eff}}/M^2$. Because the neutron star is not expected to be rotating rapidly (i.e., $\mathbf{S}_{\text{NS}} \approx 0$), the effective spin of the spacetime is

diminished, which also means that the merger properties are not as sensitive to the precise black hole spin as they would be if the neutron star mass were negligible. For example, even if the black hole is rotating maximally ($S_{\text{BH}} = m_{\text{BH}}^2$), the maximum \hat{a}_{eff} for an equal-mass binary is only $\hat{a}_{\text{max}} = 0.44$ and for $m_{\text{BH}} = 2m_{\text{NS}}$ is only $\hat{a}_{\text{max}} = 0.61$.

Unfortunately, the approximations used by Damour (2001) get progressively worse with larger \hat{a}_{eff} , so it is difficult to draw definite conclusions. The abrupt shift in the ISCO near $\hat{a}_{\text{eff}} = 1$ is likely to mean that with a comparable-mass binary the ISCO is well outside its theoretical minimum radius of M ; for example, in a Kerr spacetime $r_{\text{ISCO}} = 4.46M$ when $\hat{a}_{\text{eff}} = 0.44$, and $r_{\text{ISCO}} = 3.79M$ when $\hat{a}_{\text{eff}} = 0.61$ (for the relevant expressions, see, e.g., Shapiro & Teukolsky 1983). The exact numbers will change for objects of comparable mass, but we expect the qualitative effect to be similar: the effective spin will not be near extremal for comparable-mass objects, so dynamical instability may still set in prior to tidal stripping.

Counterintuitively, therefore, somewhat *higher* mass ratios could be more likely to result in accretion disks and stable mass transfer for a rapidly spinning black hole, because the effective spin could be closer to maximal. It might, however, still be difficult to generate stable mass transfer. For example, consider an extreme Kerr black hole of mass $m_{\text{BH}} = 7 M_{\odot}$ orbited by a neutron star of mass $m_{\text{NS}} = 1.4 M_{\odot}$ and radius $R_0 = 5m_{\text{NS}}$. The higher mass ratio means that the full spacetime is likely to be reasonably close to Kerr, making extrapolations more reliable. We have $\hat{R}_0 = 1.71$ and $\hat{r}_{\text{tide}} = 2.3$. Then $r/m_{\text{BH}} = 3.93$, so $r/M = 3.28$. Using the Damour (2001) approximation, $\hat{a}_{\text{eff}} = 0.80$, so in the Kerr spacetime $r_{\text{ISCO}}/M = 2.91$. Taking these numbers at face value, even in this case gravitational radiation losses would cause the neutron star to plunge before it was stripped.

However, this is close enough to the tidal radius that details are important: for example, if in reality the effective spin parameter is $\hat{a}_{\text{eff}} = 0.90$, then $r_{\text{ISCO}}/M = 2.32$ and stable mass

transfer might ensue. We are therefore in a domain where existing approximations are inadequate to determine the fate of the merging neutron star. Full no-approximation general relativistic calculations will be necessary. Nonetheless, it appears possible that *no* plausible combination of neutron star and black hole masses, spins, and orbital inclinations will result in an accretion disk or stable mass transfer.

4. CONCLUSIONS

Mergers of neutron stars with black holes are of great interest for models of short gamma-ray bursts and as sources of high-frequency gravitational radiation. The likely signatures of such events depend strongly on whether the star is swallowed whole in a direct merger or whether it is tidally stripped far enough away from the black hole that an accretion disk forms or stable mass transfer occurs. We have shown that for slowly rotating black holes, current calculations of tidal stripping and of the spacetimes of binary compact objects suggest that direct merger will almost always take place. This is in agreement with recent general relativistic numerical simulations and could actually enhance the likelihood of a baryon-free environment for a short gamma-ray burst (e.g., Mészáros & Rees 1992). A prograde encounter of a neutron star with a rapidly rotating black hole is more likely to result in a disk, but the current understanding of such close mergers is uncertain enough that it could be that *any* NS-BH merger will be direct. Future fully general relativistic numerical simulations will be required to resolve this issue.

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