QPO Constraints on Neutron Stars

M. Coleman Miller [a]

[a] *University of Maryland, Department of Astronomy, College Park, MD 20742-2421, USA*

Abstract

The kilohertz frequencies of QPOs from accreting neutron star systems imply that they are generated in regions of strong gravity, close to the star. This suggests that observations of the QPOs can be used to constrain the properties of neutron stars themselves, and in particular to inform us about the properties of cold matter beyond nuclear densities. Here we discuss some relatively model-insensitive constraints that emerge from the kilohertz QPOs, as well as recent developments that may hint at phenomena related to unstable circular orbits outside neutron stars.

Key words:
dense matter — equation of state — gravitation — stars: neutron — X-rays: binaries

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1 Introduction

The cores of neutron stars are nonrelativistic and at several times nuclear density, meaning that laboratory experiments cannot probe directly their state of matter. It instead falls to astronomical observations to determine whether matter at these high densities has only nucleonic degrees of freedom, or whether there are more exotic components such as hyperons, condensates, or quark matter (see Lattimer & Prakash 2001 for a discussion of models and laboratory constraints).

There are in principle several ways that observations of neutron stars can constrain the equation of state (EOS) of cold matter beyond nuclear density:

- A given EOS implies a maximum gravitational mass as a function of spin frequency. An observed mass above this maximum rules out that equation of state.
- Similarly, each EOS sets a maximum spin frequency, beyond which the EOS can be eliminated.
- Each EOS has a relation between gravitational mass, equatorial radius, and spin frequency. These relations can intersect in one place for two different EOSs, but measurement at two different masses with high precision would suffice to determine the EOS uniquely.

- Given a minimum neutron star mass (based on formation mechanisms), a given EOS implies a maximum equatorial radius for a given spin frequency.

In practice, neutron star spin frequencies are too low to pose any interesting constraints, and radius measurements have been difficult to make precise because of systematic uncertainties (although see contribution by Lattimer, these proceedings). Currently, mass measurements are the most reliable of the constraints. Double neutron star systems have masses up to $1.44\,M_\odot$ (e.g., Thorsett & Chakrabarty 1999), which rule out some of the softest early equations of state. Recent observations of neutron stars in binaries with white dwarfs have suggested some have masses $> 1.7\,M_\odot$ (e.g., Nice, Splaver, & Stairs 2003), and the neutron star in Vela X-1 could have a mass $> 1.9\,M_\odot$ (Quaintrell et al. 2003). If so, this suggests a relatively hard equation of state at high densities, and argues against substantial contributions from exotic degrees of freedom.

Here we focus on the constraints obtained from timing phenomena. The launch of the *Rossi* X-ray Timing Explorer (*RXTE*) in 1995 allowed high-signal observation of variation frequencies of hundreds of Hertz and more, and led to the discovery of the so-called kilohertz quasi-periodic brightness oscillations (kHz QPOs) from more than twenty neutron stars in low-mass X-ray binaries (LMXBs). The high observed frequencies (up to 1330 Hz) requires that they be influenced by the strongly curved spacetime near neutron stars, and gives hope that the properties of the stars also affect the QPOs.

In § 2 we give an overview of the main characteristics of the kHz QPOs. In § 3 we discuss the various models proposed to explain them, with an emphasis on the substantial agreement in key areas. In § 4 we show that, in essentially all currently viable models, important constraints on the mass and radius of neutron stars follow from the kHz QPOs. In § 5 we explore the evidence for effects of the innermost stable circular orbit (ISCO), and the implications this has for neutron stars in LMXBs. We discuss future observations and theory in § 6.

2 Kilohertz QPOs

Low-frequency (∼few to tens of Hertz) quasi-periodic brightness oscillations from neutron star LMXBs were discovered as early as 1985 (see van der Klis 1989 for a review). However, it was not until the launch of *RXTE* that it became possible to probe with sensitivity the high-frequency realm. Although the lower-frequency QPOs may have parallels in QPOs from black holes (see Psaltis, Belloni, & van der Klis 1999) and possibly even white dwarfs (Mauche 2002), the higher-frequency kHz QPOs appear to have properties unique to neutron stars. We will therefore focus on the high frequencies because these have the best chance to inform us about neutron stars in particular.

There are several trends apparent in the population of kHz QPO sources (see van der Klis 2000 for a review). The kHz QPOs are:

- Paired. It is common to observe a pair of strong kHz QPOs in a given power density spectrum. On occasion, much weaker sidebands are also observed (Jonker, Méndez, & van der Klis 2000, 2005).
- High frequency. The highest frequency so far observed is 1330 Hz (from 4U 0614+091; van Straaten et al. 2000). The highest observed frequencies from many sources are comparable to the highest possible frequencies of stable circular orbits around neutron stars (see § 3 for more detail).
- Highly coherent. Although not strictly periodic, the QPOs are very sharp features in power density spectra. The quality factor $Q \equiv \nu/\mathrm{FWHM} \sim 20-200$, where ν is the centroid frequency and FWHM is the full width at half maximum. The most coherent kHz QPOs are dramatically sharper than either the lower-frequency QPOs in neutron stars or any QPOs from black hole candidates.
- Variable. The upper and lower kHz QPOs have both been observed to vary by hundreds of Hertz in a given source. Remarkably, the coherence of the oscillations stays high, a fact of great significance to models as we discuss further in § 3. Over short times, the frequency correlates positively with countrate, but over long times the behavior is more complicated (see § 5).
- Harmonically related to the spin frequency in their separation. The frequency separation between the upper and lower kHz QPO is not constant, but its value changes by only tens of Hertz as the individual frequencies change by hundreds of Hertz. More importantly, in all eight cases in which the spin frequency is known (from millisecond pulsations in burst oscillations or accretion-powered emission) and there are two clear kHz QPOs, the separation frequency is approximately consistent with either the spin frequency or half the spin frequency (see the contribution by Wijnands, these proceedings).

3 Models of kHz QPOs

3.1 Dynamical Importance of Magnetic Fields

The harmonic relation observed in eight sources between the spin frequency and the kHz QPO separation frequency implies that the stellar spin is somehow communicated to the accretion flow. There is thus some nonaxisymmetry that rotates with the star and affects the flow in some fashion. The plausible candidates for such a nonaxisymmetry would be magnetic or radiative in nature. That is, either a tilted stellar magnetic field influences the flow directly, or radiation from a "hot spot" on the star sweeps over the accretion disk. Even if it is radiation that is the driving nonaxisymmetry, we can ask how the hot spot is attached to the star. This would require that the accreting matter be funneled to a region that rotates with the star. This can only plausibly happen if the stellar magnetic field does the funneling, hence in any realistic scenario the appearance of the spin in the QPO properties implies that the stellar magnetic field has a dynamical effect on the accretion flow.

However, the most obvious signature of a dynamically important magnetic field — the appearance of coherent oscillations at the stellar spin frequency — is absent from most kHz QPO sources. How can this be reconciled with the importance of magnetic fields in QPO production?

3.2 Resonances

An answer suggested by several researchers is that resonances play a role. The attraction of this concept is that even if the magnetic field is too weak to channel enough matter to the magnetic poles to produce pulsations, if the driving frequency of the magnetic field matches a resonance in the disk then the disk could still be affected strongly. In the context of black hole QPOs, Abramowicz, Kluźniak, and colleagues (e.g., Abramowicz et al. 2002; Lee, Abramowicz, & Kluźniak 2004) have suggested that a parametric resonance between the vertical epicyclic frequency and radial epicyclic frequency of geodesic motion could lead to a strong QPO when the frequencies are in a 3:2 ratio. Wijnands et al. (2003) proposed that in the case of neutron stars, there could be a strong QPO when the vertical and radial epicyclic frequencies are separated by the spin frequency or half the spin frequency. It remains to be seen, however, whether this idea is compatible with the drift of hundreds of Hertz in QPO frequency seen in many sources (e.g., the millisecond pulsar SAX J1807, where $\Delta\nu \approx \nu_{\rm spin}$ over $\sim$ 200 Hz; see the contribution by Wijnands, these proceedings), because the separation between geodesic epicyclic frequencies varies smoothly with radius and is therefore commensurate with the spin frequency only in a narrow range of QPO frequencies.

An alternate idea, proposed by Lamb & Miller (2005), is to extend general relativistic beat frequency models to accommodate frequency separations of $\nu_{\rm spin}/2$. The motivation behind the original beat frequency models was to explain the relation between the separation frequency and spin frequency in several sources, while also explaining how the kHz QPOs can remain coherent even as their centroids change by hundreds of Hertz. The solution suggested by Miller, Lamb, & Psaltis (1998) was that radiation from the star will extract angular momentum from the flow, leading to a sharp transition in the inward radial speed from subsonic to supersonic. This effect would not occur in purely Newtonian gravity, because the specific angular momentum of circular orbits decreases monotonically and steeply with decreasing orbital radius, hence loss of angular momentum to the radiation field does not produce a sharp change in the radial speed.

In contrast, in general relativity the specific angular momentum of circular orbits reaches a minimum at the ISCO, and thus decreases only slowly with decreasing radius near but outside the ISCO (see Figure 1). For example, the specific angular momentum at the ISCO around a nonrotating star of gravitational mass M is $\sqrt{12}\,GM/c$, whereas at double the ISCO radius the specific angular momentum is $4\,GM/c$, just 15% larger. Therefore, removal of even a small amount of angular momentum can produce a large change in radius and thus a rapid increase in inward radial speed. This process produces a sharply defined radius at which QPOs are preferentially generated (see Miller et al. 1998 for details), which is consistent with the observed quality factors of the kHz QPOs.

The precise radius depends on a competition between the radiation field and the optical depth of the accretion flow. When the radiation is generated by accretion the tendency is to increase the frequency with increasing accretion rate, because the sonic radius (marking the sharp transition in inward radial speed) is pushed closer to the star. However, for variations in the luminosity not tied to the accretion, the expectation was that higher luminosity would imply a larger radius and therefore a lower frequency. Support for this picture was provided by Yu & van der Klis (2002), who found that the frequency of the kHz QPO in 4U 1608–52 was anticorrelated with the intensity of millihertz oscillations.

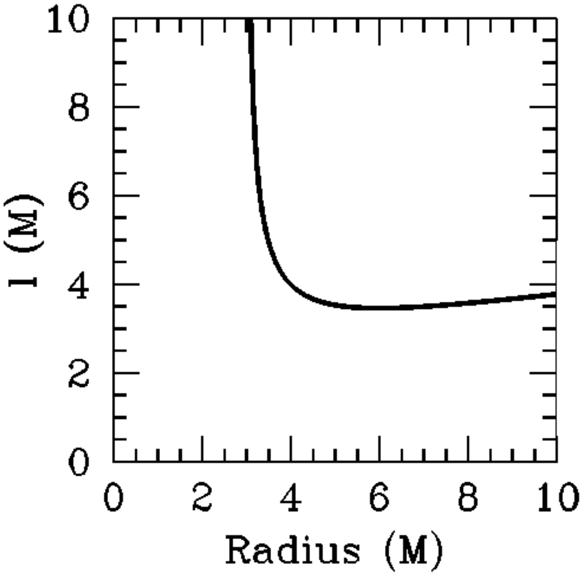

Fig. 1. Specific angular momentum l, in units of the gravitational mass M, of a circular orbit in a Schwarzschild spacetime. The flatness of the curve near the ISCO at $r = 6M$ means that a small change in angular momentum leads to a large change in radius.

The observed frequency of the upper kHz QPO also depends on the inspiral of gas towards the stellar surface (see Lamb & Miller 2001). Specifically, in this model the observed frequency is the frequency of the footpoint of accretion onto the stellar surface of streams of gas from clumps near the sonic point. As clumps drift inwards, the total azimuthal phase that gas from the clumps traverses before it reaches the surface will change. The total phase decreases as a clump moves towards the star, hence the net result is that the observed frequency is close to, but usually less than, the orbital frequency ν_{sonic} at the sonic-point radius r_{sonic}. Lamb & Miller (2001) propose that this can explain the lack of exact commensurability between $\Delta\nu$ and ν_{spin}.

In the original beat frequency model of Miller et al. (1998), the lower kHz QPO was thought to be produced by modulation of the accretion rate from the sonic point via interaction with the stellar magnetic field. This, however, would imply that the separation frequency should always be close to the stellar spin frequency. Instead, a number of sources show $\Delta\nu \approx \nu_{\rm spin}/2$, with the clinching evidence coming from the analysis of SAX J1808–3658 by Wijnands et al. (2003), who showed that $\Delta\nu = 195 \pm 6$ Hz and $\nu_{\rm spin} = 401$ Hz, with any pulsations at 200 Hz having an amplitude at least 300 times less than the amplitude at 401 Hz.

Lamb & Miller (2005) therefore proposed a variant on the original general relativistic sonic-point model to explain $\Delta\nu \approx \nu_{\rm spin}/2$ in some sources. In this scenario, the upper kHz QPO is generated in exactly the same way as before, so flattening of the angular momentum curve as in general relativity is still a fundamental requirement. However, the production of the lower kHz QPO involves a resonance with the stellar spin.

The key concept involves driving of vertical motions in the disk by the stellar magnetic field. Suppose that the magnetic field is tilted relative to the spin axis. As the star rotates, plasma in the disk is therefore subjected to time-varying magnetic stresses. As seen in a static frame at infinity, these stresses vary with frequency $\nu_{\rm spin}$. These stresses will in principle drive motion both vertically and radially, but the radial motion will be damped out much more rapidly than the vertical (see Marković & Lamb 2000), hence we will focus on the vertical motion. If the vertical driving frequency matches the natural frequency of vertical motion, there is a vertical resonance and the plasma can be driven relatively far out of the disk plane. The natural frequency is just the vertical epicyclic frequency $\nu_{\rm vert}$, and the driving frequency as seen in a frame orbiting with the plasma is $\nu_{\rm spin} - \nu_{\rm orb}$, where $\nu_{\rm orb}$ is the orbital frequency. Thus the condition for resonance is

$$\nu_{\rm spin} - \nu_{\rm orb} = \nu_{\rm vert} \ . \tag{1}$$

Note, however, that for realistic neutron star spin frequencies $\nu_{\rm vert} \approx \nu_{\rm orb}$, hence the resonance condition is satisfied when $\nu_{\rm orb} \approx \nu_{\rm spin}/2$, at the spin resonance radius r_{sr}.

The beat frequency mechanism then takes the form of shadowing of the clumps or flow at r_{sr} by the clumps at the sonic-point $r_{\rm sonic}$. The dominant beat frequency depends on how smooth the flow is at r_{sr}. If the flow is dominated by a small number of clumps, then the orbital motion of the clumps at r_{sr} produces the primary modulation. In this case, we have $\nu_{\rm lower} \approx \nu_{\rm sonic} - \nu_{\rm spin}/2$.

This changes if the flow is relatively smooth. The reason is that a periodic force on clumps will rapidly drive the motion of the clumps into phase with the force. Therefore, e.g., the phase of maximum vertical excursion of gas above the disk will correspond to the phase of maximum downward force exerted by the magnetic stresses. This will happen to every plasma element in the disk, hence the phase of maximum height above the midplane will track the phase of the magnetic field, meaning simply that there is a wave that moves at the spin frequency even if individual elements at that radius orbit at half the spin frequency. When the flow is relatively smooth, this pattern dominates over the individual motions, and $\nu_{\rm lower} \approx \nu_{\rm sonic} - \nu_{\rm spin}$. It could be that the flow tends to be smoother at larger radii, accounting for the observation that when $\nu_{\rm spin} < 400$ Hz then $\Delta\nu \approx \nu_{\rm spin}$, whereas when $\nu_{\rm spin} > 400$ Hz then $\Delta\nu \approx \nu_{\rm spin}/2$ (see Muno et al. 2001).

4 Model-Insensitive Constraints on Neutron Stars

Although various models disagree in detail, almost all models agree that the upper kHz QPO is close in frequency to the orbital frequency $\nu_{\rm orb}$ at some special radius. This is even true of the models that identify the upper kHz QPO with the vertical epicyclic frequency $\nu_{\rm vert}$ at some radius, because $\nu_{\rm vert} \approx \nu_{\rm orb}$ outside a neutron star: to first order in the dimensionless spin parameter $j \equiv cJ/GM^2$,

$$\nu_{\rm vert} \approx \nu_{\rm orb}(1 - 2j\hat{r}^{-3/2}) \; , \tag{2}$$

where $\hat{r} \equiv r/M$. For $j \lesssim 0.2$, appropriate for neutron stars with $\nu_{\rm spin} \lesssim 600$ Hz, $\nu_{\rm vert}$ and $\nu_{\rm orb}$ differ by at most 3% even at $r = 6M$.

With this in mind, Miller, Lamb, & Psaltis (1998) showed that the joint constraints that (1) the upper QPO must be generated outside the star, and (2) the upper QPO must be generated outside the ISCO (because otherwise the quality factor would be low) limits both the mass and radius of the star. To first order in rotation, the constraints from a maximum inferred orbital frequency $\nu_{\rm orb,max}$ are

$$\begin{aligned} M_{\rm max} &= 2.2\, M_\odot \left(\tfrac{1000\ \rm Hz}{\nu_{\rm orb,max}}\right)(1 + 0.75j) \\ R_{\rm max} &= 19.5\ {\rm km} \left(\tfrac{1000\ \rm Hz}{\nu_{\rm orb,max}}\right)(1 + 0.2j) \; . \end{aligned} \tag{3}$$

In addition, for a given mass there is a maximum allowed radius, meaning that the mass-radius curve derived from a given high-density equation of state must intersect the allowed region to be viable. Higher observed frequencies pose stronger constraints. The highest frequency yet observed with confidence is 1330 Hz from 4U 0614+091 by van Straaten et al. (2000), and Figure 2 shows the resulting mass-radius diagram along with several candidate equations of state.

This figure shows that the constraints from kHz QPOs tend to argue against the hardest equations of state. Higher frequencies would be even more constraining, but the amplitudes of the QPOs tend to decrease with increasing frequency, and *RXTE* does not have sufficient collecting area to detect the highest available frequencies. M. van der Klis (personal communication) has estimated that a 10 m^2 timing satellite would be able to detect QPOs

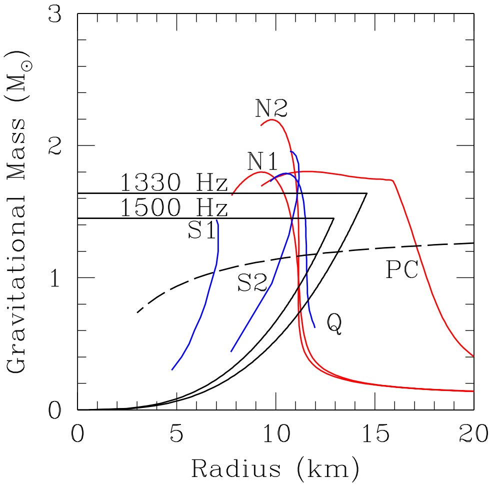

Fig. 2. Constraints from orbital frequencies. The 1330 Hz curve is for the highest kilohertz quasi-periodic oscillation frequency yet measured (for 4U 0614+091, by van Straaten et al. 2000). The 1500 Hz curve shows a hypothetical constraint for a higher-frequency source. The dashed line shows the gravitational mass for a baryonic mass of 1.4 $M_\odot$, consistent with the iron core of a neutron star (the actual mass will likely be higher than this). Equations of state are for strange stars (S1 and S2), quark stars (Q), purely nucleonic stars (N1 and N2), and an EOS with a pion condensate transition (PC). All curves are drawn for nonrotating stars; the constraint wedges would be enlarged slightly for a rotating star (see Miller, Lamb, & Psaltis 1998).

up to $\sim$ 1500 Hz in several specific sources, if the frequencies get that high. Such detections would rule out conclusively the hardest equations of state. If the frequencies never reach that level, however, it would be good evidence for the presence of a limit. The most likely cause of the limit would be spacetime geometry effects related to the ISCO. Even in current *RXTE* data there is intriguing evidence for phenomena related to the ISCO, which we now discuss.

5 Evidence for the Innermost Stable Circular Orbit

Inside the ISCO the specific angular momentum of geodesic circular orbits increases with decreasing radius, hence unless there are significant pressure gradients (which happens only near the Eddington limit), gas spirals in rapidly and thus it is not possible to maintain a highly coherent oscillation inside the ISCO. As suggested by Kluźniak, Michelson, & Wagoner (1990) prior to the discovery of kHz QPOs, one therefore expects abrupt changes in the disk flow near the ISCO, even if other forces do not force such a change farther out (such as the radiation force effects discuss earlier). With the discovery of kHz QPOs, Miller et al. (1998) suggested several possibilities. Updated in light of recent observations and models, these are:

- The orbital frequency, and hence the upper kHz QPO, should have an upper limit. If the lower kHz QPO is related to the upper via a beat frequency mechanism, it should also have a limit, but this limit will be less than that for the upper QPO by an amount roughly equal to the spin frequency or half of it.
- The quality factor of the oscillations should decrease sharply when the radius of QPO generation gets

very near the ISCO. Depending on other effects influencing the quality factor, the lower and upper kHz QPO could have somewhat different behaviors in this respect.

- The amplitude of the oscillations should also decrease towards the ISCO.

Evidence for the ISCO in QPO data would be extremely important, because (1) it would reflect a feature of strong gravity with no Newtonian parallel, and (2) knowledge of the orbital frequency at the ISCO allows a direct estimate of the mass of the neutron star, and if the mass is large enough it would pose strong constraints on the properties of matter beyond nuclear densities. It is therefore important to require an especially high standard of evidence.

5.1 *Initial reports of ISCO phenomena*

Zhang et al. (1998) analyzed data from 4U 1820–30 and discovered what they believed was evidence for the ISCO. This evidence came from the relation between *RXTE* countrate and QPO frequency. It had been established for many sources that the QPO frequency tends to increase with increasing countrate. What Zhang et al. (1998) showed is that in 4U 1820–30, this frequency increase saturates for both the upper and lower kHz QPOs; the countrate continues to increase, but the frequencies remain roughly stable. They identified the maximum frequency of $\sim$ 1100 Hz for the upper peak with the orbital frequency of the ISCO, suggesting that the mass of the neutron star exceeded $2-2.2\,M_\odot$. If true, this would be extremely constraining on the properties of neutron stars; for example, it would suggest that there is an insignificant contribution from non-nucleonic components, and that the three-body nucleonic interaction is towards the high end of repulsive (see Lattimer & Prakash 2001).

However, Méndez et al. (1999) cautioned that the countrate is not a good measure of accretion rate, and that the frequency and countrate are not one to one with each other. This is most clearly shown in the "parallel tracks" phenomenon (for a discussion and possible explanation see van der Klis 2001 and also Figure 3), in which over short times the frequency does track the countrate, but observations separated by longer periods have similar frequencies but different countrates, hence in a countrate versus frequency diagram the QPOs appear to produce parallel tracks. Therefore, the evidence for saturation is not so clean, although the maximum observed frequency is the same on each track. Other measures of accretion rate, such as hard color or the so-called S_a index of position in a color-color diagram also appear to display frequency saturation in 4U 1820–30 (Kaaret et al. 1999; Bloser et al. 2000), but the situation is still unclear because of subtleties in how the duration of a power density spectrum affects the inferred properties of QPOs (M. Méndez, private communication).

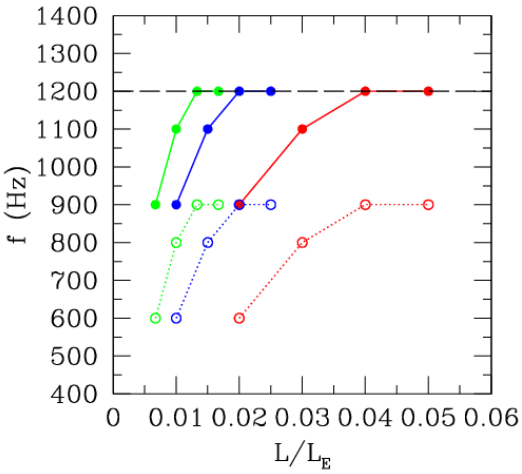

Fig. 3. Theoretical version of the parallel tracks phenomenon. The different colors represent different observations, and the dotted lower QPO frequency corresponds to the solid upper QPO frequency in each case. In short intervals, both the upper and lower QPO increase in frequency with increasing luminosity. This increase saturates at the orbital frequency of the ISCO for the upper QPO, and for this frequency minus $\nu_{\rm spin}$ or $\nu_{\rm spin}/2$ for the lower QPO. In a different observation the frequency–luminosity relation is different, but the saturation frequencies are the same.

5.2 *Recent analysis and results*

Barret, Olive, & Miller (2005a) looked at *RXTE* data from 4U 1636–536, using the shift and add technique (Méndez et al. 1998) to maximize signal. As with 4U 1820–30 and other sources, they found parallel tracks in the frequency-countrate plane, where the maximum frequency on each track was approximately the same. Focusing on the lower kHz peak (which is the stronger and sharper of the two in this source, hence is the easier one to track and characterize observationally), they found that the rms amplitude decreases with increasing frequency.

Interestingly, they also found that the quality factor Q of the oscillation increased with increasing frequency for $\nu_{\rm lower}\sim 600-850$ Hz, but that above ~ 850 Hz the quality factor drops sharply with increasing frequency. The decrease in Q at high frequencies had been seen for other sources (the earliest being 4U 1608–52, by Berger et al. 1996), but the large quantity of data for 4U 1636–536 makes more precise characterizations possible. In particular, Barret, Olive, & Miller (2005b) show that the amplitude versus frequency and Q versus frequency curves are consistent between observations with less than the median countrate and with more than the median countrate. Therefore, the amplitude and quality factor depend on frequency but not countrate, and the maximum QPO frequency is roughly independent of countrate. What does this imply?

Let us concentrate on the behavior of the quality factor with frequency. There are many effects that influence Q, and the limiting factor will be the one that gives the lowest Q. Identifying the drop with an effect associated with the ISCO is tantamount to saying that the spacetime geometry has the dominant influence. Could it be instead that, for example, the boundary layer of the accreting matter on the star changes its properties radically, or that something about the corona that presumably upscatters photons and gives the observed high-energy tail is different when the lower kHz QPO goes above a certain frequency? Perhaps interactions with the stellar magnetic field change dramatically above some $\nu_{\rm crit}$?

Without detailed models of each of these types one cannot rule out these ideas rigorously. Qualitatively, however, the simplest expectations of these scenarios are not compatible with the observation that the countrate does not influence strongly the quality factor (or the amplitude, or that the maximum frequency appears roughly independent of countrate). The reason is that all of these mechanisms involve plasma interactions (with itself, or the stellar surface, or the stellar magnetic field). If the mass accretion rate changes, so does the plasma density (or thickness of the disk, or radius of stress balance with the stellar magnetic field).

One might therefore expect that changing the mass accretion rate would result in significant changes in the quality factor or amplitude. If the countrate is related to the mass accretion rate, this would imply that changes in countrate would affect the behavior of the QPO as well. The absence of such evidence is therefore problematic for these ideas. One way out would be to suggest that the frequency really does correlate extremely well with the mass accretion rate, but that the countrate does not (as is suggested by the parallel tracks). One way to test this is to use other proxies for the mass accretion rate, such as the hard color or position on a color-color diagram (D. Barret et al., in preparation). If there still appears to be no correlation with accretion rate as measured in several different ways, this would be challenging to explain in the above pictures.

The remaining natural option is for the QPO properties to depend directly on the spacetime, through quantities such as the specific angular momentum of nearly circular orbits. To be viable, such a scenario needs to have qualitative and quantitative agreement with the major trends in the data. Currently, the lower kHz QPO in 4U 1636–536 displays an increase in Q with frequency followed by a sharp drop, whereas the weaker and less coherent upper kHz QPO shows a steady increase in Q with frequency, with no evidence for a drop although the low amplitude at high frequencies makes definitive statements difficult. Can this behavior be explained in the ISCO scenario?

To determine this, we list some of the factors that enter into the quality factor. To be concrete, we consider a model in which clumps are generated at some radius and gas spirals from them into the star as the clumps themselves drift in. Factors influencing Q are then:

(1) The range of radii over which clumps are generated. Larger range implies smaller Q.
(2) The number of cycles each clump lasts. Smaller number implies smaller Q.
(3) The radius over which each clump drifts in its lifetime. Notice that this is different from 1,

which is the range over which the clumps are *generated*. Larger drift implies smaller Q.

We focus on *smaller* Q because obviously even if two of these mechanisms would produce large Q, the one producing the smallest Q is the overall limiting factor.

All three of these factors would apply to both the upper and lower QPOs. In a more detailed model, other considerations may enter. For example, in the sonic point and spin resonance model of Lamb & Miller (2005), there are factors applying to just the lower QPO or just the upper QPO.

(4a) For the lower QPO, there is a beat between the orbital frequency at the sonic point and the individual clumps or patterns at the spin resonance frequency (giving, respectively, differences of $\nu_{\rm spin}/2$ and $\nu_{\rm spin}$). The precise difference frequency will depend somewhat on the accretion rate, but probably will be constant to within a few Hertz, so this does not contribute much to lowering Q.

(4b) For the upper QPO, as discussed in Lamb & Miller (2001), the effective frequency involves a convolution of the orbital frequency at the sonic point *and* the total azimuthal phase traveled by the stream from the sonic point to the star. This total phase can change significantly as a clump drifts towards the star, so this *can* lower Q significantly.

What happens to each of these as the sonic point moves nearer the star?

(1) From Miller, Lamb, & Psaltis (1998), the range of radii over which clumps are generated *decreases* as the sonic point moves closer, because the flattening of the specific angular momentum curve means that the transition from slow to fast radial motion becomes more abrupt closer to the star. This mechanism would increase Q with increasing frequency if it operated alone.

(2) It is unclear what determines how many cycles each clump lasts. If the clump lifetime depends mainly on interactions with the stellar magnetic field, then since the range of radii in question is small we will simplify and assume that the number of cycles is fixed and large, so that it never limits Q.

(3) The amount by which clumps drift radially in their (assumed constant) lifetimes must increase as the orbital radius decreases, because the radial drift is caused by loss of angular momentum, and the flattening of the specific angular momentum curve produces greater radial drift for fixed angular momentum loss when the radius is close to the ISCO. Therefore, operating by itself, this mechanism would lower Q as the frequency increased, and will certainly take it to zero at the ISCO.

If these three were the only mechanisms that affected Q, then since mechanism (1) gives lower Q at lower frequency and mechanism (3) gives lower Q at higher frequency (and zero at the ISCO), one expects a Q versus frequency curve that starts low, rises to a maximum, then decreases to zero.

But these are not the only influences on Q. For the lower QPO, the additional effect (4a) should not play much a role if the spin resonance frequency is defined to within a few Hertz, because by itself this would give a quality factor well above the observed one. For the upper QPO, though, it could be different:

(4b) The change in azimuthal phase traveled by gas spiraling to the star could be large far from the star (because the total phase itself is large) but smaller close to the star. This change could lead to much lower Q values, and would thus be the overall limiting factor on Q.

Notice that because there is another mechanism that limits the quality factor to lower values, the frequency at which mechanism (3) takes over, and hence the frequency of peak Q, is higher than it is when this mechanism does not operate. At a high enough frequency, the amplitude is low, which combined with the generally lower quality factor of the upper kHz QPO in this source could make a sharp drop in Q more difficult to detect than it is for the lower kHz QPO. Note that the upper QPO and lower QPO go to $Q = 0$ at different frequencies, separated by the difference frequency, which is $\approx \nu_{\rm spin}/2$ or $\approx \nu_{\rm spin}$.

5.3 Implications and Falsifications

If the sharp drop in Q is indeed caused by an approach to the ISCO, extrapolation to $Q = 0$ gives a reasonable estimate of the orbital frequency of the ISCO, which in turn allows a measurement of the gravitational mass of the neutron star. If, as in 4U 1636–536, the lower kHz QPO is the stronger, then we would estimate the ISCO orbital frequency by (1) estimating the lower peak frequency $\nu_{\rm low,max}$ where Q would be zero, then (2) estimating the ISCO frequency from $\nu_{\rm ISCO} = \nu_{\rm low,max} + \Delta\nu$, where $\Delta\nu = \nu_{\rm spin}$ or $\nu_{\rm spin}/2$. As discussed by Barret et al. (2005a,b), the implication for 4U 1636–536 and other sources is that these neutron stars have masses $M \sim 1.8 - 2.0\,M_\odot$. This is higher than inferred for double neutron star systems (which allow the most precise neutron star mass measurements), but is consistent with accretion of a few tenths of a solar mass during spin-up, and is also in line with the relatively high masses inferred for Vela X-1 (Quaintrell et al. 2003) and several rotation-powered pulsars in binaries with white dwarfs that have presumably donated mass to the neutron stars (e.g., Nice et al. 2003). The implication, although not yet certain, is that neutron stars have a relatively high upper limit to their masses, tending to argue in favor of purely nucleonic degrees of freedom at core densities rather than dominance by an exotic component such as a condensate or quark matter.

The alternatives to the ISCO interpretation are currently unformed enough that a rigorous disproof appears impossible at this time. There are, however, observations that would falsify the ISCO scenario. An obvious one is that if in a source such as 4U 1636–536, which has a clear trend to low Q at high frequencies, a clear higher-frequency QPO were seen later, then clearly the drop in Q was caused by something else. The threshold frequency for clear falsification is not perfectly well-defined, because there is no fundamental theory that predicts the functional form of Q with frequency. However, empirically we see Q drop from ~ 200 to ~ 50 in 4U 1636–536, so extrapolation to $Q = 0$ involves at most a few tens of Hertz. In this case, therefore, a clear lower kHz QPO observed at 1000 Hz or higher would falsify the ISCO idea.

Falsification would also occur if the inferred frequency of the ISCO were too small, because this would imply an unrealistically large neutron star mass. The hardest extant equations of state are some of the mean field proposals (see Lattimer & Prakash 2001), which have maximum masses $M_{\rm max} \sim 2.8\,M_\odot$. An ISCO frequency less than ~ 800 Hz is therefore excluded. The n-body equations of state have lower maxima, up to $M_{\rm max} \sim 2.5\,M_\odot$ (Lattimer & Prakash 2001), so $\nu_{\rm ISCO} > 900$ Hz. If a sharp drop in Q is seen at a lower frequency, this would demonstrate that the drop is not caused exclusively by spacetime geometry effects.

6 Conclusions

The properties of kHz QPOs yield strong constraints on neutron star structure that are complementary to other methods involving radius estimates or redshift measurements. The emerging picture is one in which neutron stars can sustain masses of $\sim 2\,M_\odot$, so they cannot be too soft, but their radii are unlikely to be more than ~ 15 km for typical masses, so they cannot be too hard either. Although not yet conclusive, this is consistent with the range of purely nucleonic equations of state derived from n-body methods and fit to laboratory data on the binding energies of light nuclei (Lattimer & Prakash 2001).

Greater certainty will require developments on both theoretical and observational fronts. Systematic study of the dependence of quality factor on frequency in many sources will test the ISCO interpretation in current analyses. A future large-area X-ray timing mission would allow careful analysis of weaker signals, and would place stronger limits on both the hardness (from higher-frequency QPOs) and softness (from clearer measurement of ISCO-related phenomena) of equations of state. Theoretically, although the current picture appears broadly consistent with observed kHz QPOs, there is as yet no first-principles calculation of the response of an MHD accretion disk to the radiative and magnetic stresses associated with the star. Such a calculation would be a challenging foray into dynamic three-dimensional photomagnetohydrodynamics in general relativity, but the rapid advances in general relativistic MHD as applied to black hole accretion disks leads to cautious optimism that within a few years there may be more fundamental numerical treatments possible. Initial steps in this direction have been taken by Romanova and colleagues (e.g., Romanova et al. 2004), and further developments would be welcome.

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