

MERGERS OF STELLAR-MASS BLACK HOLES IN NUCLEAR STAR CLUSTERS

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ABSTRACT

Mergers between stellar-mass black holes (BHs) will be key sources of gravitational radiation for ground-based detectors. However, the rates of these events are highly uncertain, given that such systems are invisible. One formation scenario involves mergers in field binaries, where our lack of complete understanding of common envelopes and the distribution of supernova kicks has led to rate estimates that range over a factor of several hundreds. A different, and highly promising, channel involves multiple encounters of binaries in globular clusters or young star clusters. However, we currently lack solid evidence of BHs in almost all such clusters, and their low escape speeds raise the possibility that most are ejected because of supernova recoil. Here, we propose that a robust environment for mergers could be the nuclear star clusters found in the centers of small galaxies. These clusters have millions of stars, BH relaxation times well under a Hubble time, and escape speeds that are several times those of globulars; hence, they retain most of their BHs. We present simulations of the three-body dynamics of BHs in this environment and estimate that, if most nuclear star clusters do not have supermassive BHs that interfere with the mergers, tens of events per year will be detectable with the advanced Laser Interferometer Gravitational-Wave Observatory.

Key words: black hole physics – galaxies: nuclei – gravitational waves – relativity

1. INTRODUCTION

Ground-based gravitational wave detectors have now achieved their initial sensitivity goals (e.g., Abbott et al. 2007). In the next few years, these sensitivities are expected to improve by a factor of ~ 10 , which will increase the searchable volume by a factor of $\sim 10^3$ and will lead to many detections per year.

One of the most intriguing possible sources for such detectors is the coalescence of a double stellar-mass black hole (BH) binary. Such binaries are inherently invisible, meaning that we have no direct observational guide to how common they are or their masses, spin magnitudes, or orientations. Comparison of the observed waveforms (or of waveforms from merging supermassive BHs (SMBHs)) with predictions based on approximate solutions and numerical relativity will be a strong test of the predictions of strong-gravity general relativity.

The electromagnetic nondetection of these sources makes rate estimates highly challenging, because our only observational handles on BH–BH binaries come from their possible progenitors. For example, a common scenario involves the effectively-isolated evolution of a field binary containing two massive stars into a binary with two BHs that will eventually merge (e.g., Lipunov et al. 1997; Belczynski & Bulik 1999). There are profound uncertainties involved in calculations of these rates due to, for example, the lack of knowledge of the details of the common envelope phase in these systems and the absence of guides to the distribution of supernova kicks delivered to BHs. As a recent indication of the range of estimated rates, note that the advanced Laser Interferometer Gravitational-Wave Observatory (LIGO) detection rate of BH–BH coalescences is estimated to be anywhere between ~ 1 –500 yr^{-1} by Belczynski et al. (2007), depending on how common envelopes are modeled.

Another promising location for BH–BH mergers is globular clusters or super star clusters, where stellar number densities are high enough to cause multiple encounters and hardening of binaries. Even though binaries are kicked out before they merge (Kulkarni et al. 1993; Sigurdsson & Hernquist 1993; Sigurdsson & Phinney 1993, 1995; Portegies Zwart & McMillan 2000;

O’Leary et al. 2006), these clusters can still serve as breeding grounds for gravitational wave sources. Indeed, O’Leary et al. (2007) estimated a rate of 0.5 yr^{-1} for initial LIGO and 500 yr^{-1} for advanced LIGO via this channel. There is, however, little direct evidence of BHs in most globulars (albeit they could be difficult to see). In addition, at least one BH in a low-mass X-ray binary apparently received a $\gtrsim 100 \text{ km s}^{-1}$ kick from its supernova (GRO J1655–40; see Mirabel et al. 2002). This is double the escape speed from the centers of even fairly rich globulars (Webbink 1985), leading to uncertainties about their initial BH population and current merger rates.

Here we propose that mergers frequently occur in the nuclear star clusters that may be in the centers of many low-mass galaxies (Böker et al. 2002; Ferrarese et al. 2006; Wehner & Harris 2006; note that some of these are based on small deviations from smooth surface brightness profiles and are thus still under discussion). It has recently been recognized that in these galaxies, which may not have SMBHs (for a status report on ongoing searches for low-mass central BHs, see Greene & Ho 2007), the nuclear clusters have masses that are correlated with the one-dimensional velocity dispersion $\sigma_{1\text{D},\text{bulge}}$ at one galactic effective radius as $M \approx 10^7 M_{\odot} (\sigma_{1\text{D},\text{bulge}}/54 \text{ km s}^{-1})^{4.3}$ (Ferrarese et al. 2006). A BH with a mass of a factor of a few below the M – σ relation would be undetectable. Note that this velocity dispersion is typically a factor of ~ 2 larger than the measured one-dimensional volume-weighted velocity dispersion $\sigma_{1\text{D}}$ of the nuclear star cluster itself (compare $\sigma_{1\text{D}}$ and M for the clusters in Walcher et al. 2005 with the values predicted with the Ferrarese et al. 2006 relation above). Measurements of $\sigma_{1\text{D}}$ indicate that it is commonly in the range 24 – 34 km s^{-1} (this is the case for seven of the ten total nuclear star clusters described in the papers of Walcher et al. 2005 and Seth et al. 2008). If the velocity distribution is isotropic, then the three-dimensional velocity dispersion $\sigma_{3\text{D}} = \sqrt{3}\sigma_{1\text{D}}$ is often between $\sigma_{3\text{D}} \sim 40$ – 60 km s^{-1} .

At these three-dimensional velocity dispersions, the half-mass relaxation time is small enough that BHs (which have $\sim 20\times$ the average stellar mass) can sink to the center in

much less than a Hubble time. In addition, although systems with equal-mass objects require roughly 15 half-mass relaxation times to undergo core collapse (Binney & Tremaine 1987), studies show that systems with a wide range of stellar masses experience core collapse within $\sim 0.2 \times$ the half-mass relaxation time (Portegies Zwart & McMillan 2002; Gürkan et al. 2004). Combined with the Ferrarese et al. (2006) relation between the cluster mass and $\sigma_{1D, \text{bulge}}$, we find that clusters with masses less than $\sim \text{few} \times 10^7 M_\odot$ and no central SMBH (or a highly undermassive SMBH) will have collapsed by now and hence increased the escape speed from the center, allowing retention of most of their BHs.

As we show in this paper, nuclear star clusters are, therefore, excellent candidates for stellar-mass BH binary mergers because they keep their BHs while also evolving rapidly enough that the holes can sink to a region of high density. If tens of percent of the BHs in eligible galaxies undergo such mergers, the resulting rate for advanced LIGO is tens per year. In Section 2, we quantify these statements and results more precisely and discuss our numerical three-body method. We give our conclusions in Section 3.

2. METHOD AND RESULTS

2.1. Characteristic Times and Initial Setup

Our approach is similar to that of O’Leary et al. (2006), who focus on globular clusters with velocity dispersions $\sigma_{1D} \leq 20 \text{ km s}^{-1}$. Here, however, we concentrate on the more massive and tightly bound nuclear star clusters. Our departure point is the relation found by Ferrarese et al. (2006) between the masses and velocity dispersions of such clusters:

$$M_{\text{nuc}} = 10^{6.91 \pm 0.11} (\sigma_{1D, \text{bulge}} / 54 \text{ km s}^{-1})^{4.27 \pm 0.61} M_\odot. \quad (1)$$

Assuming that there is no massive central BH for these low velocity dispersions, the half-mass relaxation time for the system is (see Binney & Tremaine 1987) $t_{\text{rlx}} \approx \frac{N/2}{8 \ln N} t_{\text{cross}}$, where $N \approx M_{\text{nuc}} / 0.5 M_\odot$ is the number of stars in the system (assuming an average mass of $0.5 M_\odot$) and $t_{\text{cross}} \approx R / \sigma_{3D}$ is the crossing time. Here, $R = GM_{\text{nuc}} / \sigma_{3D}^2$ is the radius of the cluster. If we assume that $\sigma_{3D} = \sqrt{3} \sigma_{1D}$ and that typically $\sigma_{1D, \text{bulge}} \approx 2 \sigma_{1D} \approx \sigma_{3D}$, this gives

$$t_{\text{rlx}} \approx 1.3 \times 10^{10} \text{ yr} (\sigma_{3D} / 54 \text{ km s}^{-1})^{5.5}. \quad (2)$$

The relaxation time scales inversely with the mass of an individual star (Binney & Tremaine 1987), so a $10 M_\odot$ BH will settle in roughly 1/20 of this time. Also note that large N -body simulations with broad mass functions evolve to core collapse within roughly 0.2 half-mass relaxation times (Portegies Zwart & McMillan 2002; Gürkan et al. 2004); hence, in the current universe, clusters with velocity dispersions $\sigma_{3D} < 60 \text{ km s}^{-1}$ will have had their central potentials deepened significantly.

The amount of deepening of the potential, and thus the escape speed from the center of the cluster, depends on uncertain details such as the initial radial dependence of the density and the binary fraction. Given that the timescale for segregation of the BHs in the center is much less than a Hubble time, we will assume that the escape speed is roughly $5 \sigma_{1D}$, as is the case for relatively rich globular clusters (Webbink 1985). This may well be somewhat conservative, because the higher velocity dispersion here than in globulars suggests that a larger fraction of binaries will be destroyed in nuclear star clusters. This could lead to less efficient

central energy production and hence deeper core collapse than is typical in globulars.

With this setup, our task is to follow the interactions of BHs in the central regions of nuclear star clusters, where we will scale by stellar number densities of $n \sim 10^6 \text{ pc}^{-3}$ (a characteristic value near the center of the Milky Way; see Genzel et al. 2003b) because of density enhancements caused by relaxation and mass segregation. Our hypothesis is then that binary-single interactions will (1) allow BHs to swap into binaries even if they began as single objects, and (2) harden BH binaries to the extent that they can merge while still in the nuclear star cluster. If a BH starts its life with a binary companion, then the interaction time is short, because every interaction it has will be a binary-single encounter that has a high cross-section. If instead the BH begins as a single object, the binary-single interaction rate is much less because it relies on the BH encountering comparatively rare binaries. This is the case we will consider, because if there is enough time for a BH to capture into a binary and then harden, there is certainly enough time for a BH that is born into a binary to harden.

All binaries in the cluster will be hard, that is, will have internal energies greater than the average kinetic energy of a field star, because otherwise they will be softened and ionized quickly (e.g., Binney & Tremaine 1987). If, for example, we consider binaries of two $1 M_\odot$ stars in a system with $\sigma_{3D} = 50 \text{ km s}^{-1}$, then for the binary to be hard, the semimajor axis has to be less than $a_{\text{max}} \sim 1 \text{ AU}$. Studies of main-sequence binaries in globular clusters, which have $\sigma_{1D} \sim 10 \text{ km s}^{-1}$, suggest that after billions of years, roughly 5%–20% of them survive, with the rest falling victim to ionization or collisions (Ivanova et al. 2005). The binary fraction will be lower in nuclear star clusters due to their enhanced velocity dispersion, but since when binaries are born, they appear to have a constant distribution across the log of the semimajor axis from $\sim 10^{-2}$ – 10^3 AU (e.g., Abt 1983; Duquennoy & Mayor 1991), the reduction is not necessarily by a large factor. We will scale by a binary fraction $f_{\text{bin}} = 0.01$, which is likely to be somewhat low and thus we will slightly overestimate the time needed for a BH to be captured into a binary.

If a BH with mass M_{BH} gets within a couple of semimajor axes of a main-sequence binary, the binary will tidally separate and the BH will acquire a companion. The timescale on which this happens is $t_{\text{bin}} = (n \Sigma \sigma_{3D})^{-1}$, where $\Sigma = \pi r_p^2 [1 + 2GM_{\text{tot}} / (\sigma_{3D}^2 r_p)]$ is the interaction cross-section for pericenter distances $\leq r_p$ when gravitational focusing is included. Here, M_{tot} is the mass of the BH plus the mass of the binary. If we assume that $M_{\text{BH}} = 10 M_\odot$ and it interacts with a binary with two $1 M_\odot$ members and an $a = 1 \text{ AU}$ semimajor axis, then the typical timescale on which a three-body interaction and capture of one of the stars occur is

$$t_{3\text{-bod}} = (n \Sigma \sigma_{3D})^{-1} \approx 3 \times 10^9 \text{ yr} (n / 10^6 \text{ pc}^{-3})^{-1} \times (f_{\text{bin}} / 0.01)^{-1} (\sigma_{3D} / 50 \text{ km s}^{-1}) (a / 1 \text{ AU})^{-1}. \quad (3)$$

With rapid sinking, BHs can form a subcluster in the galaxy core. This will decrease the number density of main-sequence stars in the core and, hence, of main-sequence binaries (although binaries, being heavier than single stars, will be overrepresented). The exchange process might thus take somewhat longer. The timescale in Equation (3) is, however, small enough compared to a Hubble time that we start our simulations by assuming that each BH has exchanged into a hard binary, and follow its evolution from there.

Another important question is whether, after a three-body interaction, a BH binary will shed the kinetic energy of its center of mass via dynamical friction and sink to the center of the cluster before another three-body encounter. If not, the kick speeds will add in a random walk, thus increasing the ejection fraction.

To compute this we note that the local relaxation time of a binary is

$$t_{\text{rlx}} = \frac{0.34}{\ln \Lambda} \frac{\sigma_{3D}^3}{G^2 \langle m \rangle M_{\text{bin}} n} \quad (4)$$

(Spitzer 1987), where $\ln \Lambda \sim 10$ is the Coulomb logarithm, $\langle m \rangle$ is the average mass of interloping stars, n is their number density, and M_{bin} is the mass of the binary. The timescale for a three-body interaction is $t_{3\text{-bod}} = (n \Sigma \sigma_{3D})^{-1}$ as above. Note, however, that for this calculation we assume that the BH has already captured into a binary. Therefore, it can interact with every star instead of just those in binaries, and thus the factor f_{bin} is no longer applicable and the timescale is typically 100 times less than indicated by the numerical factor in Equation (3). For a gravitationally focused binary, which is of greatest interest because only these could in principle produce three-body recoil sufficient to eject binaries or singles, $r_p < G M_{\text{bin}} / \sigma_{3D}^2$. If we also assume that the total mass M_{tot} of the three-body system is close to M_{bin} because most of the interlopers have much less mass than the BH, then $\Sigma \approx 2\pi r_p G M_{\text{bin}} / \sigma_{3D}^2$ and

$$t_{3\text{-bod}} \approx \frac{\sigma_{3D}}{2\pi n r_p G M_{\text{bin}}} \quad (5)$$

If we let $r_p = q G M_{\text{bin}} / \sigma_{3D}^2$, with $q < 1$, then

$$t_{3\text{-bod}} \approx \frac{\sigma_{3D}^3}{2\pi q G^2 M_{\text{bin}}^2 n} \quad (6)$$

so that

$$t_{\text{rlx}} / t_{3\text{-bod}} \approx \frac{2q}{\ln \Lambda} \frac{M_{\text{bin}}}{\langle m \rangle} \quad (7)$$

The encounters most likely to deliver strong kicks to the binary occur when the binary is very hard, $q \ll 1$; hence, this quantity is typically less than unity, meaning that after a three-body encounter, a binary has an opportunity to share its excess kinetic energy via two-body encounters and thus settle back to the center of the cluster. We, therefore, treat the encounters separately rather than adding the kick speeds in a random walk.

In a given encounter, suppose that a binary of total mass $M_{\text{bin}} = M_1 + M_2$, a reduced mass $\mu = M_1 M_2 / M_{\text{bin}}$, and a semimajor axis a_{init} interacts with an interloper of mass m_{int} , and that the kinetic energy of the interloper at infinity is much less than the binding energy of the binary (i.e., this is a very hard interaction). If after the interaction the semimajor axis is $a_{\text{fin}} < a_{\text{init}}$, then energy and momentum conservation mean that the recoil speed of the binary is given by $v_{\text{bin}}^2 = G\mu \frac{m_{\text{int}}}{M_{\text{bin}} + m_{\text{int}}} (1/a_{\text{fin}} - 1/a_{\text{init}})$, and the recoil speed of the interloper is $v_{\text{int}} = (M_{\text{bin}}/m_{\text{int}})v_{\text{bin}}$. For example, suppose that $M_1 = M_2 = 10 M_{\odot}$, $M_{\text{int}} = 1 M_{\odot}$, $a_{\text{init}} = 0.1$ AU, and $a_{\text{fin}} = 0.09$ AU. The binary then recoils at $v_{\text{bin}} = 15$ km s⁻¹ and stays in the cluster, whereas the interloper recoils at $v_{\text{int}} = 300$ km s⁻¹ and is ejected.

We treat all three objects as point masses, but in fact main-sequence stars are extended enough that they have a good chance of being tidally disrupted in an encounter with a BH binary. Almost all of the disrupted mass is eventually ejected at speeds

comparable to the binary orbital speed; hence, we assume that tidal disruptions of main-sequence stars have the same effect on the binary energetically as a normal three-body ejection. Note, however, that since unlike for interactions with point sources, the ejected mass will not go in a single direction, it is likely that tidal disruptions will not cause the binary to recoil as much and thus such interactions are likely to result in a somewhat greater retention fraction than we calculate. We also find that close approach distances are great enough that post-Newtonian corrections are not necessary during interactions, and that a small enough fraction of stars are involved in these encounters that the effect on the mass distribution due to mergers and ejections is negligible. In addition, we assume throughout our calculations that nuclear star clusters do not have massive BHs at their centers.

2.2. Results

The central regions of the clusters undergo significant mass segregation, and thus the mass function will be at least flattened, and possibly inverted. This has been observed for globular clusters (see Table 3 of Sosin 1997 or Table 1 of De Marchi et al. 2007) and is also seen in numerical simulations (e.g., Baumgardt et al. 2008 or Gill et al. 2008). To include this effect, when we consider the mass of a BH, its companion, or the interloping third object in a binary-single encounter, we go through two steps. First, we select a zero-age main-sequence (ZAMS) mass between $0.2 M_{\odot}$ and $100 M_{\odot}$ using a simple power-law distribution $dN/dM \propto M^{-\alpha}$. While there is evidence that the upper limit of the ZAMS might be greater than $120 M_{\odot}$ (Oey & Clarke 2005), we chose the more traditional value of $100 M_{\odot}$ (Kroupa & Weidner 2003) to be conservative. We allow α to range anywhere from 2.35 (the unmodified Salpeter distribution) to -1.0 , where smaller values indicate the effects of mass segregation. Second, we evolve the ZAMS mass to a current mass. Our mapping is that for $M_{\text{ZAMS}} < M_{\text{ms,max}}$, where $M_{\text{ms,max}}$ is $1 M_{\odot}$ or $3 M_{\odot}$ depending on the model used, the star is still on the main sequence and retains its original mass; for $1 M_{\odot} < M_{\text{ZAMS}} < 8 M_{\odot}$, the star has evolved to a white dwarf, with mass $M_{\text{WD}} = 0.6 M_{\odot} + 0.4 M_{\odot} (M_{\text{ZAMS}}/M_{\odot} - 0.6)^{1/3}$; for $8 M_{\odot} < M_{\text{ZAMS}} < 25 M_{\odot}$, the star has evolved to a neutron star, with mass $M_{\text{NS}} = 1.5 M_{\odot} + 0.5 M_{\odot} (M_{\text{ZAMS}} - 8 M_{\odot})/17 M_{\odot}$; and for $M_{\text{ZAMS}} > 25 M_{\odot}$, the star has evolved to a BH with mass $M_{\text{BH}} = 3 M_{\odot} + 17 M_{\odot} (M_{\text{ZAMS}} - 25 M_{\odot})/75 M_{\odot}$. Therefore, we assume that BH masses range from $3 M_{\odot}$ to $20 M_{\odot}$.

These prescriptions are overly simplified in many ways. We, therefore, explore different mass function slopes, main-sequence cutoffs, and so on, and find that our general picture is robust against specific assumptions. Note that, consistent with O'Leary et al. (2006), we find that there is a strong tendency for the merged BHs to be biased toward high masses. Therefore, if BHs with masses greater than $20 M_{\odot}$ are common, these will dominate the merger rates. This is important for data analysis strategies, because the low-frequency cutoff of ground-based gravitational wave detectors implies that higher-mass BHs will have proportionally more of their signal in the late inspiral, merger, and ringdown.

The three-body interactions themselves are assumed to be Newtonian interactions between point masses and are computed using the hierarchical N -body code HNBODY (K. Rauch and D. Hamilton 2008, in preparation), using the driver IABL developed by Kayhan Gültekin (see Gültekin et al. 2004, 2006 for a detailed description). These codes use a number of high-accuracy techniques to follow the evolution of grav-

Table 1
Simulations of Nuclear Star Clusters^a

V_{esc} (km s ⁻¹) ^b	$M_{\text{ms,max}}$ ^c	α ^d	$\langle M_{\text{BH}} \rangle (M_{\odot})$ ^e	f_{merge} ^f	f_{notBH} ^g	$\langle M_{\text{bin,merge}} \rangle (M_{\odot})$ ^h	$\langle N_{\text{single,eject}} \rangle$ ⁱ
50	1 M_{\odot}	0	11.7	0.25	0.0	31.2	24.8
62.5	1 M_{\odot}	0	11.7	0.33	0.0	31.6	15.3
75	1 M_{\odot}	0	11.7	0.42	0.0	30.9	11.5
87.5	1 M_{\odot}	0	11.7	0.52	0.0	31.9	7.9
20	1 M_{\odot}	0	11.7	0.63	0.02	30.0	6.2
112.5	1 M_{\odot}	0	11.7	0.68	0.0	31.4	4.7
125	1 M_{\odot}	0	11.7	0.72	0.02	31.8	4.3
137.5	1 M_{\odot}	0	11.7	0.76	0.01	32.0	3.0
150	1 M_{\odot}	0	11.7	0.80	0.03	32.3	2.8
162.5	1 M_{\odot}	0	11.7	0.93	0.03	31.3	2.0
175	1 M_{\odot}	0	11.7	0.89	0.02	31.9	2.0
187.5	1 M_{\odot}	0	11.7	0.90	0.01	31.3	2.1
200	1 M_{\odot}	0	11.7	0.94	0.08	31.1	1.3
212.5	1 M_{\odot}	0	11.7	0.89	0.05	30.5	1.0
225	1 M_{\odot}	0	11.7	0.98	0.06	31.0	1.2
237.5	1 M_{\odot}	0	11.7	0.94	0.06	30.1	1.0
250	1 M_{\odot}	0	11.7	0.96	0.06	30.0	0.71
200	1 M_{\odot}	-1.0	13.4	0.94	0	32.4	1.3
200	1 M_{\odot}	-0.5	12.6	0.95	0.01	32.2	1.5
200	1 M_{\odot}	0.5	10.7	0.94	0.1	28.3	0.91
200	1 M_{\odot}	1.0	9.7	0.98	0.41	27.3	0.43
200	1 M_{\odot}	1.5	8.8	0.99	0.79	23.0	0.04
200	1 M_{\odot}	2.0	7.5	1.00	0.99	...	0
200	1 M_{\odot}	2.35	7.4	1.00	1.00	...	0
200	3 M_{\odot}	-1.0	13.4	0.85	0.03	33.3	1.5
200	3 M_{\odot}	-0.5	12.6	0.94	0.01	31.9	1.3
200	3 M_{\odot}	0	11.7	0.95	0.05	30.4	1.5
200	3 M_{\odot}	0.5	10.7	0.94	0.11	29.2	1.0
200	3 M_{\odot}	1.0	9.7	0.99	0.48	25.3	0.38
200	3 M_{\odot}	1.5	8.8	0.99	0.85	24.7	0.04
200	3 M_{\odot}	2.0	7.5	1.00	1.00	...	0
200	3 M_{\odot}	2.35	7.4	1.00	1.00	...	0

Notes.

^a All runs had 100 realizations.

^b Escape speed from cluster.

^c Maximum mass of main-sequence star.

^d Number distribution of stars on zero-age main sequence: $dN/dM \propto M^{-\alpha}$.

^e Average mass of all BHs given α and our evolutionary assumptions.

^f Fraction of runs in which holes merged rather than being ejected.

^g Fraction of runs in which holes merged with something other than another BH.

^h Average mass of double BH binaries that merged.

ⁱ Average number of single BHs ejected per binary that merged.

itating point masses. Between interactions, we use the Peters equations (Peters 1964) to follow the gradual inspiral and circularization of the binary via emission of gravitational radiation. This is negligible except near the end of any given evolution.

We begin by selecting the mass of the BH and of its companion (which does not need to be a BH) from the evolved mass function. We also begin with a semimajor axis that is 1/4 of the value needed to ensure that the binary is hard. We do this because soft binaries are likely to be ionized and thus become single stars rather than merge. We also select an eccentricity from a thermal distribution $P(e)de = 2ede$. We then allow the binary to interact with single field stars drawn from the evolved mass function, one at a time, until either (1) the binary merges due to gravitational radiation, (2) the binary is split apart and thus ionized (this is exceedingly rare given our initial conditions), or (3) the binary is ejected from the cluster. The entire set of interactions until merger typically takes millions to tens of millions of years, and only rarely more than a hundred million

years, so it finishes in much less than a Hubble time. This implies that the total time for an individual, initially single, BH to merge with another object is dominated by the few billion years needed to capture into a binary rather than by subsequent interactions. In the course of these interactions there are typically a number of exchanges, which usually swap in more massive for less massive members of the binary. This is the cause of the bias toward high-mass mergers that was also found by O’Leary et al. (2006). As shown in Table 1, for $\alpha < 1$, most BHs acquire a BH companion in the process of exchanges, and for $\alpha \leq 0.5$ virtually all do.

The results in Table 1 are focused on different mass function slopes and escape speeds. As expected, we find that for $V_{\text{esc}} > 150$ km s⁻¹, the overwhelming majority of BH binaries merge in the nuclear star cluster rather than being ejected (see Figure 1). This is the difference from lower- σ globular clusters, where the mergers happen outside the cluster. Note also that in addition to few binaries being ejected, there are typically only 1–2 single BHs ejected per merger, suggesting that greater than 50% of

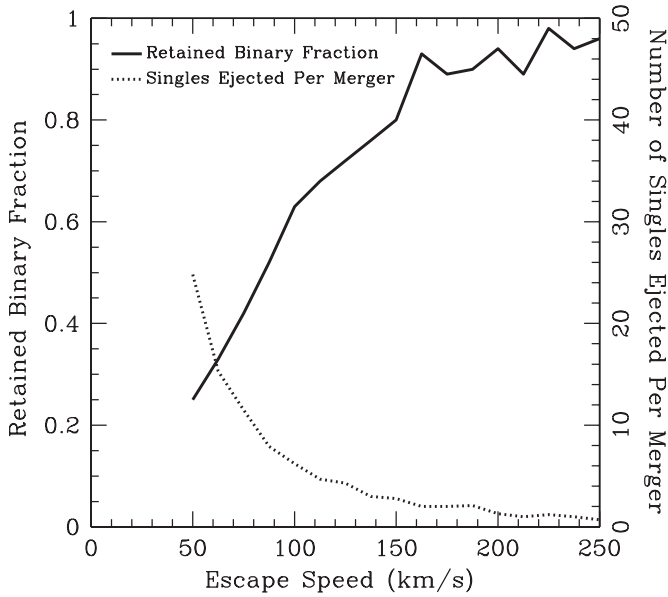


Figure 1. Fraction of binaries retained in the nuclear star cluster (solid line) and average number of BHs ejected per BH merger (dotted line) as a function of the cluster escape speed. Here, the zero-age main-sequence distribution of masses is $dN/dM \propto M^0$, to account for mass segregation in the cluster center, where most interactions occur. We also assume a maximum BH mass of $20 M_{\odot}$ and a maximum main-sequence mass of $1 M_{\odot}$, but most results are robust against variations of these quantities. All runs are done with 100 realizations, which explains the lack of perfect smoothness. We see, as expected, that the retention fraction increases rapidly with escape speed, so that for nuclear star clusters, most binaries stay in the cluster until merger. We also see that at $V_{\text{esc}} \sim 200 \text{ km s}^{-1}$ and above, tens of percent of BH singles also stay in the cluster. This suggests a high merger efficiency.

holes will merge. In contrast, at the 50 km s^{-1} escape speed typical of globulars, greater than 20 single BHs are ejected per merger, suggesting an efficiency of less than 10%. For well-segregated clusters (with $\alpha \leq 0$), the average mass of BHs that merge, binary ejection fraction and number of singles ejected, and number of BHs that merge with each other instead of other objects are all insensitive to the particular mass function slope. For less segregated clusters with $\alpha > 0$, the retention fraction of BHs rises rapidly to unity because most of the objects that interact with the holes are less massive stars. For example, in clusters with $\alpha \geq 1.0$, about 10% of BH mergers occur with neutron stars, in contrast to a few percent or less for more segregated clusters. In such clusters, there might be a channel by which the mass of the holes increases via accretion of main-sequence stars, but we expect $\alpha > 0$ to be rare for nuclear star clusters because of the shortness of the segregation times of BHs. Overall, there appears to be a wide range of realistic parameters in which fewer than 10% of binary BHs are ejected before merging.

3. DISCUSSION AND CONCLUSIONS

We have shown that nuclear star clusters with velocity dispersions around $\sigma_{3D} \sim 40\text{--}60 \text{ km s}^{-1}$ are promising breeding grounds for stellar-mass BH mergers. At significantly lower velocity dispersions, as found in globulars, the escape speed is low enough that the binaries are ejected before they merge. Significantly higher velocity dispersions appear correlated with the appearance of supermassive BHs (Gebhardt et al. 2000; Ferrarese & Merritt 2000). In such an environment, there

might also be interesting rates of BH mergers (see O’Leary et al. 2008 for a recent discussion), but the increasing velocity dispersion closer to the central object means that binary fractions are lower and softening, ionization, or tidal separation by the supermassive BH itself are strong possibilities for stellar-mass binaries (Miller et al. 2005; Lauburg & Miller, in preparation).

To estimate the rate of detections with advanced LIGO, we note that velocity dispersions in the $\sigma_{3D} \sim 40\text{--}60 \text{ km s}^{-1}$ range correspond to roughly a factor of $\sim 5\text{--}10$ in galaxy luminosity (Ferrarese et al. 2006). Galaxy surveys suggest (e.g., Blanton et al. 2003) that for dim galaxies, the luminosity function scales roughly as $dN/dL = \phi^*(L/L_*)^\beta$, where $\phi^* = 1.5 \times 10^{-2} h^3 \text{ Mpc}^{-3} \approx 5 \times 10^{-3} \text{ Mpc}^{-3}$ for $h = 0.71$, and $\beta \approx -1$. This implies that there are nearly equal numbers of galaxies in equal logarithmic bins of luminosity. A factor of 5–10 in luminosity is roughly e^2 , so the number density of relevant galaxies is approximately 10^{-2} Mpc^{-3} . To get the rate per galaxy, we note that typical initial mass functions and estimates of the mass needed to evolve into a BH combine to suggest that for a cluster of mass M_{nuc} , approximately $3 \times 10^{-3} (M_{\text{nuc}}/M_{\odot})$ stars evolve into BHs (O’Leary et al. 2007). This implies a few $\times 10^4$ BHs per nuclear star cluster. If a few tens of percent of these merge in a Hubble time, and if the rate is slightly lower now because many of the original BHs have already merged (see O’Leary et al. 2006), this suggests a merger rate of $> 0.1 \times \text{few} \times 10^4 / (10^{10} \text{ yr})$ per galaxy or few $\times 10^{-9} \text{ Mpc}^{-3} \text{ yr}^{-1}$.

Mergers of the original BHs are not expected to significantly decrease the detection rates, for two basic reasons. First, nuclear star clusters are not isolated. Instead, the cluster itself is surrounded by a stellar distribution (it is, after all, the center of the galaxy; therefore, unlike for globular clusters, nuclear star clusters are not surrounded by vacuum). This distribution will include BHs. In time, these holes will sink by dynamical friction into the nuclear star cluster itself. This helps replenish the holes that are kicked out by three-body processes in the cluster. For example, if the number density scales as $n \propto r^{-2}$ (a reasonable approximation for many galactic centers), then there are as many stars (and presumably BHs) from some radius R to $2R$ as from 0 to R , and from $2R$ to $3R$ as from R to $2R$. Under such a circumstance, the relaxation time scales as the square of the radius, so there should be an abundant supply of BHs over any reasonable timescale.

The second reason is that the timescale for BHs to capture into a binary goes up with increasing nuclear star cluster mass, because all relaxation times increase. As a result, if clusters of the particular mass we suggested have been depleted significantly, clusters of higher mass will not have been. This shifts the optimal cluster mass to a larger value. However, as in the previous point, it seems reasonable that clusters will be replenished anyway.

At the average detection distance of $\sim 1.15 \text{ Gpc}$ at which advanced LIGO is expected to be able to see mergers of two $10 M_{\odot}$ BHs (I. Mandel 2008, private communication), the available volume is $6.4 \times 10^9 \text{ Mpc}^3$, for a rate of $\gtrsim 30$ per year. Roughly 50%–80% of galaxies in the eligible luminosity range appear to have nuclear star clusters (see Ferrarese et al. 2006 for a summary). If the majority of the clusters do not have a supermassive BH, this suggests a final rate of tens per year for advanced LIGO. This could be augmented somewhat by small galaxies that originally had supermassive BHs, but had them ejected after a merger and then reformed a central cluster (Volonteri 2007; Volonteri et al. 2008).

For nearby ($z < 0.1$) events of this type, it might be possible to identify the host galaxy. However, for more typical $z \sim 0.5 \Rightarrow d \approx 1.15$ Gpc events, the number of candidates is too large. We can demonstrate this by adopting extremely optimistic values for angular localization and distance accuracy. Even assuming angular localization of $\Delta\Omega = (1^\circ)^2$ and a distance accuracy of $\Delta d/d = 1\%$, the number of galaxies in the right luminosity range is $N \sim 4\pi(1150 \text{ Mpc})^3(\Delta\Omega/4\pi)(\Delta d/d)(0.01 \text{ Mpc}^{-3}) \approx 45$. Therefore, barring some unforeseen electromagnetic counterpart, the host will usually not be obvious.

We anticipate that tens per year is a somewhat conservative number, because (unlike in a globular cluster) the central regions of galaxies are not devoid of gas; hence, more BHs could form in the vicinity of the cluster and fall in. In addition, if stellar-mass BHs with masses beyond $20 M_\odot$ are common, this also increases the detection radius and hence the rate. Even for total masses $\sim 30 M_\odot$ and at redshifts $z \sim 0.5$, the observer frame gravitational wave frequency at the innermost stable circular orbit is $f_{\text{ISCO}} \sim 4400 \text{ Hz}/[30(1+z)] \sim 100 \text{ Hz}$. This is close enough to the range where the frequency sensitivity of ground-based gravitational wave detectors declines that detection of many of these events will strongly rely on the signal obtained from the last few orbits plus merger and ringdown. In much of this range, numerical relativity is essential.

As a final point, we note that for the same reason that nuclear star clusters are favorable environments for retention and mergers of stellar-mass BHs, they could also be good birthplaces for more massive BHs. This could be prevented, even for the relatively high escape speeds discussed here, if recoil from gravitational radiation during the coalescence exceeds $\sim 200 \text{ km s}^{-1}$. The key uncertainty here is the spin magnitudes of the holes at birth. Numerous simulations demonstrate that high spins with significant projections in the binary orbital plane can produce kicks of up to several thousand kilometers per second (Gonzalez et al. 2007). If there is significant processing of gas through accretion disks, the spins are aligned in a way that reduces the kick to below 200 km s^{-1} (Bogdanović et al. 2007), but stellar-mass BHs cannot pick up enough mass from the interstellar medium for this to be effective. For example, the Bondi–Hoyle accretion rate is $\dot{M}_{\text{Bondi}} \approx 10^{-13} M_\odot \text{ yr}^{-1} (\sigma_{3D}/50 \text{ km s}^{-1})^{-3} (n_{\text{gas}}/100 \text{ cm}^{-3}) (M/10 M_\odot)^2$, where n_{gas} is the particle number density in the gas. This means that to accrete the $\sim 1\%$ of the BH mass needed to realign the spin (Bogdanović et al. 2007) would require at least a trillion years. Current estimates of stellar-mass BH spins suggest $a/M > 0.5$ in many cases (Shafee et al. 2006; McClintock et al. 2006; Miller 2007; Liu et al. 2008). If the spins are isotropically oriented and uniformly distributed in the range $0 < a/M < 1$, and the mass ratios are in the $m_{\text{small}}/m_{\text{big}} \sim 0.6\text{--}0.8$ range typical in our simulations, then use of the Campanelli et al. (2007) or Baker et al. (2008) kick formulae imply that roughly 84% of the recoils exceed 200 km s^{-1} and 78% exceed 250 km s^{-1} . This suggests that multiple mergers are rare unless there is initially an extra-massive BH as a seed (e.g., Holley-Bockelmann et al. 2008 for a discussion of the effects of gravitational wave recoil), but further study is important.

In conclusion, we show that the compact nuclear star clusters found in the centers of many small galaxies are ideal places to foster mergers between stellar-mass BHs. It is not clear whether multiple rounds of mergers can lead to runaway, but this is a new potential source for ground-based detectors such

as advanced LIGO, where numerical relativity will play an especially important role.

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