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Constraining neutron star masses and radii using thermonuclear X-ray bursts

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Abstract. Precisely measured neutron star masses and especially radii would provide unique constraints on the properties of cold matter at several times nuclear density. Observations using the Rossi X-ray Timing Explorer suggest that such measurements might be possible using thermonuclear X-ray bursts. Here we discuss the prospects for mass and radius constraints, with a particular focus on potential systematic errors.

Keywords. dense matter — stars: neutron — X-rays: bursts

1. Introduction

The composition of cold matter at several times nuclear saturation density, as exists in the cores of neutron stars, cannot be determined using terrestrial experiments. Instead the stars themselves must be examined for clues. Masses have been obtained for several neutron stars in binaries, but precise and accurate radii have remained elusive.

Three decades ago it was proposed by van Paradijs (1979) that the recently-discovered phenomenon of thermonuclear X-ray bursts from accreting neutron stars could be used to constrain radii as well as masses. It was not, however, until the launch of the *Rossi* X-ray Timing Explorer (RXTE) in 1995 that data of sufficient quality existed to test the van Paradijs assumptions. The revolution in our understanding of bursts afforded by high-precision RXTE data has led to renewed interest in the van Paradijs method and other methods based on careful characterization of burst spectra and evolution.

Here we discuss methods that have been proposed to constrain neutron star masses and radii using analyses of thermonuclear burst data. We begin by discussing why neutron star radii are so much more difficult to measure than their masses, and why the radii would be key in constraining properties of dense matter. After a quick review of other proposed methods for measuring radii, we focus on the van Paradijs method, which involves observations of spectra during bursts and has been suggested as a method that can yield highly precise neutron star radii. We discuss the strong indications that critical assumptions in this method are not valid for real burst sources, and hence that these assumptions need to be modified and that current radius estimates using this method are unreliable. We conclude with a look ahead to proposed X-ray missions such as LOFT and AXTAR, and the prospects these have for yielding neutron star constraints. We use "mass" to refer to the gravitational mass (i.e., the mass that would be inferred from the orbit of a distant satellite using Kepler's laws) rather than the baryonic mass (the sum of the rest masses of all the constituent particles of the star); the gravitational mass is typically $\sim 20-30\%$ less than the baryonic mass for a neutron star. We also adopt the typical practice and use "radius" to mean the circumferential radius of a nonrotating star, i.e., the circumference at the equator divided by 2π, rather than different measures of the radius such as the proper distance from the center to the surface.

2. The importance and difficulty of determining radii

There are comparatively few macroscopic indicators of the equation of state of cold high-density matter. Soft equations of state tend to yield neutron stars with lower maximum masses and smaller radii at a given mass than do stiff equations of state. Thus the higher a measured mass is, the stiffer the equation of state must be at the relevant densities. There are also other implications for high-density matter. For example, Demorest *et al.* (2010) showed that the mass $M = 1.97 \pm 0.04\ M_\odot$ they measured for PSR J1614–2230 constrains the maximum mass density of matter within a neutron star to be $< 3.7 \times 10^{15}$ g cm^{-3}. There is still, however, a wide range ($\sim 9-15$ km) of radii allowed for plausible masses, even for equations of state that can support a 2 $M_\odot$ star. Even if the mass of a star is not known, Lattimer & Prakash (2007) show that knowledge of the radius to a precision of ~ 1 km would be highly informative, because neutron star radii are nearly constant over a wide range of masses for most viable equations of state.

Radius measurements are, however, extremely challenging. In contrast to the stellar mass, which can be inferred from the orbit of a companion and in the best cases by relativistic pulse delay effects, the stellar radius has no effects at a distance. Less direct methods must therefore be used, and these are often subject to substantial systematic errors. For example, one might think of using a method similar to one that is used successfully for normal stars: assume that the surface emits as a blackbody, then measure the temperature T from the spectrum and the luminosity L from the observed flux and the distance, assuming isotropic emission. Then $R = [L/(4\pi\sigma T^4)]^{1/2}$. However, application of this method to neutron stars tends to yield unreasonable radii, often 5 km or less. The reason, as we discuss below, is that atmospheric effects (primarily scattering) lower the efficiency of emission well below the value for a blackbody, while producing a spectrum with a shape very close to the shape of a Bose-Einstein spectrum. Thus, radius estimates require a deep understanding of the spectrum, to estimate efficiencies, and excellent data, to validate our understanding and discriminate between models.

3. Thermonuclear X-ray bursts

So-called Type 1 X-ray bursts were discovered by Belian *et al.* (1976) and Grindlay *et al.* (1976), and explained soon thereafter as being caused by runaway nuclear fusion of matter donated to a neutron star by a close binary companion [e.g., Lamb & Lamb (1977), Joss (1977), Lamb & Lamb (1978), Joss (1978), and Woosley & Wallace (1982); see also Woosley & Taam (1976)]. Observations in the succeeding decades have revealed that these bursts can have a variety of durations: the shortest (lasting for seconds) are thought to be produced by fusion of a helium layer, whereas the longest (lasting for hours, and called superbursts) are believed to be produced by fusion in a much deeper layer of carbon. The maximum surface fluxes produced by all types of bursts are comparable. The fluxes from some bursts are high enough to cause cause photospheric radius expansion (these are called PRE bursts), which is thought to occur when the surface flux temporarily exceeds the Eddington critical flux. Indeed, the flux we observe when the inferred photospheric radius retreats to the surface of the star after a radius expansion and the atmosphere becomes static is often used to estimate the Eddington luminosity of the star. The moment when this occurs is called the time of touchdown and is assumed to be the moment when the fitted spectral temperature is a maximum. See Galloway *et al.* (2008) for a summary of burst observations.

Although the nuclear physics and possibly the ignition and spreading of hot gas may be significantly different in normal, few-second bursts and in few-hour superbursts, the spectrum we see depends only on the propagation of the radiation through the atmosphere. The spectra of both types of bursts are therefore expected to be the same, other things being equal. Therefore, we can use observations of superbursts with thousands of times as many counts as a normal burst to determine with high precision the spectra characteristic of all bursts.

An ideal observed spectrum from the neutron star surface would have identifiable atomic lines, which would immediately allow us to determine the gravitational redshift at the stellar surface as well as much other valuable information. The high surface gravity of neutron stars means that heavy elements, which might not be fully ionized, sink in seconds [Alcock & Illarionov (1980)], but it was hoped that continuously accreting stars such as bursters might nevertheless have a significant abundance of heavy elements such as iron in their photospheres. Indeed, Cottam *et al.* (2002) found evidence of iron lines at a redshift $z = 0.35$ in burst spectra from EXO 0748−676. However, a later observation [Cottam *et al.* (2008)] found the source in a different state with no lines (even the abundant zero-redshift lines that had been apparent in the earlier observation were absent), so the redshifted could not be confirmed. This neutron star was subsequently found to have a spinning at 552 Hz [Galloway *et al.* (2010)], fast enough that any lines were expected to be much broader than those reported by Cottam *et al.* [Lin *et al.* (2010)]. However, there is a recent suggestion that second-order rotational effects could make a portion of each line appear narrow [Baubock *et al.* (2012)]. In any case, the lack of confirmed atomic lines from any neutron star surface means that for now we must work with the continuum spectrum. This is a much subtler game that is far more susceptible to systematic error.

4. The van Paradijs method and recent applications

van Paradijs (1979) proposed that thermonuclear bursts could be used to constrain both the masses and radii of bursting neutron stars. Updated slightly to reflect our current understanding, his proposal requires the following assumptions:

- The Eddington luminosity of the star is known. This requires knowledge of the distance to the star and knowledge that a fiducial observed flux (say, the observed flux at touchdown) corresponds to the Eddington critical flux at the stellar surface. It also requires that the emission from the star at the fiducial moment be isotropic, and that the composition and hence opacity of the atmosphere be known.
- The entire surface of the star radiates uniformly and isotropically at the fiducial moment.
- The emission spectrum is fully understood, so that we can correct for the inefficiency of the emission.

The first assumption gives us, effectively, a redshifted mass, because for a slowly rotating and isotropically emitting star the highest luminosity at infinity that allows a static atmosphere is $L = L_E(1 - 2GM/Rc^2)^{1/2}$, where $L_E = 4\pi GMc/\kappa$ is the standard Newtonian Eddington luminosity, M and R are respectively the mass and radius of the star, and κ is the radiative opacity. The second and third assumptions allow us to determine the redshifted surface area of the star. Combining all these assumptions allows us to determine both the mass and the radius of the star.

Only RXTE, with its large area, broad spectral coverage, and superb calibration could be used to collect data with the precision required to potentially obtain useful constraints on stellar masses and radii. It was therefore reasonable to analyze RXTE data using the van Paradijs assumptions and see what resulted; that is, an appropriate first step was to estimate only the statistical uncertainties, leaving an assessment of systematic errors for future work. Some such studies, such as the analysis of 4U 1820−30 by Güver *et al.* (2010) gave promising results. This star is ideal in many ways: its location in the globular cluster NGC 6624 allows its distance to be determined much more precisely than those of most burst sources, while the tightness of its orbit (which has an 11.5-minute period) indicates that the donor is a helium star and hence that the atmosphere of the neutron star is nearly pure helium. Using the van Paradijs approach, Güver *et al.* (2010) found statistical uncertainties of 4% in both the mass and the radius of 4U 1820−30. Such small uncertainties are encouraging, but a closer examination of this approach by Steiner *et al.* (2010) showed that these apparently tight constraints actually indicate the presence of significant systematic errors.

In the van Paradijs method, M and R are related to one theoretical and three observed input quantities (the computed color correction factor and the observed flux at touchdown, apparent emitting area, and distance to the star) via quadratic equations. The expressions for M and R therefore involve square roots of combinations of the four input quantities; the argument of these roots must be nonnegative in order for the expressions to give real numbers for M and R. Each of the four input quantities has a probability distribution. Steiner *et al.* (2010) showed that if the standard van Paradijs assumptions are used to analyze the 4U 1820−30 bursts, only $\sim 1.5 \times 10^{-8}$ of the parameter space spanned by the joint probability distribution of these quantities produces M and R values that are real numbers. Indeed, this is the reason the inferred statistical uncertainties in M and R are so small for this star: the van Paradijs assumptions, rather than the data, tightly constrain the allowed parameter values. This is a strong indication that these assumptions are not valid for this star, and hence possibly for other stars. Steiner *et al.* (2010) suggested that the standard van Paradijs assumptions should be relaxed by allowing the radius of the photosphere to be larger than the stellar radius, but they retained the assumption that the emission is uniform over a spherical photosphere. As we discuss below, fits of the best current spectral models to the 4U 1820−30 superburst spectrum suggest that even this assumption may not be correct: the *fraction* of the stellar surface that is emitting during the superburst changes and may therefore change during normal bursts. Moreover, there is no evidence that the emitting area is ever the entire stellar surface during the superburst.

5. Spectral models and implications

Clearly, an absolute requirement for obtaining reliable information from analyses of burst continuum spectra is a good model for the continuum. As we discussed earlier, although burst spectra may have shapes similar to blackbody spectra, the neutron star radii inferred from blackbody fits to the observed spectra are unreasonably high. To see why this is, consider a thought experiment in which the outward radiative flux is $F = \sigma T_{\mathrm{eff}}^4$ and constant in time, and the photons in the atmosphere of the neutron star initially have a Planck distribution with the effective temperature T_{eff} at and below a certain depth. Assume that above this thermalization depth photons interact with the gas only via Thomson scattering, i.e., there is no photon production or absorption. Finally, suppose that 80% of the photons moving outward from the thermalization depth are scattered back to the thermalization depth and absorbed, whereas 20% escape. As time passes, the matter at the thermalization depth will heat up until it reaches a temperature T_{col} such that $0.2\sigma T_{\mathrm{col}}^4 = \sigma T_{\mathrm{eff}}^4$. Once the atmosphere has reached a steady state, the spectrum of the emerging radiation will be a Planck spectrum with this higher "color" temperature T_{col} and the temperature derived by fitting a Planck spectrum to the spectrum of this radiation will be greater than the effective temperature by a "color factor" $f \equiv T_{\mathrm{col}}/T_{\mathrm{eff}} > 1$. Hence the neutron star radius inferred by assuming the stellar surface emits like a blackbody with the temperature $T = T_{\mathrm{col}}$ inferred by correcting the observed color temperature for the redshift will be a factor f^{-2} smaller than the actual radius.

The dominance of electron scattering over other forms of opacity in the $kT > 1$ keV light-element atmospheres relevant to burst spectra creates spectral shapes that are very close to the shape of a Bose-Einstein spectrum [Lo *et al.*, in preparation; see also Illarionov & Sunyaev (1975)]. Unfortunately, Bose-Einstein spectral shapes contain no information about the efficiency of the emission or the mass and radius of the star. Thus, very high quality spectral data and very accurate model atmosphere spectra are needed to extract information about the stellar mass and radius.

A first step in this program was performed by Boutloukos *et al.* (2010). They used 64 seconds of RXTE Proportional Counter Array (RXTE PCA) data from a superburst from 4U 1820−30, and showed that although the models used in most previous burst analyses describe these data very poorly, Bose-Einstein spectral shapes provide an excellent fit. Our subsequent analysis (in preparation) shows that a fit at least as good is provided by the detailed model atmosphere spectra of Suleimanov *et al.* (2012), which improved upon the earlier treatment by Suleimanov *et al.* (2011) by treating the electron scattering fully relativistically. Moreover, our preliminary joint fits (in preparation) of 102 16-second segments of data from this superburst, starting ∼160 seconds after the PRE phase, shows that the Suleimanov *et al.* (2012) spectral models fit much better than Bose-Einstein spectral shapes. For example, fits to RXTE PCA Standard2 channels 3–32 inclusive give $\chi^2/\mathrm{dof} = 5238/5098$ for the best-fit Suleimanov *et al.* (2012) models, versus $\chi^2/\mathrm{dof} = 5770/4998$ for the best-fit Bose-Einstein models.

This is encouraging. This result represents the first validation of detailed model atmosphere spectra using data with enough precision to discriminate between models. One might hope that one could implement the van Paradijs approach using these models, but we find instead that our fit of the 4U 1820−30 superburst data argue against one of the key assumptions in this method. This is the assumption that the entire surface emits uniformly during the burst tail. If this assumption were true, we would have found that the best-fit emitting area remained constant throughout the ∼1600 seconds that we analyzed. Instead, we find that the emitting area decreased steadily throughout the portion of the burst that we analyzed, with a total drop of ∼20% (see Figure 1). This drop is due to a decrease in the *fraction* of the stellar surface that is emitting, rather than a ∼ 10% decrease in the radius of a spherical photosphere entirely surrounding the star. We know this because, given that the scale height of an atmosphere supported only by gas pressure is $\sim kT/mg \sim$ few cm $\ll R \sim 10$ km, any palpable increase in the radius of the photosphere requires a surface radiative flux extremely close to Eddington; e.g., a flux of 0.9999 F_E would be necessary to increase the stellar radius by ∼ 10%. We

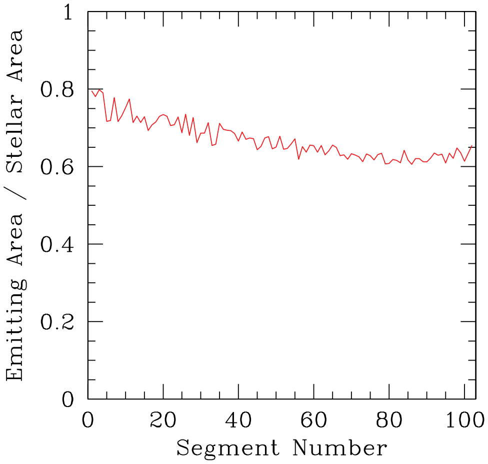

Figure 1. Emitting area relative to the total area of the stellar surface inferred by fitting of the Suleimanov *et al.* (2012) models to 16 second segments of the RXTE PCA data from 4U 1820–30. The illustrative fit we use for this figure yields a stellar radius of 10.2 km, but other masses and radii give equally good fits. We assume the distance to this source is 8.4 kpc [Valenti *et al.* (2007)]. This figure shows that the fit is reasonable, because the required emitting area is never more than the stellar area, but also shows that the fraction of the surface that emits decreases systematically and significantly during the entire ∼ 1600 seconds of data we have analyzed. It is possible that the emitting area is never the whole stellar surface at any time during the burst. Thus for this source, and possibly others, one cannot assume that the entire surface emits uniformly during a burst.

have verified that fluxes anywhere near this high give terrible fits to the data. Thus it is the emitting fraction of the stellar surface that is changing, and hence the standard assumption that the entire stellar surface is emitting uniformly is wrong for this burst. One must therefore consider the possibility that it is also wrong for shorter bursts (as was already suggested by the existence of burst oscillations, which cannot be produced by uniform emission from the entire stellar surface). This is also a concern for other methods that assume that the entire surface is emitting uniformly, such as the proposal by Suleimanov *et al.* (2012) that fits of the inferred color factors during the initial decay phases of bursts can yield M and R.

A different proposal, made first by Majczyna & Madej (2005), is to use fits of the spectral shape to sufficiently precise data to obtain both M and R directly. Because it is the shape of the spectrum that matters, rather than the absolute flux, this method does not require knowledge of the distance to the star or the fraction of the stellar surface that is emitting, so long as the emitting portion has a uniform spectrum, and it is even immune to photon energy-independent shifts in the calibration of the instrument (although energy-dependent shifts do affect this method). What Majczyna & Madej (2005) noted is that because the surface gravity g and the surface redshift z have different dependences on M and R, if g (which affects the atmospheric structure and thus the spectrum as it would be measured on the surface) and z (which determines the spectrum as seen by a distant observer given the surface spectrum) can both be constrained precisely, then M and R can be determined using the relations

$$R = (c^2/2g)(1 - 1/(1+z)^2)(1+z) \tag{5.1}$$

and

$$M = (Rc^2/2G)(1 - 1/(1+z)^2)\ . \tag{5.2}$$

Our preliminary results suggest that for the RXTE PCA data from the 4U 1820−30 superburst, which are the most precise data that we currently have, it is not possible to constrain g and z independently. We find that for a given g, $1 + z$ is constrained to within a few percent, which appears to be because although electron scattering dominates the opacity, at the lowest accessible photon energies free-free absorption becomes important enough to affect the spectrum. Full constraints on both g and z apparently require additional high-quality data, similar to what would be provided by proposed future X-ray timing missions missions, such as LOFT [Feroci *et al.* (2012)] or AXTAR [Ray *et al.* (2011)].

6. Conclusions

We have shown that the model atmosphere spectra of Suleimanov *et al.* (2012) fit RXTE PCA data from the 4U 1820−30 superburst well, and better than approximate solutions, such as the Bose-Einstein spectral shape, or competing detailed model atmosphere spectra. This is the first time that such a comparison has been possible. At the same time, fits of the Suleimanov *et al.* (2012) models to current data show that the emitting fraction of the neutron star surface changes significantly. This casts doubt on a key assumption that has been used for previous mass and radius estimates, which is that the entire stellar surface emits uniformly after touchdown. However, the validation of the Suleimanov *et al.* (2012) models inspires cautious optimism that fits of these models to data from future large-area X-ray satellites such as LOFT or AXTAR will yield reliable constraints on both the masses and radii of bursting neutron stars. Even if, as with current data, we can only establish a link between the surface redshift and surface gravity, this constraint will be useful in combination with other measurements. Overall, the excellent fit of the data by the best available detailed spectral models represents an important step forward in our understanding of both neutron stars and dense matter.

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