THE ASTROPHYSICAL JOURNAL, 509:793–801, 1998 December 20

EFFECTS OF RAPID STELLAR ROTATION ON EQUATION-OF-STATE CONSTRAINTS DERIVED FROM QUASI-PERIODIC BRIGHTNESS OSCILLATIONS

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Received 1998 May 4; accepted 1998 July 23

ABSTRACT

Quasi-periodic X-ray brightness oscillations (QPOs) with frequencies $\gtrsim 1$ kHz have now been discovered in more than a dozen neutron stars in low-mass X-ray binary systems using the *Rossi X-Ray Timing Explorer*. There is strong evidence that the frequencies of some kilohertz oscillations are the orbital frequencies of accreting gas in nearly circular orbits around these stars. Some stars that produce kilohertz QPOs may have spin frequencies $\gtrsim 400$ Hz. For spin rates this high, first-order analytic treatments of the effects of the star's rotation on its structure and the spacetime are inaccurate. Here we use the results of a large number of fully relativistic, self-consistent numerical calculations of the stellar structure of rapidly rotating neutron stars and their interior and exterior spacetimes to investigate the constraints on the properties of such stars that can be derived if stable circular orbits of various frequencies are observed. We have computed the equatorial radius of the star, the radius of the innermost stable circular orbit, and the frequency of the highest frequency stable circular orbit as functions of the stellar spin rate for spin rates up to the maximum possible and for several illustrative equations of state. Our calculations show that the upper bounds on the stiffness of neutron star matter implied by a given orbital frequency are typically significantly stricter for stars with spin frequencies $\gtrsim 400$ Hz than for slowly rotating stars.

Subject headings: accretion, accretion disks — dense matter — equation of state — relativity — stars: neutron — stars: oscillations

1. INTRODUCTION

The successful launch of the *Rossi X-Ray Timing Explorer* (*RXTE*) has made it possible to investigate, for the first time, the X-ray variability of neutron stars and black holes at frequencies $\gtrsim 300$ Hz. One of the most important discoveries made with *RXTE* is that many neutron stars in low-mass X-ray binaries produce high-frequency brightness oscillations with frequencies in the range ~ 300–1200 Hz (see van der Klis 1998). High-frequency oscillations are observed both during type I (thermonuclear) X-ray bursts and in the persistent X-ray emission. The discovery of these oscillations has made possible derivation of interesting upper bounds on the masses and radii of these neutron stars and significant new constraints on the equation of state of neutron star matter (Miller, Lamb, & Psaltis 1998a, 1998b; Lamb, Miller, & Psaltis 1998; Miller & Lamb 1998; Strohmayer et al. 1998).

Only a single oscillation has been observed from each source during a type I X-ray burst, and the oscillations in the tails of bursts appear to be highly coherent (see, e.g., Smith, Morgan, & Bradt 1997), with frequencies that are always the same in a given source (see, e.g., Strohmayer 1997). The burst oscillations are thought to be caused by the existence of one or two brighter regions on the stellar surface that produce oscillations at the stellar spin frequency or its first overtone as the star spins (see Strohmayer, Zhang, & Swank 1997b for compelling evidence in favor of this interpretation). The frequency of the burst oscillations ranges from ~ 330 Hz in 4U 1702$-$42 (Swank 1997) to 589 Hz in an unidentified source in the direction of the Galactic center (Strohmayer et al. 1997a).

The kilohertz quasi-periodic oscillations (QPOs) observed in the persistent emission have high amplitudes and relatively high coherences. The frequencies of the two QPOs often observed simultaneously in a given source have a frequency separation that is almost constant in many sources (see Wijnands & van der Klis 1997; Psaltis et al. 1998a, 1998b), although the frequencies of the QPOs themselves vary by hundreds of hertz. The separation frequencies of the two kilohertz QPOs seen in 4U 1728$-$34 (Strohmayer et al. 1996, 1997b) and 4U 1702$-$42 (Swank 1997) are consistent with the frequencies of their burst oscillations. The separation frequencies in 4U 1636$-$536 (Wijnands et al. 1997; Zhang et al. 1997) and KS 1731$-$260 (Smith et al. 1997; Wijnands & van der Klis 1997) are approximately one-half the frequencies of their burst oscillations.

The presence of only two simultaneous kilohertz QPOs in a given source, the approximately constant frequency separation $\Delta\nu$ between them, and the consistency of $\Delta\nu$ with the stellar spin frequency inferred from burst oscillations are strong evidence that the stellar spin is generating the frequency difference, that only one sideband of the primary QPO frequency is being generated, and that one of the two QPOs is therefore caused by the beat of the spin frequency with the other frequency (see Lamb et al. 1998; Miller et al.

1998a). This implies that in addition to the spin frequency there is only one other primary frequency and that this frequency is a rotational frequency such as an orbital frequency. This excludes neutron star surface and photon bubble oscillations as explanations for the primary kilohertz QPO frequency and makes disk oscillations an improbable explanation (see van der Klis 1998; Lamb et al. 1998).

In all sources the frequencies of the kilohertz QPOs fall within the expected range of orbital frequencies near a neutron star and can vary by several hundred hertz in a few hundred seconds (see Wijnands et al. 1998 and van der Klis 1995) while remaining highly coherent ($\nu/\Delta\nu \sim 100$). This is strong further evidence against disk oscillations and in favor of orbital motion of inhomogeneities in the accretion disk as the cause of the primary kilohertz QPO (see Lamb et al. 1998). The accreting gas exerts a strong torque on the neutron star and hence the star is expected to be spinning in the same sense as the orbital motion of the accreting gas. The beat frequency must therefore be the lower of the two kilohertz QPO frequencies, whereas the orbital frequency is the higher.

There are two candidates for the orbital frequency: the frequency at the radius where the accreting gas first couples strongly to the magnetic field of the neutron star and the frequency at the sonic radius where radiation forces or general relativistic effects cause the radial motion of the gas to increase sharply and become supersonic. The general properties of the kilohertz QPO sources and the specific properties of the kilohertz QPOs themselves strongly indicate that the relevant frequency is the orbital frequency at the sonic point (Miller et al. 1998a; see also Lamb et al. 1998). In either case, the frequency $\nu_{\rm QPO2}$ of the higher frequency kilohertz QPO is the orbital frequency of gas in a nearly circular orbit around the neutron star, whereas the frequency $\nu_{\rm QPO1}$ of the lower frequency QPO is the beat of the neutron star spin frequency with an orbital frequency near $\nu_{\rm QPO2}$.

In the following discussion we shall describe a circular orbit as stable or unstable according to its properties as determined by solving the geodesic equation for a test particle moving in that orbit in the spacetime of interest. However, it is important to bear in mind that *there are no closed, circular orbits in the vicinity of an accreting neutron star*, because the motion of gas near such a star is affected not only by the curvature of spacetime but also by radiation, magnetic, and viscous forces, which cause the gas to spiral inward even at radii where, in their absence, closed, stable orbits would be possible (Miller & Lamb 1993, 1996). However, the distinction between stable circular orbits (SCOs) and unstable circular orbits is still relevant. In particular, the innermost stable circular orbit (ISCO) is still physically significant when the effects of radial gas pressure forces on the ISCO can be neglected (which should be valid when the luminosity of the source is much less than the Eddington critical luminosity), because under these conditions gas inside the ISCO spirals inward so quickly that it cannot produce a wavetrain with the coherence observed for the kilohertz QPOs, regardless of whether it is acted on by radiation, magnetic, and viscous forces (Miller et al. 1998a).

Identification of the higher frequency kilohertz QPO with the frequency of an SCO has made it possible to derive upper bounds on the masses and radii of the neutron stars in the kilohertz QPO systems (Miller et al. 1998a, 1998b). These bounds follow from the requirement that the radius $R_{\rm orb}$ of the orbit be larger than the radius $R_{\rm ms}$ of the ISCO as well as larger than the equatorial radius $R_{\rm eq}$ of the star (if $R_{\rm eq} < R_{\rm orb} < R_{\rm ms}$, orbits with the required frequency exist but are unstable).

For nonrotating stars, observation of a given orbital frequency can be used to derive upper bounds on the mass and radius that are independent of the equation of state assumed (Miller et al. 1998a). For rotating stars, the situation is more complicated. In general, both the structure of the star and the spacetime are affected by the star's rotation, and there are no general analytical expressions for the relevant quantities. However, the exterior spacetime of a slowly and uniformly rotating fluid star is unique to first order in the dimensionless angular momentum $j \equiv cJ/GM^2$, where J and M are the star's angular momentum and gravitational mass and can be expressed analytically to this order (Hartle & Thorne 1968). The leading corrections to the expressions for the orbital frequency and the radius of the ISCO are first order in j. Using these expressions, one can demonstrate that observation of a given orbital frequency also implies upper bounds on the mass and radius of a slowly rotating star (Miller et al. 1998a). For a given stellar spin frequency, these upper bounds depend on the moment of inertia and hence on the equation of state assumed.

Many of the kilohertz QPO sources appear to have spin frequencies $\sim$250–350 Hz (see Miller et al. 1998a). Examples of such sources include 4U 0614+091, 4U 1608−52, 4U 1820−30, Cyg X-2, Sco X-1, GX 5−1, and GX 17+2, all of which have kilohertz QPO separation frequencies in this range, as well as 4U 1728−34 and 4U 1702−42, which not only have kilohertz QPO separation frequencies in this range but also have burst oscillation frequencies that are consistent with these separation frequencies. The $\sim$520 and $\sim$580 Hz frequencies of the burst oscillations seen in KS 1731−260 and 4U 1636−536 are thought to be twice their spin frequencies, although this is not certain. For spin frequencies in this range, j is $\sim$0.1–0.3, depending on the assumed equation of state and the mass of the star, and hence an analysis that is first order in j is quite accurate for such stars. Such an analysis shows that spin rates $\sim$300 Hz can increase the upper bound on the stellar mass by as much as $\sim$10%–20% but typically increase the upper bound on the radius by only $\sim$2%–5% (Miller et al. 1998a, 1998b).

On the other hand, some neutron stars that show kilohertz QPOs may turn out to have spin frequencies $\gtrsim$400 Hz. For example, oscillations with frequencies $\sim$550 and $\sim$590 Hz have been seen during X-ray bursts from, respectively, Aql X-1 and the unknown source in the direction of the galactic center, possibly indicating that these neutron stars have spin frequencies this high (Miller et al. 1998a). The recent discovery using *RXTE* that the source SAX J1808.4−3658 has a coherent 401 Hz oscillation indicates that this accreting neutron star is spinning rapidly (Wijnands & van der Klis 1998a, 1998b). For stars spinning this fast, the effect of the star's spin on its equilibrium structure (which is second order) and on the spacetime can be substantial. In order to obtain accurate results for such high spin rates, the equilibrium stellar structure and the interior and exterior spacetime must be computed self-consistently, which can be done only numerically.

Here we use the results of a large number of fully relativistic, self-consistent numerical calculations of the structure of rapidly rotating neutron stars and the interior and exterior spacetime to investigate the constraints on the properties of such stars that can be derived if SCOs of various frequencies are observed. We have computed the equatorial radius of the star, the radius of the ISCO, and the frequency of the highest frequency SCO as functions of the stellar spin rate and gravitational mass, for spin rates up to the maximum possible and for several illustrative equations of state. Comparison of these results with the highest observed kilohertz QPO frequency in a given source can be used to derive bounds on the mass and radius of the neutron star in that source, for a given equation of state. We also report the frequency of the highest frequency SCO as a function of the stellar spin rate, for stars of any mass constructed using a given equation of state. These curves can be used to check whether a particular equation of state is consistent with the frequency of a given kilohertz QPO.

Our calculations show that the upper bounds on the stiffness of neutron star matter implied by a given orbital frequency are typically significantly stricter for stars with spin frequencies $\gtrsim 400$ Hz than for slowly rotating stars.

In § 2 we describe our assumptions and calculational method. In § 3 we present our results and discuss the implications for constraining the properties of neutron star matter. Our conclusions are summarized in § 4.

2. ASSUMPTIONS AND METHOD

In deriving bounds on the masses and radii of the neutron stars with kilohertz QPOs, we assume that the higher frequency of the two simultaneous kilohertz QPOs is the frequency of a stable circular orbit around the neutron star, for the reasons discussed in § 1. Hence the orbital radius $R_{\rm orb}$ that corresponds to the QPO frequency must be larger than both the equatorial radius $R_{\rm eq}$ of the neutron star and the radius $R_{\rm ms}$ of the ISCO (Miller et al. 1998a).

We compute the equilibrium stellar structure and the interior and exterior spacetime using the numerical code described in Cook, Shapiro, & Teukolsky (1992, 1994a, 1994b). This code solves the full general relativistic equation of hydrostatic equilibrium for a star with a given spin rate using a variation of the metric potential method of Komatsu, Eriguchi, & Hachisu (1989a, 1989b). It gives accurate solutions even for stars that are spinning very rapidly.

In any stationary, axisymmetric spacetime, the orbital frequency at a given coordinate radius, as measured at infinity, is $\Omega = d\phi/dt$, where ϕ and t are, respectively, the global azimuthal and time coordinates based on the spacelike and timelike Killing vector fields of the spacetime. The time interval required for one orbit of an element of gas is the same everywhere, as measured in the global time coordinate. Given the metric of the exterior spacetime, the orbital frequency at a given radius is the solution of the geodesic equation for circular orbits (see Lightman et al. 1973, p. 469):

$$g_{\phi\phi,r}\Omega^2 + 2g_{t\phi,r}\Omega + g_{tt,r} = 0 \;, \tag{1}$$

where $g_{\phi\phi}$, $g_{t\phi}$, and g_{tt} are the metric components indicated and commas denote partial derivatives.

2.1. *Masses and Equations of State*

We have explored the constraints implied by observation of an SCO of given frequency for a variety of neutron star equations of state. These restrictions are most significant if the equation of state is hard rather than soft. Hence, in this report we present results for four relatively hard equations of state. For completeness, we consider both baryonic masses that are stable for nonrotating stars (the so-called "normal" sequences of Cook et al. 1994b) and the higher baryonic masses that are stable only for rotating stars (the "supramassive" sequences of Cook et al. 1994b). Whether the supramassive sequences are accessible depends on how the specific angular momentum of the accreting gas varies with time.

In order to facilitate comparisons with previous studies of neutron star properties (see, e.g., Pethick & Ravenhall 1995), we consider the Friedman-Pandharipande-Skyrme (FPS) equation of state (Friedman & Pandharipande 1981; Lorenz, Ravenhall, & Pethick 1993). The FPS equation of state is based on the Urbana v_{14} two-nucleon potential plus the density-dependent three-nucleon interaction model of Lagaris & Pandharipande (1981) and gives a maximum gravitational mass for a nonrotating star of about 1.8 $M_\odot$, compared with a maximum mass of 2.12 $M_\odot$ for a rotating star. The maximum spin frequency for stars in the normal sequence is 1411 Hz, and the maximum spin frequency for stars in the supramassive sequence is 1878 Hz.

As an example of later realistic equations of state, we consider the UU equation of state (Wiringa, Fiks, & Fabrocini 1988), which is based on the Urbana v_{14} two-nucleon potential plus the Urbana VII three-nucleon potential (Schiavilla, Pandharipande, & Wiringa 1986) and gives a maximum mass for a nonrotating star of about 2.2 $M_\odot$. Although it is based on older scattering data, the UU equation of state is similar to the recent A18 + UIX′ + δv_b equation of state (Akmal, Pandharipande, & Ravenhall 1998), which is based on the modern Argonne v_{18} two-nucleon potential and the Urbana IX three-nucleon potential and takes into account the nonzero momentum of the interacting nucleons (see Pandharipande, Akmal, & Ravenhall 1998). Like the A18 + UIX′ + δv_b equation of state, the UU equation of state gives a maximum mass of about 2.2 $M_\odot$ for a nonrotating neutron star. The maximum mass for a rotating neutron star is 2.61 $M_\odot$, and the maximum rotation frequencies for the normal and supramassive sequences are, respectively, 1561 Hz and 1989 Hz.

In order to illustrate the generic effects of significant softening of a hard equation of state at a critical density, we consider the tensor interaction (TI) equation of state of Pandharipande & Smith (1975a; "M" in the Arnett & Bowers 1977 survey). Although the TI equation of state is itself no longer of interest to nuclear physicists, this equation of state demonstrates the effects of a very strong, first-order phase transition, such as may occur at the transition from nucleon matter to quark matter (see Glendenning 1992; Heiselberg, Pethick, & Staubo 1993; Pandharipande et al. 1998). The maximum mass of a nonrotating star constructed using the TI equation of state is 1.8 $M_\odot$, and the maximum mass of a rotating star is 2.1 $M_\odot$. The maximum rotation frequency for the normal sequence is 707 Hz, and for the supramassive sequence it is 1229 Hz.

Finally, as an example of the relatively stiff equations of state often given by mean field theories, we consider the mean-field equation of state of Pandharipande & Smith (1975b; "L" in the Arnett & Bowers 1977 survey). The maximum mass of a nonrotating star constructed using this equation of state is 2.7 $M_\odot$, compared with 3.27 $M_\odot$ for a rotating star. The maximum rotation frequency is 1031 Hz for the normal sequence and 1321 Hz for the supramassive sequence.

2.2. *First-Order Expressions*

In § 3 it will be instructive to compare the behavior of the orbital frequencies and radii computed using our numerical models of rapidly rotating stars with the behavior given by the analytical expressions valid for slowly rotating stars. As noted in § 1, the spacetime around a rotating fluid star is unique to first order in the dimensionless angular momentum j. To this order in j, the frequency of a prograde orbit at circumferential radius r around a star with gravitational mass M is (see Lightman et al. 1973, p. 469; Miller et al. 1998a)

$$\Omega = [1 - j(M/r)^{3/2}](M/r^3)^{1/2} , \tag{2}$$

and the circumferential radius of the ISCO is

$$R_{\rm ms}(M, j) \approx 6M[1 - j(2/3)^{3/2}] , \tag{3}$$

in units in which $G \equiv c \equiv 1$. In the present work we always quote circumferential radii [defined as the proper circumference in the equatorial plane at some radius, divided by 2π, or equivalently $(g_{\phi\phi})^{1/2}$], in contrast to Miller et al. (1998a), in which we quoted Boyer-Lindquist radii. The two radii are identical to first order in j, but to higher orders in j the circumferential radius is the physically meaningful radius, which is the reason we use it here. Combining equations (2) and (3), one can show that to first order in j, the frequency $\nu_{K,\rm ms}$ of the innermost stable prograde orbit is (see Kluźniak, Michelson, & Wagoner 1990; Miller et al. 1998a)

$$\nu_{K,\rm ms} \approx 2210(1 + 0.75j)(M_\odot/M)\ \text{Hz} . \tag{4}$$

Thus, for slowly rotating stars the frequency of the ISCO *increases linearly* with the star's spin rate.

Using equations (2), (3), and (4), one can show (Miller et al. 1998a) that the mass and radius of a slowly rotating star are bounded above by

$$M_{\rm max} \approx [1 + 0.75j(\nu_{\rm spin})]M^0_{\rm max} \tag{5}$$

and

$$R_{\rm max} \approx [1 + 0.20j(\nu_{\rm spin})]R^0_{\rm max} . \tag{6}$$

Here $j(\nu_{\rm spin})$ is the value of j for the observed stellar spin rate at the maximum allowed mass for the equation of state being considered, and

$$M^0_{\rm max} = 2.2(1.0\ \text{kHz}/\nu^*_{\rm QPO2})\ M_\odot \tag{7}$$

and

$$R^0_{\rm max} = 19.5(1.0\ \text{kHz}/\nu^*_{\rm QPO2})\ \text{km} \tag{8}$$

are the upper bounds on the mass and radius of a nonrotating star in terms of $\nu^*_{\rm QPO2}$, the highest observed frequency of the higher frequency kilohertz QPO. The precise upper bounds on the mass and radius depend on the equation of state and can be determined by searching a grid of neutron star models for the one that gives the maximum allowed mass. Equations (5) and (6) show that the bounds are always greater for a slowly rotating star than for a nonrotating star, regardless of the equation of state assumed.

No expressions similar to equations (2)–(6) are available for rapidly rotating stars.

3. RESULTS AND DISCUSSION

We first show how the radius of the ISCO and the equatorial radius vary with stellar spin rate for stars constructed using the FPS equation of state. The behavior of these radii makes clear why the frequency of the highest frequency SCO around a star of given mass generally first increases as the star is spun up and then decreases. Considering this behavior for stable stars with different masses makes the behavior of the maximum frequency of an SCO for stars of any mass and equation of state understandable.

Next, we present mass-radius relations for stars with a wide range of spin rates, constructed using the FPS and UU equations of state. We then show how to derive limits on the mass and radius of a rapidly rotating star from the frequency of an SCO around it and discuss the constraints on the equation of state of neutron star matter implied by such constraints.

3.1. *Radii and Orbital Frequencies*

Figures 1*a* and 1*b* show how the circumferential radius of the ISCO and the circumferential radius of the stellar equator vary with $\nu_{\rm spin}$, the stellar spin frequency measured at infinity, for a star constructed using the FPS equation of state. These stars have constant baryonic masses equal to those of nonrotating stars with gravitational masses of 1.4 $M_\odot$ and 1.6 $M_\odot$. Because the gravitational mass increases only slightly with increasing spin frequency, the curves for stars of constant gravitational mass are almost identical to these curves (the largest stable equatorial radii are very slightly smaller). As expected, the dimensionless angular momentum j increases linearly with spin rate for slowly rotating stars but more steeply for rapidly rotating stars: for the 1.4 $M_\odot$ model, $j = 0.23$ at $\nu_{\rm spin} = 0.5$ kHz and 0.52 at 1.0 kHz; for the 1.6 $M_\odot$ models, $j = 0.20$ at 0.5 kHz and 0.43 at 1.0 kHz; for the 1.8 $M_\odot$ models, $j = 0.16$ at 0.5 kHz and 0.36 at 1.0 kHz. Figure 1*c* shows how the frequency of the highest frequency SCO varies with the spin rates of 1.4 $M_\odot$, 1.6 $M_\odot$, and 1.8 $M_\odot$ stars.

The circumferential radius $R_{\rm ms}$ of the ISCO decreases linearly with spin rate for slowly rotating stars, in agreement with the first-order expression (eq. [3]), but decreases more slowly as the spin rate increases. For the 1.4 $M_\odot$ star, the deviation from equation (3) is significant at $\nu_{\rm spin} \approx 300$ Hz (see Fig. 1*a*). For the 1.6 $M_\odot$ star, the deviation is significant at 500 Hz and at about 1065 Hz $R_{\rm ms}$ reaches a minimum and then *increases* with increasing spin rate, until an ISCO no longer exists (see Fig. 1*b*). In contrast, the circumferential equatorial radius of the stellar models increases quadratically with the spin rate from $\nu_{\rm spin} = 0$, exceeding $R_{\rm ms}$ at about 580 Hz for the 1.4 $M_\odot$ star and at about 1220 Hz for the 1.6 $M_\odot$ star. For spin rates above these critical rates, all circular orbits with radii larger than the star's equatorial radius are stable: there is no ISCO.

The Kerr spacetime can be expressed analytically and is therefore sometimes used as a convenient approximation to the exterior spacetime of a spinning neutron star. For this reason, in Figures 1*a* and 1*b* we compare $R_{\rm ms}$(Kerr), the circumferential radius of the ISCO in a Kerr spacetime with the same gravitational mass and angular momentum as the stellar models, with the actual radius $R_{\rm ms}$ of the ISCO. Unlike the actual radius, $R_{\rm ms}$(Kerr) decreases monotoni-

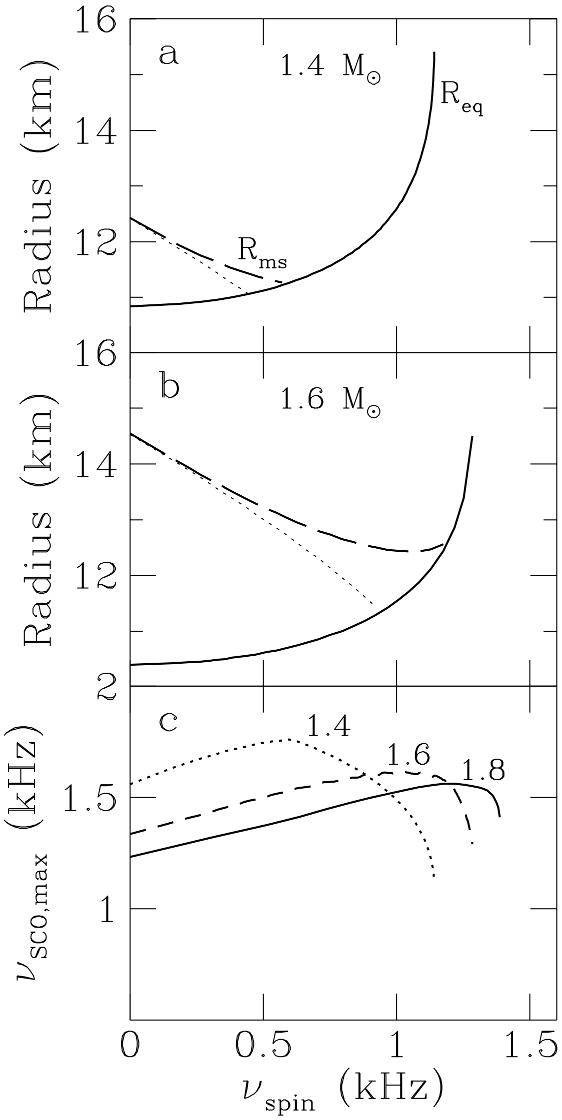

FIG. 1.—Typical variations of important radii and frequencies with stellar spin rate. (*a*) Circumferential radii of the innermost stable circular orbit (*dashed line*) and the stellar equator (*solid line*) as a function of spin rate, for a 1.4 $M_\odot$ star. Also shown for comparison is the circumferential radius of the innermost stable circular orbit in a Kerr spacetime with the same gravitational mass and angular momentum (*dotted line*). (*b*) Same radii as in (*a*), but for a 1.6 $M_\odot$ star. (*c*) Frequency of the highest frequency stable circular orbit as a function of stellar spin rate, for 1.4 $M_\odot$ (*dotted line*), 1.6 $M_\odot$ (*dashed line*), and 1.8 $M_\odot$ (*solid line*) stars. All stellar models were constructed using the FPS equation of state.

cally and nearly linearly with spin rate even at high spin frequencies. Indeed, at high spin rates R_{ms}(Kerr) decreases *faster* than linearly with increasing spin rate, and hence the exact Kerr expression for R_{ms} is a worse approximation than the first-order approximation (eq. [3]).

R_{ms}(Kerr) is significantly smaller than R_{ms} at high spin rates. As a result, when R_{ms} is larger than the equatorial radius of the star, the frequency of the highest frequency SCO is significantly *lower* than one would estimate using the Kerr spacetime, and the constraints on the mass and radius of the star are correspondingly tighter. For both the 1.4 $M_\odot$ and 1.6 $M_\odot$ stars, the critical spin rate at which the ISCO disappears in the Kerr approximation is about 23% smaller than in the actual spacetime. Thus, for stellar spin rates $\gtrsim$400 Hz, the exterior spacetime of a spinning black hole is generally an inaccurate approximation to the exterior spacetime of a neutron star.

Figure 1*c* shows why the constraints on the mass and radius of a slowly rotating star implied by a given SCO frequency are generally looser for a slowly rotating star than for a nonrotating star of the same mass, whereas the constraints on a rapidly rotating star are usually much tighter. For slowly rotating stars with gravitational masses of 1.4, 1.6, and 1.8 $M_\odot$ constructed using the FPS equation of state, the equatorial radius of the star is smaller than the radius of the ISCO. Hence, at low spin rates the highest frequency SCO is the ISCO, which at these spin rates shrinks linearly as the spin rate increases (see eq. [3]), causing the frequency of the highest frequency SCO to *increase* linearly with the star's spin rate. However, at a certain critical spin rate the equatorial radius of the star becomes larger than the radius of the ISCO for a star with the given gravitational mass and angular momentum. For spin rates above this critical spin rate, the highest frequency SCO is the orbit that just skims the stellar surface. At high spin rates, the equatorial radius of the star increases rapidly with increasing spin rate, causing the frequency of the highest frequency SCO to *decrease* rapidly.

For the $M = 1.4\ M_\odot$ star, the frequency of the highest frequency SCO is maximized at the spin frequency for which the radius of the ISCO is equal to the radius of the stellar equator (see Fig. 1*a*). However, the highest frequency SCO for a star of given mass but any possible spin rate is not necessarily the ISCO with radius equal to the equatorial radius of the star. This is illustrated by the $M = 1.6\ M_\odot$ star. For this star, the frequency of the highest frequency SCO has its maximum at the spin frequency at which the radius of the ISCO is a minimum (see Fig. 1*b*). At this frequency the radius of the ISCO is larger than the equatorial radius of the star.

3.2. *Maximum SCO Frequency*

The maximum frequency of the highest frequency SCO for stable stars of any mass can be determined by constructing curves like those shown in Figure 1*c*, for a dense sequence of stellar masses. The curve of maximum frequency as a function of spin rate is then the upper envelope of these curves. Figure 2 shows curves of maximum SCO frequency as a function of stellar spin rate, for stable stars of any mass constructed with the four indicated equations of state. If the measured spin frequency of the star and the frequency of a nearly circular orbit correspond to a point that lies above the curve for a given equation of state, that equation of state is excluded for all neutron stars.

As Figure 2 shows, the maximum frequency of the highest frequency SCO for stars of any mass typically *decreases* with increasing spin rate, even though the frequency of the highest frequency SCO for a fixed gravitational mass *increases* with increasing spin rate over a wide range of spin rates. The reason is that, at low spin rates, the mass that gives the highest frequency SCO is the mass at which the ISCO coincides with the stellar equator. This mass increases with increasing spin rate, causing the maximum frequency to decrease.

Stars constructed with the TI equation of state M are an exception to this rule. This equation of state is extremely stiff at the low densities characteristic of the centers of lower

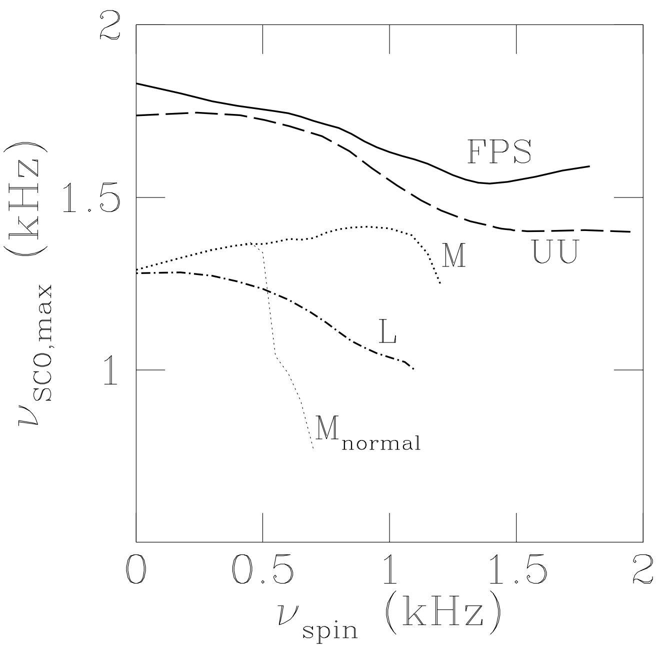

FIG. 2.—Maximum frequency of a stable circular orbit as a function of spin rate for stable stars of any mass, for the four indicated equations of state, which are discussed in the text. The curve labeled "M_{normal}" includes only normal sequences, whereas the other curves include both normal and supramassive sequences (see § 3.1 for a discussion).

mass stars, but becomes very soft at the density reached at the center of a nonrotating $M \approx 1.75\ M_{\odot}$ star, because a pion condensate forms at this density. For this equation of state, the maximum orbital frequency occurs for a stellar mass near the maximum mass allowed by the star's spin frequency (e.g., 1.8 $M_{\odot}$ for a slowly rotating star).

Consider first the normal sequences. At low spin frequencies, the surface of the 1.8 $M_{\odot}$ star is well inside the ISCO, the highest frequency SCO is therefore the ISCO, and the frequency of the maximum-frequency SCO therefore *increases* with increasing spin rate. However, at about 500 Hz, the maximum frequency stops increasing and then plummets, as shown by the curve labeled "M_{normal}" in Figure 2. The reason is a general relativistic effect first pointed out by Cook et al. (1994b): as the angular momentum of a star increases, the spin frequency first increases, then decreases, and finally increases again, producing a local maximum in the spin frequency versus angular momentum relation. For normal sequences using equation of state M, this local maximum occurs at a spin frequency slightly greater than 500 Hz. Hence, in order to have an observed spin frequency higher than this, the star must have a much higher angular momentum, but a star with this much angular momentum has a much larger equatorial radius, larger than the ISCO for a star of its mass and angular momentum, so the highest frequency SCO is at the stellar surface and has a smaller frequency.

Consider now the supramassive sequences. For a fixed baryonic mass, there is a local maximum in the spin frequency versus angular momentum relation, just as for the normal sequences. A curve of $\nu_{SCO,max}$ versus ν_{spin} constructed for a supramassive sequence with a fixed baryonic mass would therefore look similar to the "M_{normal}" curve in Figure 2, except that it would start at a positive spin frequency and would plummet at a higher spin frequency. The local maximum of the spin frequency increases with increasing baryonic mass. The envelope of these curves produces the curve labeled "M." The rapid downturn of this curve at a spin frequency of approximately 1100 Hz occurs because above this frequency there is no ISCO, and consequently the highest frequency orbit is the one that skims the surface. For $\nu_{spin} > 1100$ Hz, the equatorial radius increases rapidly with increasing spin frequency, and therefore the maximum frequency of a circular orbit decreases rapidly.

Comparison of the two curves in Figure 2 for equation of state M demonstrates the potential importance of the supramassive sequences. For example, if only normal sequences are physically accessible (e.g., if the gas accretes with low specific angular momentum), then observation of a 1100 Hz SCO from a star with a spin frequency in excess of 550 Hz would rule out equation of state M, whereas equation of state M would still be viable if supramassive sequences are accessible.

3.3. *Mass and Radius Bounds*

As explained in § 1, observation of a given SCO frequency around a nonrotating star allows one to derive upper bounds on the mass and radius that are independent of the equation of state, whereas for a rotating star one must consider a specific equation of state in order to derive bounds on the mass and radius.

Given an equation of state and a stellar spin rate, the mass of the star must be between the mass-shedding limit and the radial instability limit, regardless of the values of any orbital frequencies. The radius of the star is bounded by the extreme values of the radii given by the mass-radius relation over this mass interval. Observation of an SCO with a certain frequency may allow one to restrict further the allowed mass and radius intervals, depending on the equation of state and the frequency of the SCO. The possible further restrictions are of two types: a lower bound on the mass, imposed by the requirement that the radius of the orbit be greater than the radius of the star, and an upper bound on the mass, imposed by the requirement that the radius of the orbit be equal to or greater than the radius of the ISCO. If either of these bounds restrict further the allowed mass range, the radius of the star is bounded by the extreme values of the radii given by the mass-radius relation over this reduced mass interval.

Figure 3 shows the constraints on neutron star masses and radii imposed by stellar stability and observation of a stable circular orbit, for stars constructed using the FPS and UU equations of state (see § 2.2). The mass-radius relations (curves of gravitational mass vs. equatorial circumferential radius) shown in this figure were constructed by generating several sequences of stellar models. Each sequence consisted of stellar models with the same baryon number but a range of spin frequencies. The grid of models constructed in this way was then used to generate the mass-radius relations shown in Figure 3. These relations are tabulated in Tables 1 and 2, where for each spin frequency the boundary between normal and supramassive stars is indicated by the dashed lines. The relations shown for rapidly spinning stars are much flatter than the usual mass-radius relations for nonrotating stars.

The high-mass end of each constant spin-frequency sequence shown in Figure 3 is the gravitational mass above which the star is unstable to a radial instability. This mass

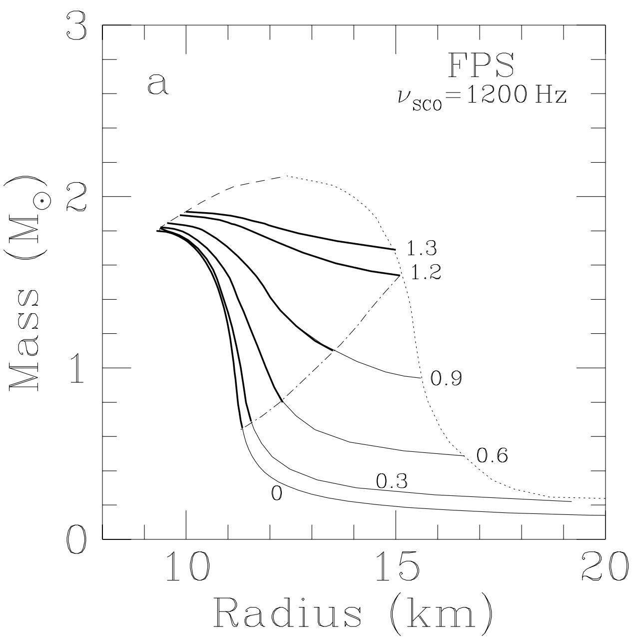

FIG. 3*a*

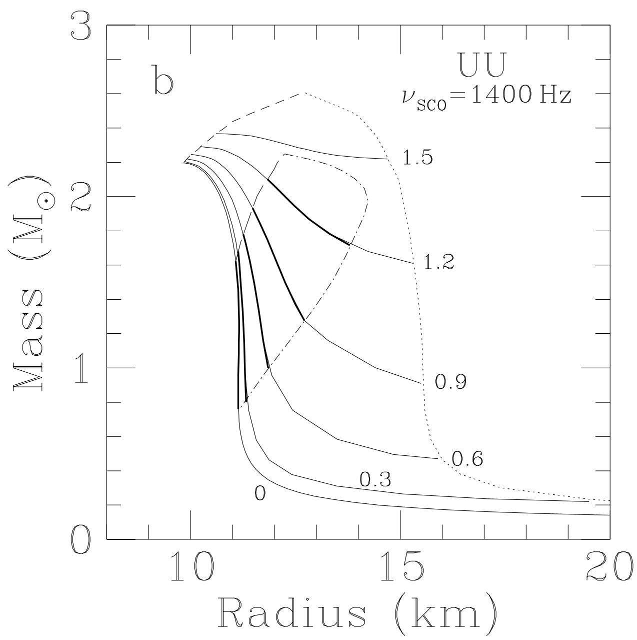

FIG. 3*b*

FIG. 3.—Constraints on neutron star masses and radii imposed by stellar stability and observation of a stable circular orbit. (*a*) Solid lines show the mass-radius relations for FPS stars with the spin frequencies indicated (in kHz). The high-mass end of each mass-radius relation shown is the radial instability limit (*dashed line*) whereas the low-mass end is the mass-shedding limit (*dotted line*). The dot-dashed line shows the lowest stellar mass (largest stellar radius) consistent with the requirement that the radius of the star be smaller than the radius of a 1200 Hz orbit. For FPS stars and this orbital frequency, the requirement that the radius of the orbit also exceed the radius of the ISCO does not constrain the mass or radius of the star. The bold portion of each mass-radius curve highlights the region allowed by both the physical limits of the equation of state and by the observation of an SCO with the indicated frequency. (*b*) Same as in (*a*) but for UU stars and an orbital frequency of 1400 Hz. Although the dot-dashed line curves strongly to the left at high stellar spin rates, it always shows the smallest stellar mass (largest stellar radius) consistent with the requirement that the radius of the star be smaller than the radius of a 1400 Hz orbit. The line of long dashes shows the largest stellar mass (smallest stellar radius) consistent with the requirement that the radius of the orbit exceed the radius of the ISCO.

TABLE 1

MASS-RADIUS RELATIONS FOR THE FPS EQUATION OF STATE AND DIFFERENT SPIN RATES

0 Hz		300 Hz		600 Hz		900 Hz		1200 Hz		1300 Hz	
M ($M_\odot$)	*R* (km s^{-1})	*M* ($M_\odot$)	*R* (km s^{-1})	*M* ($M_\odot$)	*R* (km s^{-1})	*M* ($M_\odot$)	*R* (km s^{-1})	*M* ($M_\odot$)	*R* (km s^{-1})	*M* ($M_\odot$)	*R* (km s^{-1})
0.096	50.79	0.22	19.2	0.486	16.64	0.94	15.6	1.54	15.1	1.69	15.0
0.117	26.38	0.26	15.3	0.51	15.2	0.95	15.2	1.56	14.51	1.74	13.4
0.142	19.54	0.30	13.9	0.56	13.7	0.97	14.8	1.60	13.53	1.79	12.6
0.170	16.30	0.34	13.1	0.64	13.0	1.02	14.2	1.68	12.70	1.82	12.15
0.203	14.50	0.41	12.4	0.72	12.6	1.14	13.1	1.73	12.2	1.835	11.94
0.242	13.39	0.48	12.0	0.80	12.3	1.25	12.6	1.78	11.74	1.842	11.87
0.286	12.68	0.56	11.8	0.87	12.1	1.34	12.2	1.81	11.45	– – –	– – –
0.336	12.20	0.64	11.6	1.01	11.9	1.41	12.0	1.825	11.31	1.848	11.80
0.392	11.87	0.72	11.5	1.13	11.7	1.48	11.85	1.832	11.23	1.851	11.67
0.456	11.65	0.80	11.4	1.24	11.5	1.54	11.67	– – –	– – –	1.857	11.64
0.527	11.49	0.87	11.4	1.33	11.4	1.58	11.52	1.837	11.16	1.873	11.36
0.607	11.38	1.01	11.3	1.41	11.2	1.66	11.2	1.841	11.07	1.889	11.09
0.693	11.30	1.13	11.2	1.47	11.14	1.71	10.99	1.847	11.05	1.902	10.73
0.882	11.20	1.24	11.1	1.53	11.04	1.76	10.7	1.864	10.81	1.912	9.995
1.078	11.11	1.33	11.0	1.57	10.92	1.79	10.49	1.879	10.52	…	…
1.264	10.98	1.40	10.9	1.65	10.7	1.81	10.36	1.892	9.851	…	…
1.427	10.81	1.47	10.8	1.70	10.51	1.815	10.30	…	…	…	…
1.559	10.60	1.52	10.74	1.75	10.25	– – –	– – –	…	…	…	…
1.661	10.34	1.64	10.5	1.78	10.05	1.82	10.21	…	…	…	…
1.734	10.05	1.75	10.04	1.805	9.82	1.825	10.15	…	…	…	…
1.778	9.746	1.79	9.65	– – –	– – –	1.829	10.09	…	…	…	…
1.790	9.594	1.80	9.54	1.811	9.71	1.846	9.547	…	…	…	…
1.797	9.444	– – –	– – –	1.815	9.61	…	…	…	…	…	…
1.799	9.295	1.814	9.39	1.821	9.39	…	…	…	…	…	…

NOTE.—The dashed lines indicate the boundary between normal and supramassive stars for each spin frequency.

TABLE 2

Mass-Radius Relations for the UU Equation of State and Different Spin Rates

0 Hz		300 Hz		600 Hz		900 Hz		1200 Hz		1500 Hz	
M ($M_\odot$)	R (km s^{-1})	M ($M_\odot$)	R (km s^{-1})	M ($M_\odot$)	R (km s^{-1})	M ($M_\odot$)	R (km s^{-1})	M ($M_\odot$)	R (km s^{-1})	M ($M_\odot$)	R (km s^{-1})
0.135	21.17	0.220	19.50	0.470	15.90	0.910	15.50	1.610	15.30	2.220	14.70
0.151	18.24	0.231	16.83	0.480	15.08	0.965	14.54	1.664	14.16	2.225	14.01
0.172	16.06	0.265	15.07	0.553	13.32	1.167	13.02	1.777	13.32	2.242	13.40
0.199	14.49	0.299	13.31	0.751	12.27	1.347	12.53	1.873	12.77	2.256	13.10
0.232	13.37	0.377	12.27	0.959	11.88	1.507	12.24	1.955	12.43	2.265	12.89
0.273	12.58	0.462	11.82	1.159	11.72	1.641	12.01	2.021	12.18	2.274	12.81
0.320	12.05	0.552	11.56	1.339	11.63	1.754	11.83	2.073	11.96	2.278	12.69
0.376	11.69	0.749	11.34	1.498	11.53	1.851	11.65	2.115	11.77	2.282	12.64
0.440	11.45	0.956	11.30	1.631	11.41	1.933	11.48	2.148	11.61	2.284	12.60
0.511	11.30	1.156	11.28	1.743	11.30	1.999	11.34	2.174	11.49	– – –	– – –
0.593	11.21	1.335	11.25	1.840	11.19	2.051	11.20	2.194	11.38	2.286	12.56
0.686	11.16	1.493	11.20	1.921	11.09	2.093	11.10	2.209	11.29	2.299	12.36
0.789	11.14	1.626	11.16	1.986	11.00	2.126	10.98	2.220	11.22	2.312	12.17
1.022	11.15	1.738	11.09	2.038	10.87	2.152	10.89	2.228	11.18	2.324	12.02
1.271	11.16	1.834	11.00	2.080	10.77	2.172	10.79	2.233	11.13	2.334	11.80
1.513	11.12	1.915	10.90	2.113	10.67	2.187	10.72	2.240	11.08	2.345	11.66
1.726	11.02	2.032	10.71	2.159	10.49	2.198	10.66	– – –	– – –	2.354	11.45
1.907	10.85	2.106	10.52	2.185	10.36	2.206	10.61	2.256	10.95	2.362	11.24
2.041	10.63	2.152	10.34	2.199	10.26	2.212	10.57	2.269	10.82	2.366	10.80
2.129	10.38	2.179	10.19	2.206	10.19	2.219	10.52	2.275	10.75	2.367	10.61
2.177	10.13	2.190	10.06	– – –	– – –	– – –	– – –	2.281	10.67	…	…
2.189	10.01	2.199	9.96	2.207	10.16	2.235	10.39	2.288	10.38	…	…
2.195	9.898	– – –	– – –	2.214	10.04	2.247	10.01	2.291	10.23	…	…
2.196	9.814	2.205	9.86	2.219	9.92	…	…	…	…	…	…..

Note.—The dashed lines indicate the boundary between normal and supramassive stars for each spin frequency.

limit is indicated by the short-dashed lines in Figures 3*a* and 3*b*. The maximum gravitational mass increases with increasing spin rate, both because the gravitational mass corresponding to a given baryonic mass increases with increasing spin and because the maximum stable baryonic mass increases with increasing spin. The low-mass end of each mass-radius relation shown is the gravitational mass below which the star is subject to mass-shedding at the equator. This mass limit is indicated by the dotted lines in Figures 3*a* and 3*b*.

Consider now the possible further restrictions on the allowed mass and radius intervals imposed by observation of an SCO with a high frequency. The requirement that the radius of an orbit be larger than the equatorial radius of the star places a lower bound on the mass of the star given the frequency of an SCO. If, for that star's spin rate, there are stable stars with masses smaller than this lower bound, then the observation of an SCO raises the lower bound on the mass of a star. Whether observation of an SCO with a given frequency raises the lower bound on the stellar mass depends on the equation of state and the star's spin rate as well as the frequency of the SCO. For example, for the FPS equation of state and a spin rate of 600 Hz, observation of a 1200 Hz SCO imposes a lower bound on the mass of 0.8 $M_\odot$, whereas the mass-shedding limit is 0.56 $M_\odot$ (see Fig. 3*a*). Hence, in this case the limit imposed by the SCO is stricter. On the other hand, if $\nu_{\rm spin} > 1200$ Hz, there is always an SCO with a frequency of at least 1200 Hz around any FPS star that is stable against mass-shedding and hence the SCO observation does not further restrict the allowed mass interval. In contrast, the lower mass limit imposed by observation of an SCO with a frequency of 1400 Hz is always stricter than the mass shedding limit for a UU star, regardless of its spin rate.

Observation of an SCO lowers the upper bound on the mass of a star if, for that star's spin rate, there are stable stars with ISCOs with radii larger than that permitted by the requirement that the radius of the orbit be larger than the radius of the ISCO. Again, whether observation of an SCO with a given frequency lowers the upper bound on the stellar mass depends on the equation of state and the star's spin rate as well as the frequency of the SCO. For example, for the FPS equation of state, observation of a 1200 Hz SCO frequency would not lower the upper bound on the stellar mass imposed by the radial instability limit, regardless of the star's spin rate. In contrast, for the UU equation of state observation of a 1400 Hz SCO frequency would lower the upper bound on the stellar mass imposed by the radial instability limit, regardless of the star's spin rate.

Figure 3 shows clearly that observation of an SCO with a frequency $\gtrsim 1200$ Hz usually reduces greatly the area of the radius-mass plane allowed for stars constructed using a given equation of state. For example, the area allowed for the UU equation of state if a 1400 Hz SCO is observed is only a small fraction of the area allowed by the requirement of stellar stability (see Fig. 3*b*).

If the spin frequency of a star is known, the upper and lower bounds on its mass and radius imposed by observation of an SCO of a given frequency can be read off Figure 3 by looking for the intersections of the relevant bounding curves with the mass-radius curve for that spin frequency. For example, if the spin frequency is 600 Hz and the SCO frequency is 1200 Hz, then a neutron star with the FPS equation of state must have a mass between 0.80 and 1.82 $M_\odot$ and a radius between 9.39 and 12.26 km.

Even if the spin frequency of a neutron star is unknown, one can still extract upper and lower bounds on the mass and radius of the star for a given equation of state, using the highest observed frequency of an SCO from the source and the appropriate figure, such as Figure 3*a* or 3*b*. For example, if a 1200 Hz SCO is observed, then a neutron star with the FPS equation of state must have a mass between 0.64 and 2.12 $M_\odot$ and a radius between 9.28 and 15.1 km.

Figures 3*a* and 3*b* also show that, for a given equation of state, knowledge of any two of the mass, radius, and spin frequency fixes the value of the third quantity, which may or may not be consistent with the kilohertz QPO frequency. Thus, if the spin frequency of a neutron star in a low-mass X-ray binary is known and the radius or mass can be determined by means other than observation of a kilohertz QPO (e.g., by measuring the emitting area during a thermonuclear X-ray burst or by measuring the mass dynamically), then observation of a kilohertz QPO will overdetermine the properties of the star, providing a check on the consistency of the mass and radius estimates.

4. CONCLUSIONS

Our results show that deviations from a first-order treatment of the effects of spin on the structure of neutron stars and on circular orbits around them are typically significant for spin frequencies $\gtrsim 400$ Hz. The Kerr spacetime is generally a poor approximation to the exterior spacetime of neutron stars spinning this fast or faster.

Our results demonstrate that the upper bounds on the stiffness of neutron star matter implied by the high frequencies and coherences of the kilohertz QPOs are tightened significantly if the star is rotating rapidly. The constraints on the equation of state become much tighter if observations conclusively identify a QPO frequency as the orbital frequency at the ISCO. For example, if the orbital frequency at the ISCO is 1100 Hz, the required mass is 2.0 $M_\odot$ for nonrotating stars and substantially more for rotating stars. Such a high mass would rule out many of the softer equations of state and would imply that the three-nucleon interaction is strongly repulsive at high densities (see Pandharipande et al. 1998).

As shown in § 3.3, even if the spin rate of the star is unknown, the area of the mass-radius plane allowed by observation of a high-frequency SCO can be quite small. If the spin rate is known, the range of radii allowed for a given equation of state is usually very small.

Results of the kind presented in this work will be still more constraining if measurements of SCO frequencies can be combined with constraints on other quantities, such as the stellar compactness M/R (see Strohmayer 1997; Miller & Lamb 1998; Lamb et al. 1998; Strohmayer et al. 1998) or the radius of the star (see Strohmayer et al. 1997b; Strohmayer et al. 1998).

Additional observations of kilohertz QPO sources are extremely important, because these observations could provide very tight and robust constraints on the fundamental properties of neutron stars and on the equation of state of neutron star matter.

This work was supported in part by NSF grant AST 96-18524, NASA grant NAG 5-2925, and NASA *RXTE* grants at the University of Illinois, NASA grant NAG 5-2868 at the University of Chicago, and through the *Compton Gamma-Ray Observatory* Fellowship Program, by NASA grant NAS 5-2687.

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