[07] Conservation Laws and Gravity (9/19/17)

Upcoming Items

1. Homework #3 due now.
2. Homework #4 due in one week (start early!).
3. Read Ch. 4.5 by next class and do the self-study quizzes

Special Announcement

…Considering adding the Astronomy major and want to receive Astronomy-related emails (such as for the undergraduate research showcase next semester)?

Please email Dr. Melissa Hayes-Gehrke at mhayesge@umd.edu

Special day today!

- Anyone know which group has their international day today?

Special day today!

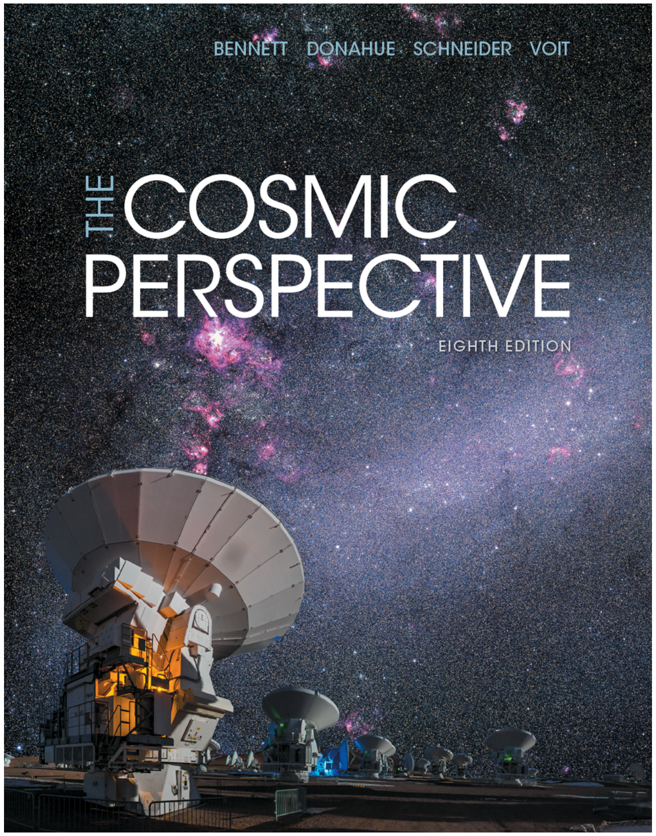
It’s International Talk Like a Pirate Day, savvy?

LEARNING GOALS

Chapters 4.3–4.4

For this class, you should be able to…

- *… use conservation of momentum, angular momentum, and/or energy to solve certain types of astrophysical problems;*
- *… Understand the basics of Newtonian gravity*



Any astro questions?

In-class quiz!

A gas cloud contracts under its own gravity, but the total energy (of all types) of the cloud remains constant. Which of the following statements is **true**?

A. The cloud’s gravitational potential energy becomes more negative and its thermal energy decreases.
B. The cloud’s gravitational potential energy becomes more positive and its temperature increases.
C. The cloud’s thermal energy increases and the kinetic energy of its gas particles decreases.
D. The cloud’s temperature increases and its gravitational potential energy becomes more negative.

Consider an object orbiting a star in an ellipse. Which of the following **must** violate conservation of angular momentum?

A. The object comes closer to the star but speeds up while maintaining a constant mass.
B. The object loses some mass and moves farther from the star, but remains at the same speed.
C. The object increases in mass, but slows down and comes closer to the star.
D. The object moves farther from the star and speeds up, staying at a constant mass.
E. The object loses mass, comes closer to the star, but speeds up.

Conservation of Momentum

- Newton's laws can be recast as a statement of momentum conservation
- This means the total momentum of a system of bodies is unchanged unless there is a net external force:

Momentum after = Momentum before

Proof for two-body system

- Suppose we have only two particles in the system, A and B
  Force of A on B is $\mathbf{F}_{AB}$; of B on A is $\mathbf{F}_{BA}$
- What is the rate of change of momentum of A?
  $d\mathbf{p}_A/dt=\mathbf{F}_{BA}$; similarly, $d\mathbf{p}_B/dt=\mathbf{F}_{AB}$
- What is the rate of change of momentum of the *system*?
  $d(\mathbf{p}_A+\mathbf{p}_B)/dt=d\mathbf{p}_A/dt+d\mathbf{p}_B/dt=\mathbf{F}_{BA}+\mathbf{F}_{AB}$
- But from Newton's Third Law,
  $\mathbf{F}_{BA} = -\mathbf{F}_{AB}$
- Therefore, $d(\mathbf{p}_A+\mathbf{p}_B)/dt=-\mathbf{F}_{AB}+\mathbf{F}_{AB}=0$
- Without a net *external* force, linear momentum is conserved!

Challenge Problem 1

- These are completely voluntary, but if you want to pursue these topics in greater depth you might want to try them
- In the proof we just did we assumed that we have only two particles. Can you generalize the proof to an arbitrary number of particles?

Using Momentum Conservation

- Suppose that an astronaut is drifting away from her spaceship, without a tether. Luckily, she is holding a wrench, and throws it [what direction?] to push her back. Suppose the wrench has a mass of 1 kg and the astronaut (with all her equipment) has a mass of 100 kg. What is the ratio of her speed to the wrench's speed after she throws it?

$$m_{\text{astronaut}} v_{\text{astronaut}} + m_{\text{wrench}} v_{\text{wrench}} = 0$$

$$\frac{v_{\text{astronaut}}}{v_{\text{wrench}}} = -\frac{m_{\text{wrench}}}{m_{\text{astronaut}}}$$

$$= \underline{1\%} \text{ (in opposite direction).}$$

Group Exercise

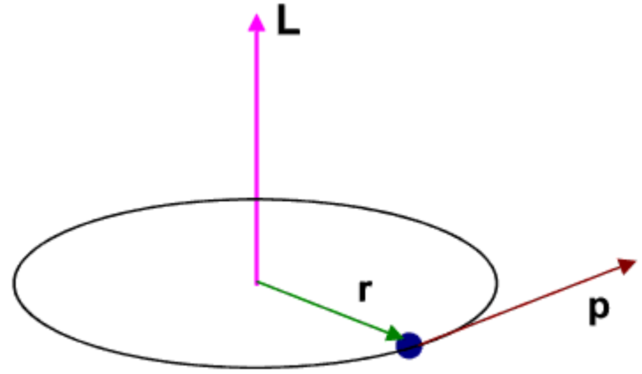
A 200 kg spacecraft fires its rockets for 20 seconds, ejecting propellant in a direction opposite the spacecraft's motion at a rate of 0.001 kg per second at 100 m/s.

1. What is the change in speed of the spacecraft?
2. What is the force (thrust) on the spacecraft?

(Assume the mass change of the spacecraft is negligible.)

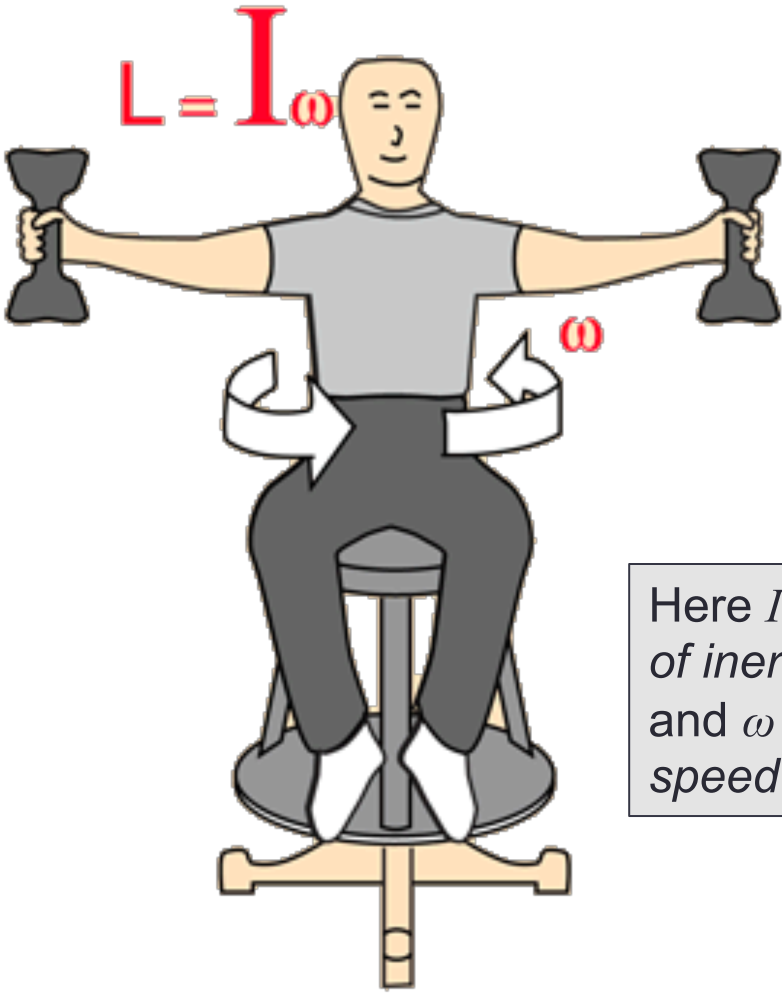
Conservation of Angular Momentum

- Angular momentum $L = mv_{\perp}r$, where m is the object mass.
  - Here $v_{\perp}$ is the component of v perpendicular to r.
  - In vector notation, $\mathbf{L} = \mathbf{r} \times m\mathbf{v} = \mathbf{r} \times \mathbf{p}$.

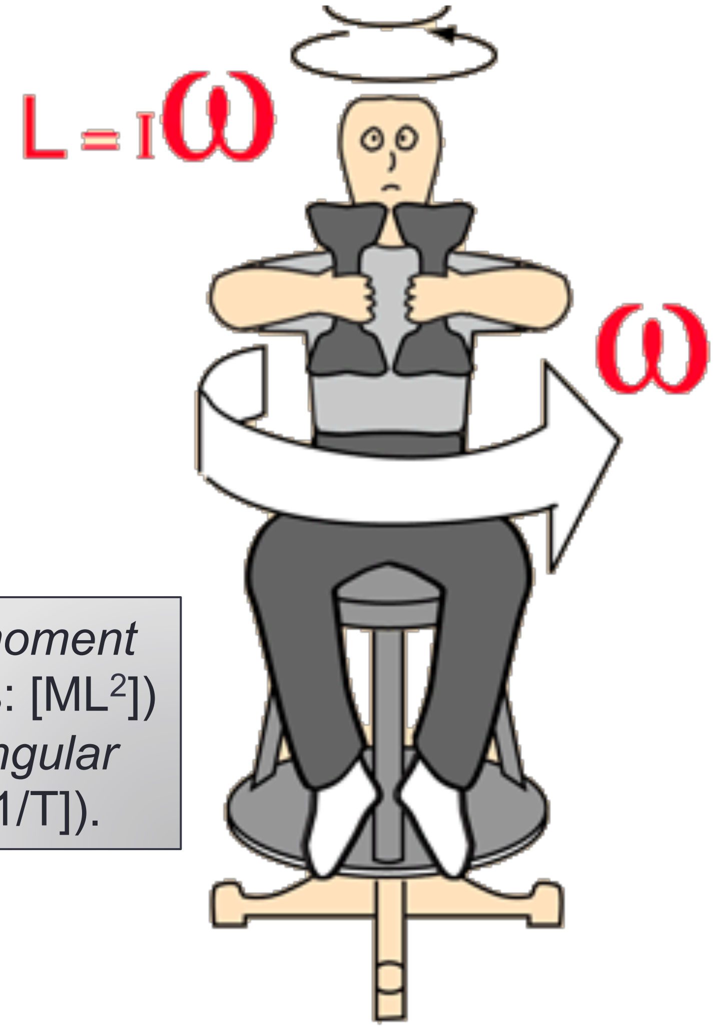


- The angular momentum of an object cannot change unless a net external twisting force (torque) is acting on it.
- Earth experiences no significant torque as it orbits the Sun, so its orbit will continue indefinitely (well, until our planet is swallowed by the Sun $4\ \mathrm{Gyr}$ from now…).

- Angular momentum conservation also explains why objects rotate faster as they shrink in radius:



Here I is the *moment of inertia* (units: $[ML^2]$) and ω is the *angular speed* (units: [1/T]).



Challenge Problem 2

- Can you do a proof, similar to our proof of linear momentum conservation, to show that a system with no net torques on it has an unchanging total angular momentum? For this you also need to know that the force between two points is along the line between them, $\mathbf{F}_{AB} \sim (\mathbf{r}_A - \mathbf{r}_B)$
- You can look up the answer, of course, but you'll learn more if you do these problems yourself. Good luck!

Why do cats always land on their feet?

Group Exercise

A particular asteroid has an semimajor axis of $2.5\ \mathrm{AU}$ and an eccentricity of 0.5. How many times faster is the asteroid moving at perihelion compared to at aphelion?

Hint: At perihelion and aphelion, $v_\perp$ is the orbital speed.

Energy Conservation

- Energy is a slipperier concept than linear momentum or angular momentum
- Why? Because energy comes in different forms, and can change forms
  Example: if you let a ball drop, its energy transitions from gravitational potential energy to kinetic energy
- What if you then let it bounce a bunch of times? It eventually comes to rest on the ground, so it has no kinetic energy and no potential energy
- What form does the energy take, then?

Energy Forms and Equations

Form	Equation	Notes
Kinetic	$\mathbf{KE = mv^2/2 = p^2/(2m)}$	Work = force × distance.
Thermal	$E_{th} = 3Nk_BT/2$	N particles, ideal gas.
Gravitational Potential near surface of planet	$GPE = mgh$	h << radius of planet.
Gravitational Potential general form	$\mathbf{GPE = -GMm/r}$	Gradient gives gravity—see next lecture.
Radiative	$E = hf$	Chapter 5.
Mass Equivalence	$E = mc^2$	Einstein's formula.

Group Exercise

You drop a ball on Mars from a height h onto the ground.

1. If the ball loses half its total energy to sound and heat when it bounces, what is its rebound height?
2. What if instead the ball rebounds with 90% of its impact speed?

(Assume h is small, and ignore the finite size of the ball.)


Gravity Fail

Newton's Law of Gravity

- You have a mass m_A and a mass m_B
- The radial vector from A to B is $\mathbf{r}_{AB}$
- Then the gravitational force on B from A is
  $\mathbf{F}_{grav} = -[(Gm_Am_B)/r^2](\mathbf{r}_{AB}/r)$
  where r is the magnitude of $\mathbf{r}_{AB}$
- This is universal and always attractive
  Called the "inverse square law"
- Notice that because $\mathbf{F}_{grav}=m_B\mathbf{a}_B$, $\mathbf{a}_B$ is independent of the mass m_B.
- This is amazing! It means that all objects fall the same way in the same gravitational field
  Let's think about this some more...

The Weak Equivalence Principle

- Consider object (mass m) accelerating in the gravitational field of the Earth.
  - Newton's 2nd law says…

$$F = m_{\mathrm{I}} a.$$

m_I is the "inertial mass" of the object

  - Newton's law of gravitation says…

$$F = \frac{GMm_{\mathrm{G}}}{r^2}.$$

m_G is the "gravitational mass" of the object

  - Acceleration is then…

$$a = \left(\frac{m_{\mathrm{G}}}{m_{\mathrm{I}}}\right)\frac{GM}{r^2}.$$

- When all other forces are eliminated, we find that the acceleration of an object in a gravitational field does not depend on ANY property of the body (Galileo).
- Means that $m_I = m_G$
- Has been verified to 1 part in 10^{13}.
- Known as the ***Weak Equivalence Principle***.

The Acceleration of Gravity (g)

- Galileo showed that g is the *same* for all falling objects, regardless of their mass.

Apollo 15 demonstration.