Thursday September 21, 2017

Upcoming Items

1. Homework #4 due next class
2. Finish Ch. 4 and do the self-study exercises
3. Midterm #1 will be Tue Oct 10

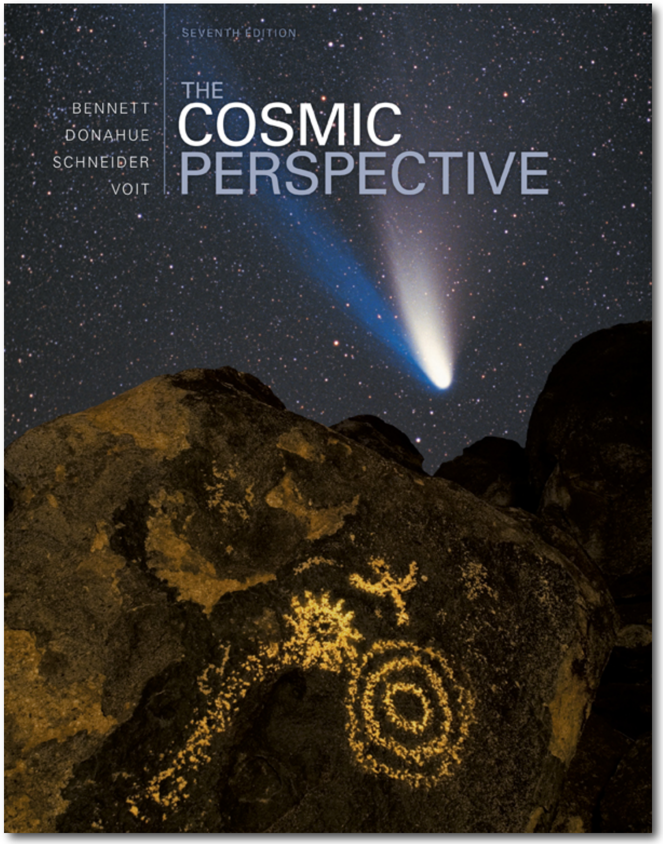
http://www.astro.umd.edu/~tamarab/Site/Research/97187CB4-2B6A-40B8-9940-9EE36CABC885_files/tidal_disruption.jpg

GRAVITY AND ORBITS

Chapter 4.4

By the next lecture, you should be able to…

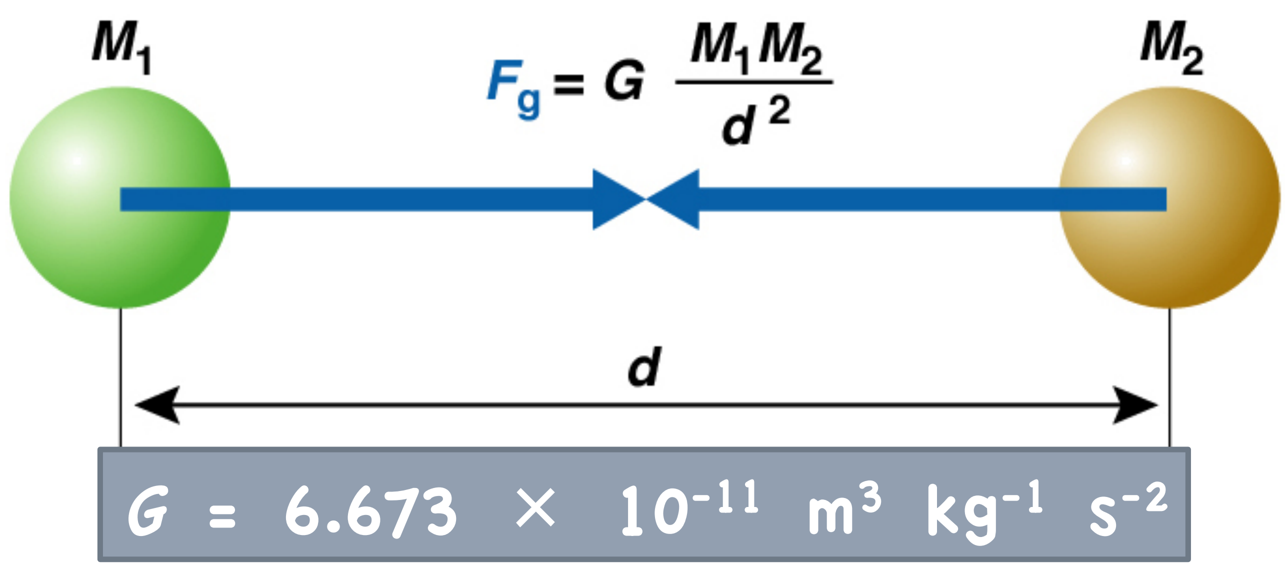
- *determine when it is valid to approximate the two-body gravity problem using (1) point masses (Gauss' law), (2) Kepler's laws as a simplified form of Newton's laws, and/or (3) motion around one body instead of the center of mass, justify your decision to use these approximations, and calculate relevant quantities (orbital period, semimajor axis, etc.);*



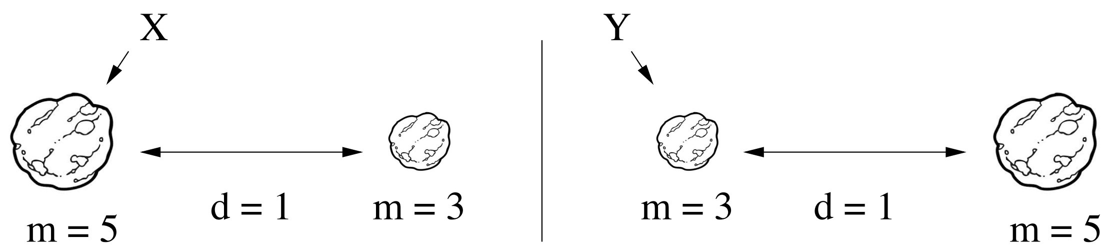
Any astro questions?

The Universal Law of Gravitation

- Every mass (particle) attracts every other mass (particle) in the universe.
- The force of attraction is directly proportional to product of masses (by symmetry & Newton's 3rd law).
- Attraction is inversely proportional to square of distance between centers.



Concept Quiz 1



Which of the following correctly describes how the gravitational force exerted **by** asteroid X on its "partner" asteroid compares to the gravitational force exerted **by** asteroid Y on its "partner"?

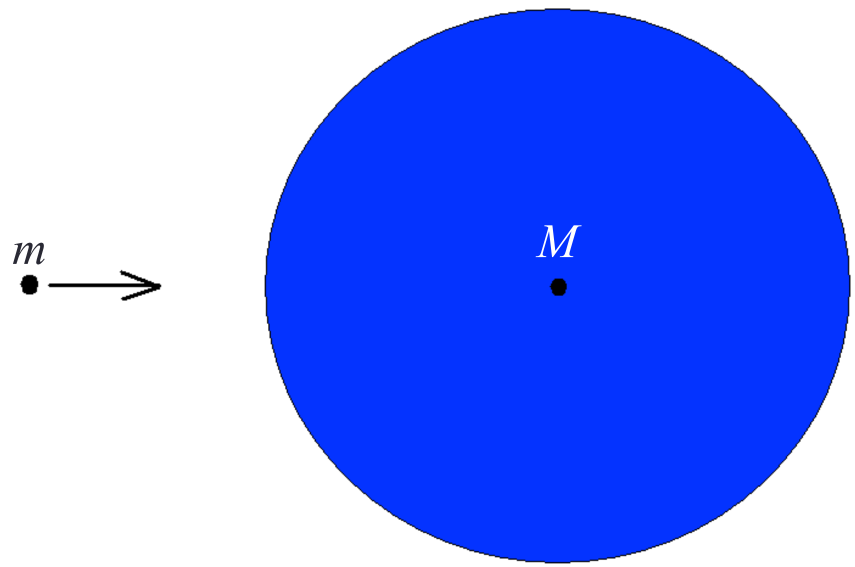
A. The force of X on its partner is greater than the force of Y on its partner.

B. The force of Y on its partner is greater than the force of X on its partner.

C. **The force of X on its partner is equal to the force of Y on its partner.**

Gauss' Law for Gravity

- Using integral calculus (or, if you're Newton, using geometry!), it can be shown that a spherical object with mass M acts gravitationally on external objects like a *particle* of mass M at the sphere's center.



$$F = \frac{GMm}{r^2}$$

Crucial point: we can treat planets as point masses!

Weight

- From Gauss' law, it follows that the gravity force you feel on the surface of a planet of mass M and radius R is

$$W = \frac{GMm}{R^2} = mg,$$

where

$$g \equiv \frac{GM}{R^2}.$$

- On Earth, $M = M_E \sim 6.0\times10^{24}$ kg and $R = R_E = 6{,}400$ km, so $g_E = \underline{9.8 \text{ m/s}^2}$. (Often see ⊕ symbol to denote Earth.)

- Kepler's third law: Let's apply Newton's second law to a planet in a circular orbit with period *P* about the Sun…

$$F = ma \Rightarrow \frac{GM_* m}{r^2} = \frac{mv^2}{r}$$

$$\Rightarrow \frac{GM_*}{r^2} = \left(\frac{2\pi r}{P}\right)^2 \frac{1}{r}$$

$$\Rightarrow P^2 = \frac{4\pi^2}{GM_*} r^3$$

Centripetal force

Remember v = circumference/period

  - In fact, same formula also holds for elliptical orbits (proof is longer).
  - Can see that constant of proportionality depends on mass of the central object. If P is in years and r is in AU, get $P^2 = r^3$ (try it!).

Newton's Form of Kepler's 3rd Law

A more general form for *any* two objects in mutual orbit:

$$P^2 = \frac{4\pi^2}{G(m_1 + m_2)} a^3$$

P = orbital period (for example, in seconds).

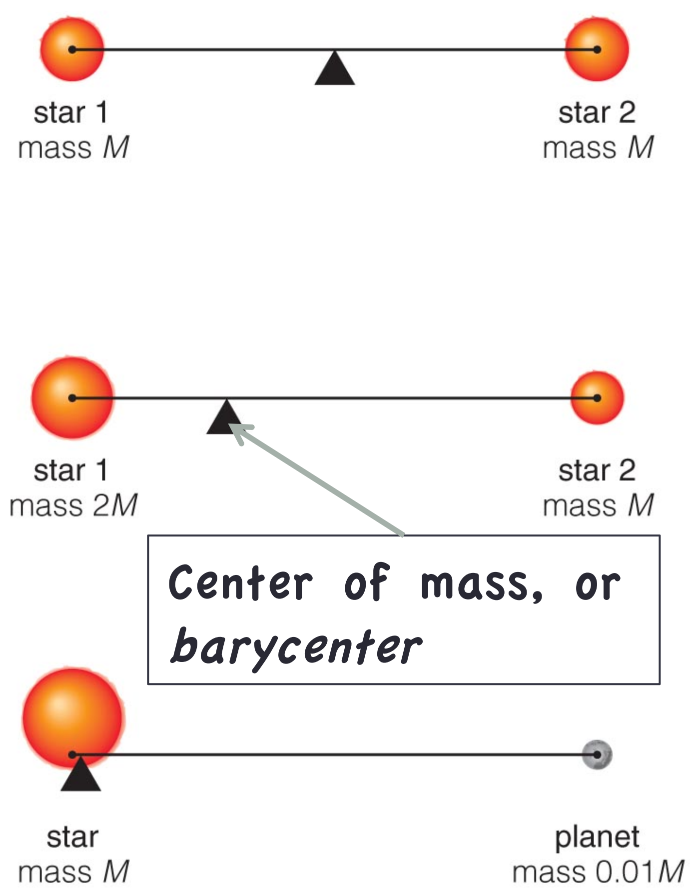
a = semimajor axis (e.g., in meters).

$(m_1 + m_2)$ = sum of object masses (e.g., in kilograms).

Newton's Form of Kepler's 3rd Law

- *If* a small object orbits a larger one ($m_1 >> m_2$), *and* you measure the orbital period (P) *and* the average separation (a), *then* you can calculate the mass of the larger object.
- Examples:
  - Can calculate Sun's mass from Earth's orbital period (1 yr) and average distance (1 AU).
  - Can calculate Earth's mass from orbital period and distance of a satellite.
  - Can calculate Jupiter's mass from orbital period and distance of one of its moons.

Center of Mass

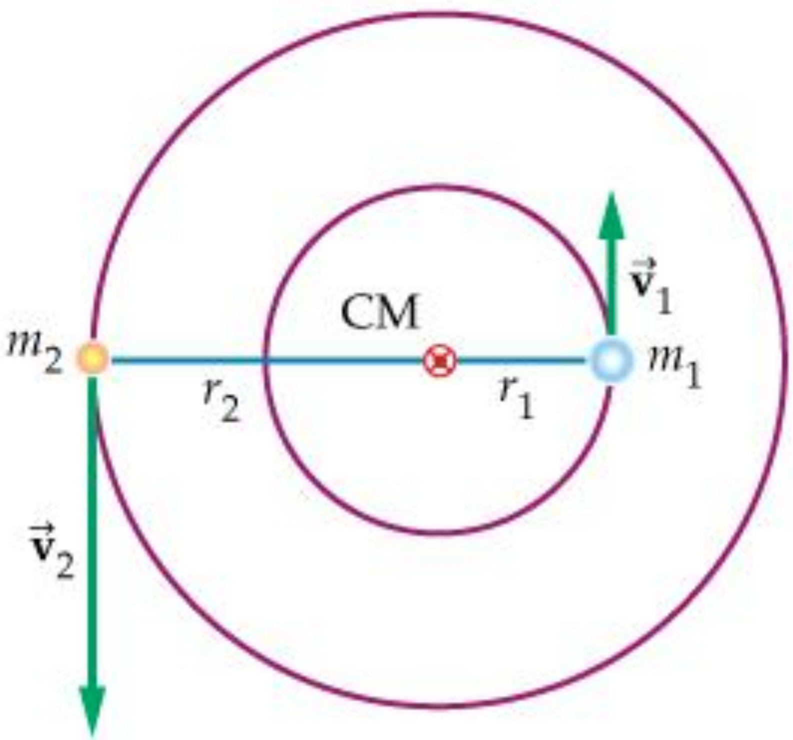


- Because of momentum conservation, orbiting objects orbit around their *center of mass*.

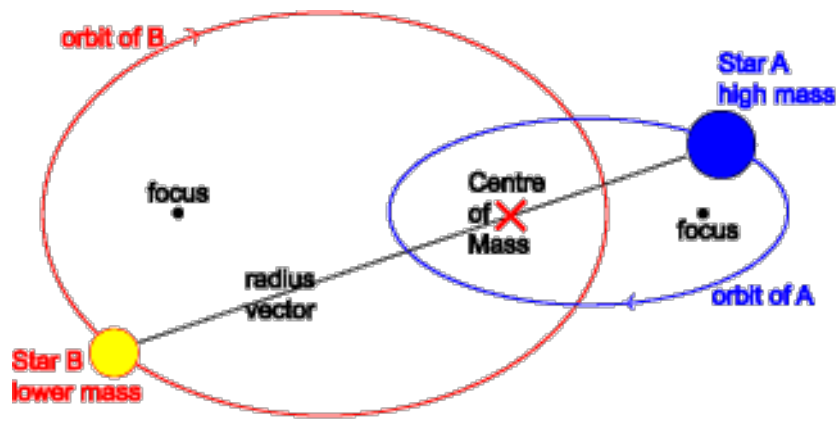
$$m_1 r_1 = m_2 r_2$$

Example Two-body Closed Orbits

Circular Orbit



Eccentric Orbit



How much does the Sun wobble?

- All the planets tug on the Sun, but none more than Jupiter.
- Data: $M_J \sim 0.001\ M_\odot$, $r_J = 5.2$ AU, $P_J = 12$ yr.
- From the mass-balance equation, $M_\odot r_\odot = M_J r_J$.
- So $r_\odot$ ~ 0.0052 AU (about $1.1\ R_\odot$, just outside the Sun).
- Mass balance also applies to orbital speed: $M_\odot v_\odot = M_J v_J$.
- For Jupiter, in a nearly circular orbit, $v_J = 2\pi r_J / P_J = 13$ km/s.
- So $v_\odot$ = 13 m/s (easily detectable by exoplanet searches).

Orbital Energy

- The energy of an orbit is one of its most important defining quantities: consider a particle of mass m in an orbit about a (much larger) mass M. Then…
  - ***Kinetic energy*** of particle is

$$KE = \frac{1}{2}mv^2.$$

  - ***Gravitational potential energy*** of particle is

$$GPE = -\frac{GMm}{r}.$$

  - ***Total orbital energy*** is just the sum…

$$E = KE + GPE = \frac{1}{2}mv^2 - \frac{GMm}{r}.$$

  - Total energy remains unchanged (it is conserved) as particle orbits… but it can trade between kinetic and potential form.

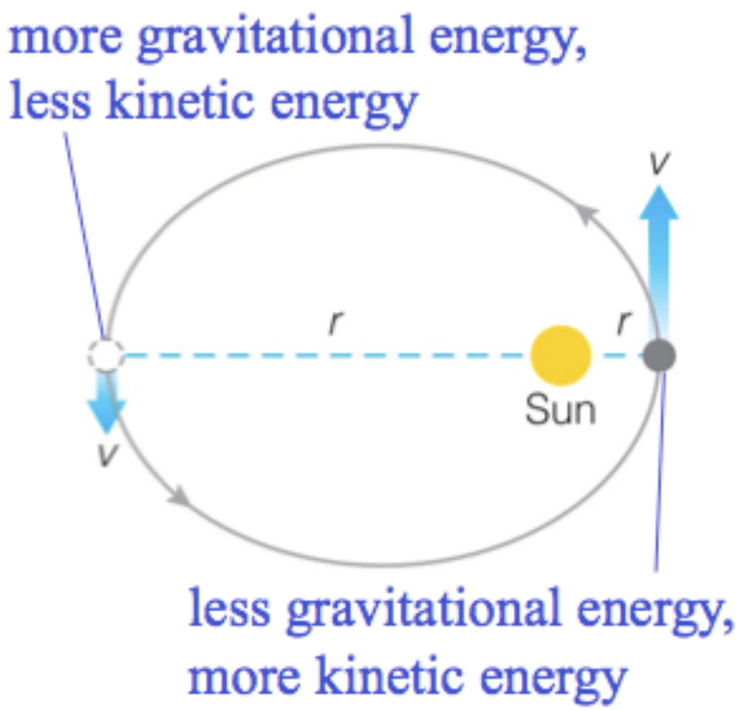
How does this relate to $GPE = mgh$? That was a special case:

$$\Delta GPE = -GMm\left(\frac{1}{R+r_1} - \frac{1}{R+r_2}\right)$$

$$\approx -\frac{GMm(r_2 - r_1)}{R^2} = mgh,$$

where $h = (r_2 - r_1)$.

Orbital Energy

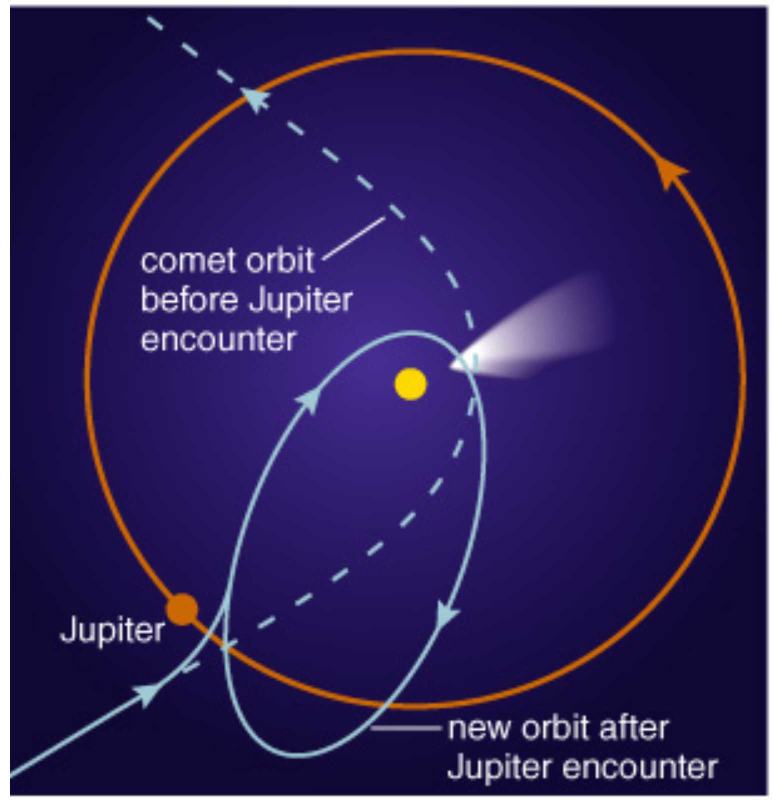


- Total orbital energy stays constant if there is no external force.
- Orbits cannot change spontaneously.

Total orbital energy stays constant.

Changing an Orbit

- So what can make an object gain or lose orbital energy?
  - Thrust.
  - Friction or atmospheric drag.
  - A gravitational encounter.



Elliptical Orbit ($E < 0$)

- The semimajor axis of *any* orbit is related to the total energy…

$$E = -\frac{GMm}{2a}, \text{ or } a = -\frac{GMm}{2E}.$$

Can derive from E & L conservation, and properties of ellipses...

- If $E < 0$, you have a ***bound orbit*** ($a > 0$).
- A special case is a circular orbit ($a = r =$ constant)…

$$E = -\frac{GMm}{2r} = \frac{1}{2}mv^2 - \frac{GMm}{r}, \text{ so } v = \sqrt{\frac{GM}{r}}.$$

Constant orbital speed.

Parabolic Orbit ($E = 0$)

- Particle starts "at rest at infinity," falls in, swings by central object and heads back out to "infinity at rest."
- Orbit is ***marginally unbound***.
- Particularly simple relationship between speed and position:

$$E_{\text{tot}} = \frac{1}{2}mv^2 - \frac{GMm}{r} = 0$$

$$\Rightarrow v = \sqrt{\frac{2GM}{r}}.$$

This is the escape speed …more in a moment!

Hyperbolic Orbit ($E > 0$)

- Unlike parabolic case, particle has non-zero speed even when it is far from the gravitating mass (i.e., “at infinity”).
- This is an ***unbound orbit***.

Energy in the Full Two-body Problem

- If the second body's mass is not negligible compared to the first's, we need to include the kinetic energy of the larger mass when computing the total energy.
- It's easiest in this case to perform the calculation in the center-of-mass frame using *relative* position and velocity.
- The total energy can then be expressed as

$$E_{tot} = \frac{1}{2}\mu v^2 - \frac{Gm_1 m_2}{r} = -\frac{GM\mu}{2a}.$$

- Here $M = m_1 + m_2$ is the total mass, $\mu = m_1 m_2 / M$ is the *reduced mass*, v is the relative speed, and r is the separation.

Escape Speed

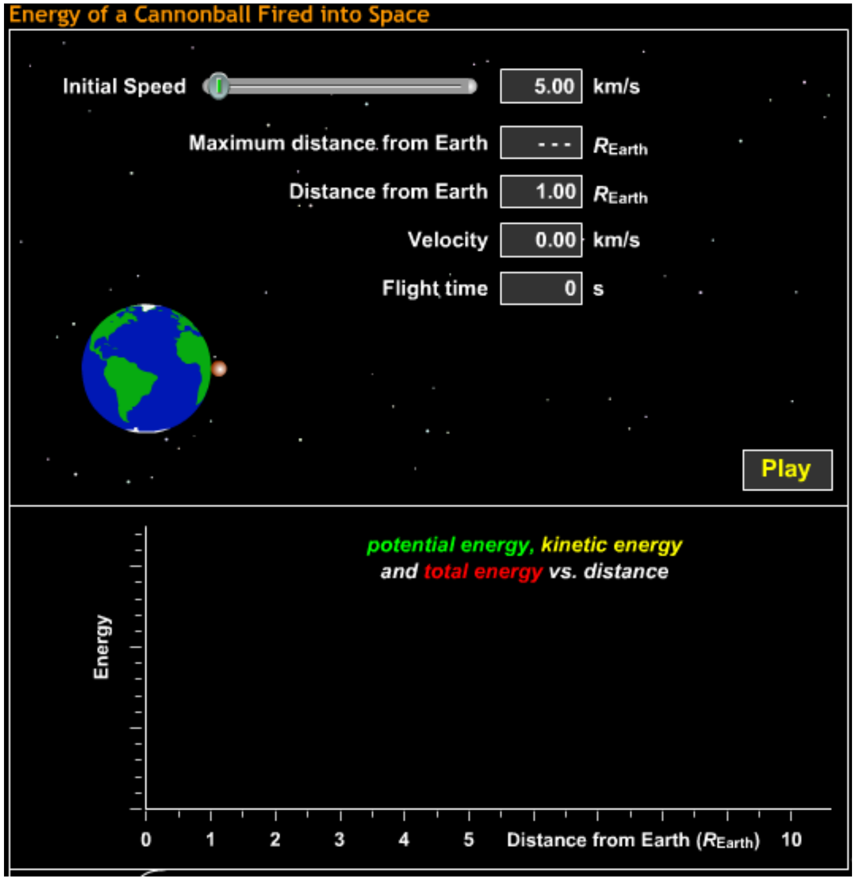
- Another important use of the energy equation is calculation of "escape speed."
  - Definition: The ***escape speed*** from a planet's surface is the speed that needs to be given to a particle in order for it to reach "infinity."
  - This is just the speed needed to put it on a parabolic orbit…

$$v_{\text{esc}} = \sqrt{\frac{2GM}{R}}.$$

M = planet mass
R = planet radius

Escape Speed

- Escape speed from Earth ≈ 11.2 km/s at sea level (about 40,000 km/hr).
- Does not depend on mass of object being launched!

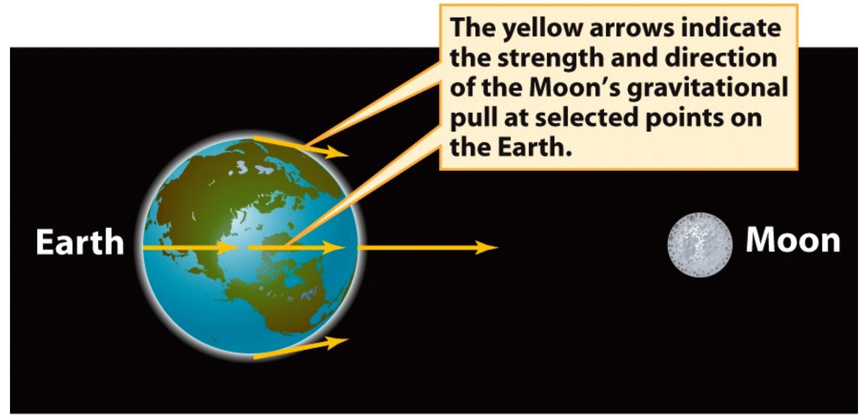


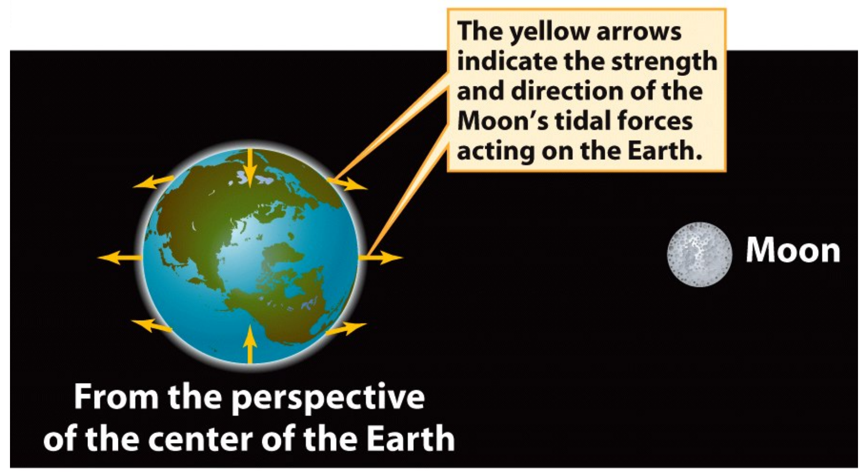
More on Escape Speed

- Launching rockets: need $\sqrt{2}$ more speed to escape than to orbit (circular *vs*. parabolic).
- Atmospheric escape: if average thermal speed of molecule approx. $> v_{\rm esc}/5$, lose that species over time.
  - E.g., if $T \approx 300$ K, H atoms have average speed around 2.2 km/s, so all atomic hydrogen escapes. H_2 molecules (~ 1.9 km/s) also escape. But N_2 molecules (~ 520 m/s) are retained. More later…
- Black holes: what if M so big or R so small that $v_{\rm esc} = c$? Nothing can escape!

Tides in the Earth-Moon System

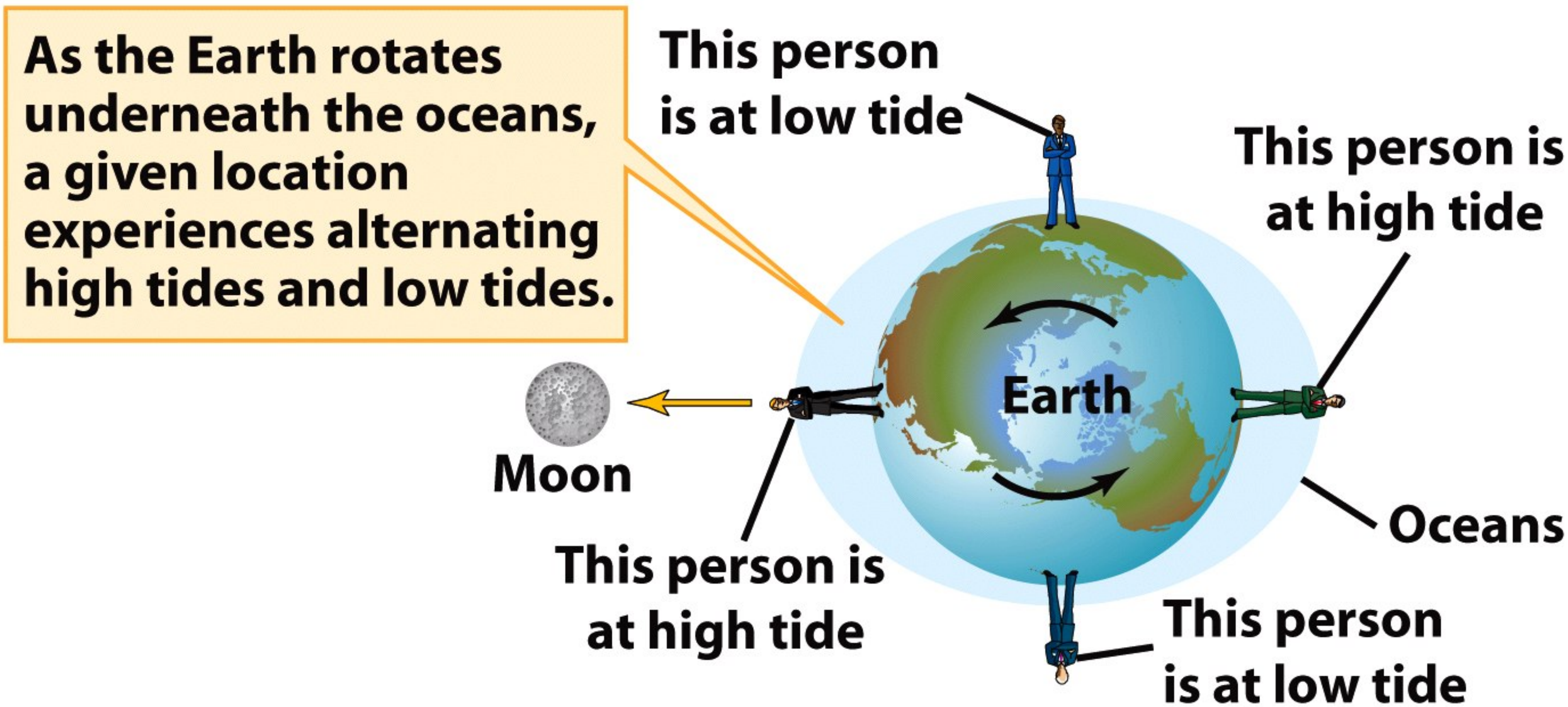
- Consider the gravitational force exerted by the Moon on various parts of the Earth…

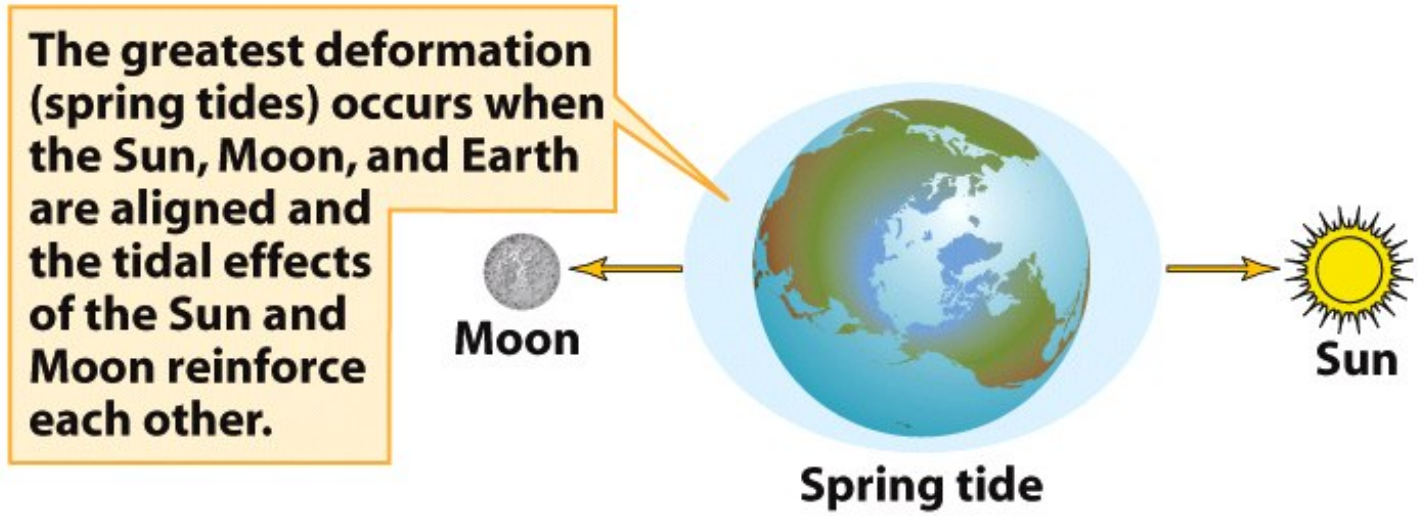




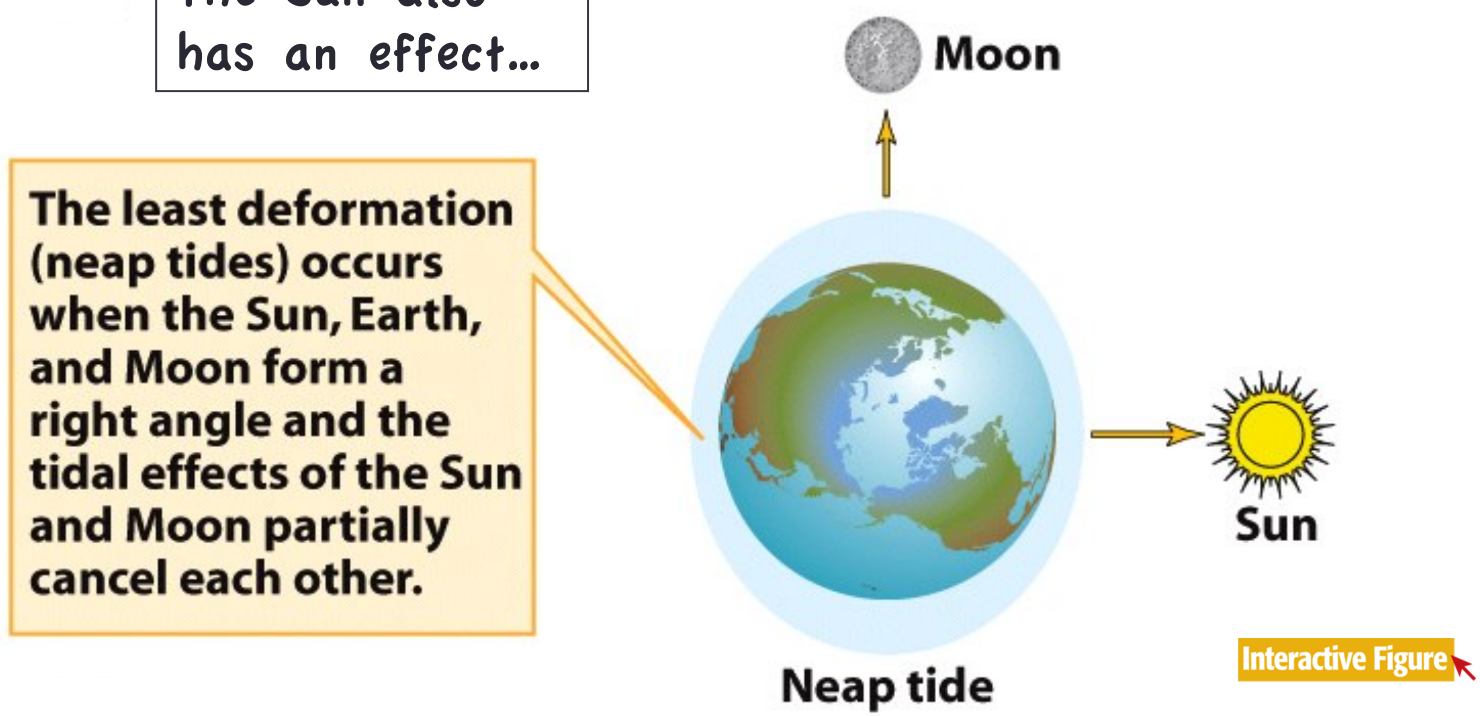
How does gravity vary over an object?

- Let's work with the *magnitude* of the force
  Then $F=-GMm/r^2$
- Suppose that we want to know the difference in the force between a distance r, and r+h, where h<<r.
  How do we do that?
- Difference is force is
  $\Delta F=(dF/dr)dr=(dF/dr)h$ in our case, because h<<r
- What is dF/dr?
  $dF/dr=d(-GMm/r^2)/dr=2GMm/r^3$
- Thus the difference in force is $2GMmh/r^3$, for h<<r
- Ex: across Earth's diameter $D=2R_E$, Moon's force varies:
  $\Delta F=4GM_Em_MR_E/d^3_{orb}$, where d_{orb}=orbital distance


As the Earth rotates underneath the oceans, a given location experiences alternating high tides and low tides.
This person is at low tide
This person is at high tide
Earth
Moon
Oceans
This person is at high tide
This person is at low tide



The Sun also has an effect…



Tides in Other Systems

- Tides or tidal effects manifest whenever the difference in gravity across an object is significant.

- For example, tidal heating of Jupiter's moon Io makes it the most volcanically active body in the solar system.

- We have even detected stars being torn apart when they wander too close to a galaxy's supermassive black hole…