

# [13] Formation of the Solar System, Part 1 (10/12/17)

## Upcoming Items

1. Read Ch. 9.1 & 9.2 by next class and do the self-study quizzes.
2. Homework #6 due next class.

## APOD 10/12/16



# Great Job on the Midterm!

- Class average  $118/150=79\%$
- Class standard deviation  $21/150$
- This was a tremendous performance! I tried to be fair in writing the exam, but also to challenge you to show your knowledge across quite a breadth of material

This was *far* deeper than previous ASTR 120 exams

You really came through! I know you have worked hard in this course. I'm very proud of you.

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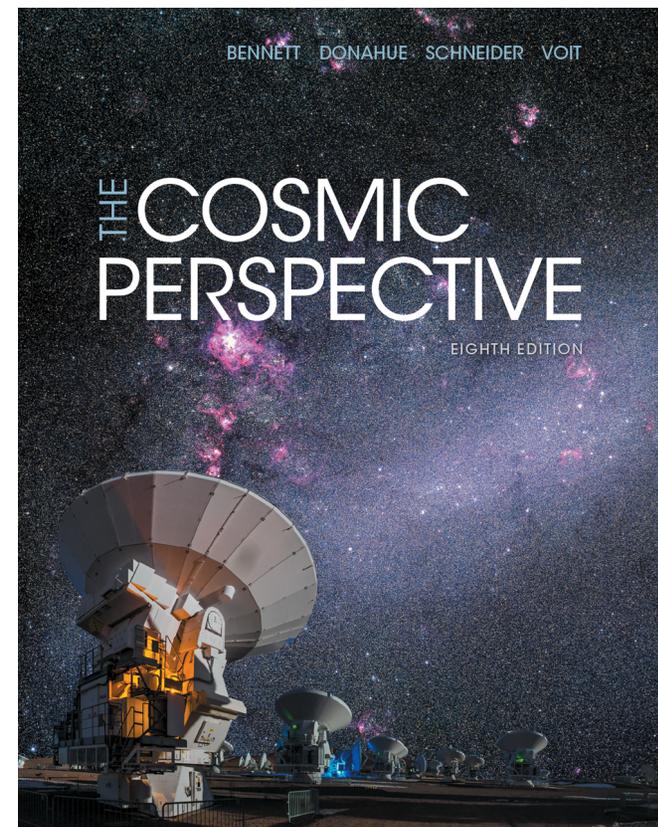
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**You really came through! I know you have worked hard in this course. I'm very proud of you.**
- What if you didn't do as well as you hoped?
  - 1. Remember, midterm was only 15% of class total**
  - 2. Consider going to see tutors, not just before HW is due**
- Again, though, I'm really impressed by how well you did!

# LEARNING GOALS

## Chapter 8.1–8.2

*For this class, you should be able to...*

- ... use the nebular theory and conservation laws to predict the configurations of planets (orbit directions and alignment) and their dominant compositions (metal, rock, ice, and/or gas as a function of distance from the star) in a solar system.*
- ... deduce how the temperature varies as a function of distance from a star based on thermal radiation laws;*
- ... demonstrate that the Sun lost most of its angular momentum after it formed.*



Any astro questions?

# LIGO Press Conference, 10 AM Mon.

- Viewing in PSC lobby
- At 3 PM on Monday, also in PSC lobby, there will be talks by Prof. Peter Shawhan (UMd Physics), Dr. Julie McEney and Dr. Brad Cenko (Goddard), and me, about implications of the announcement
- Feel free to come to either or both!

# What Trends Should We Explain?

- Planets all orbit in the same direction?
- Two types of planets?
- Planetary orbits are nearly circular?
- Small-integer ratios of many orbital periods?
- Four small planets and four large planets?
- One planet is more massive than the rest put together?
- As astronomers, we have to make choices; if we try to account for *all* the details, we'll go crazy!
- In this case, let's start with the orbits all being the same way  
**Very improbable if the orientations were random**
- Perhaps if we can explain that, then some of the other trends can also be explained...

# Derivations in Classes

- Note: to improve depth of understanding and to make contact with what we've learned so far, we'll do a number of derivations going forward including today  
**The book only has qualitative discussion of the Solar System; you're prospective majors, you can do more 😊**
- I intend to give you time to discuss derivations in groups  
**If I forget, please remind me!**
- Important that you follow the derivations and, especially, that you understand the physical content of the arguments, so have your group ask if you're not sure about a detail!

# What Does Common Direction Signify?

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- Large net angular momentum
- Why? If directions were random then the angular momentum would tend to cancel out  
Remember that angular momentum is a vector.  
**An orbit going one way, combined with the same orbit but going the opposite way, add to zero total ang. momentum**
- Thus we need to have some mechanism that will naturally give us a lot of angular momentum
- But in astronomy, “a lot” has to be with respect to something
- In this case, it means “as much as circular orbits over tens of AU” **Why?**

# The Nebular Theory

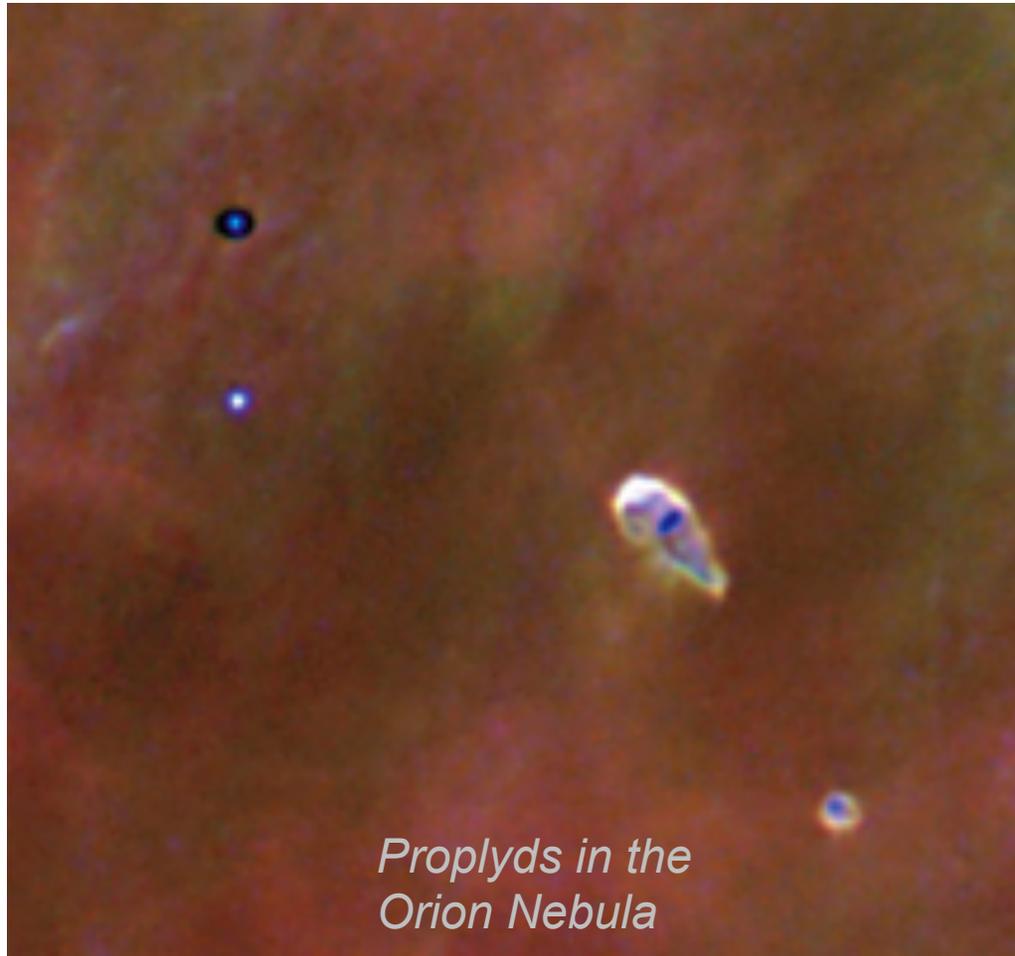


Orion Nebula

- According to the nebular theory, our solar system formed from a giant cloud of interstellar gas.

(*nebula* = cloud)

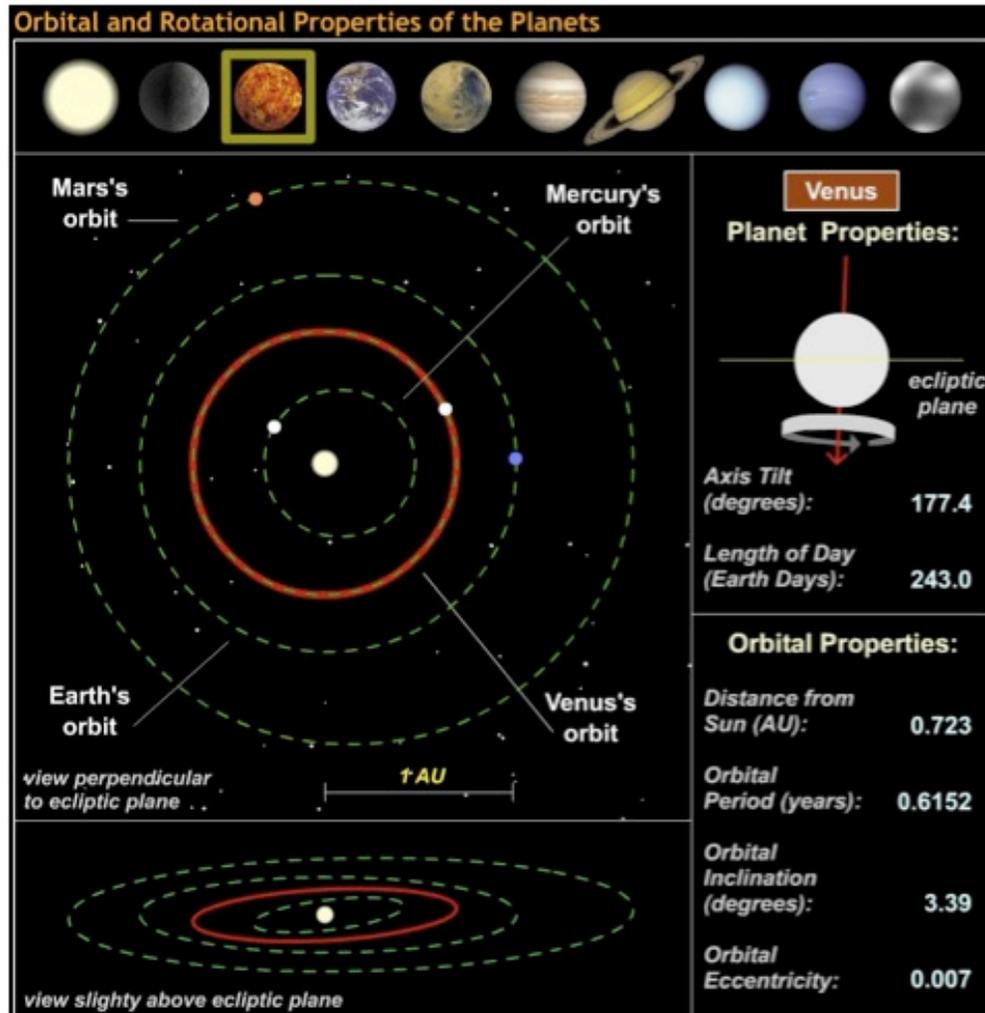
# Evidence from Other Gas Clouds



*Proplyds in the  
Orion Nebula*

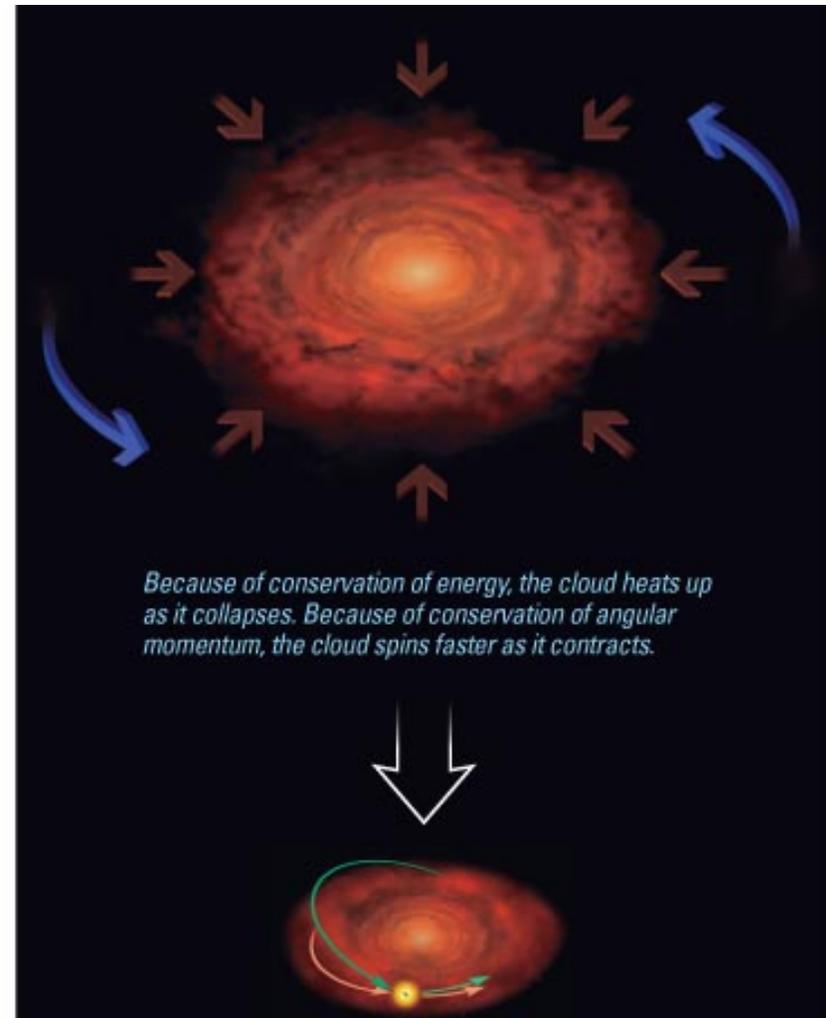
- We can see stars forming in other interstellar clouds, lending support to the nebular theory.

# What caused the orderly patterns of motion in our solar system?



# Conservation of Angular Momentum

- The rotation speed of the cloud from which our solar system formed increased as the cloud contracted.



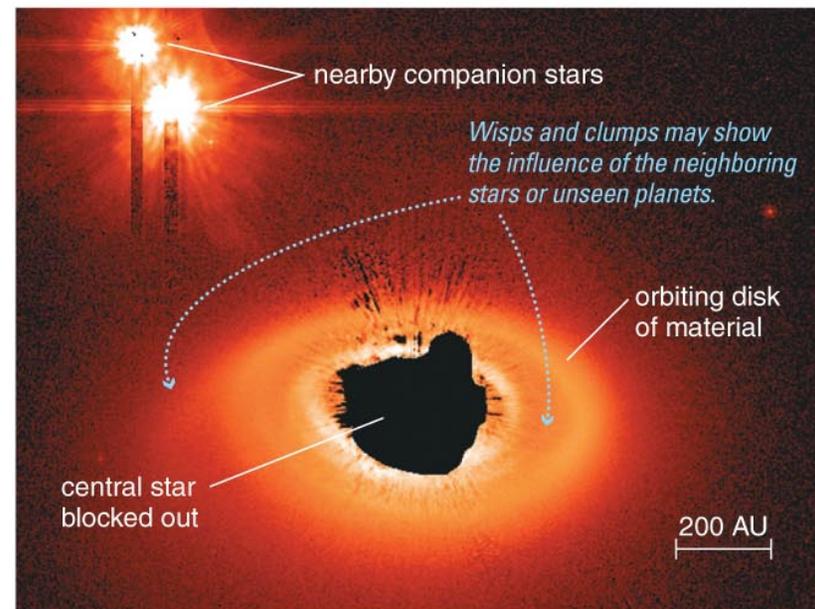
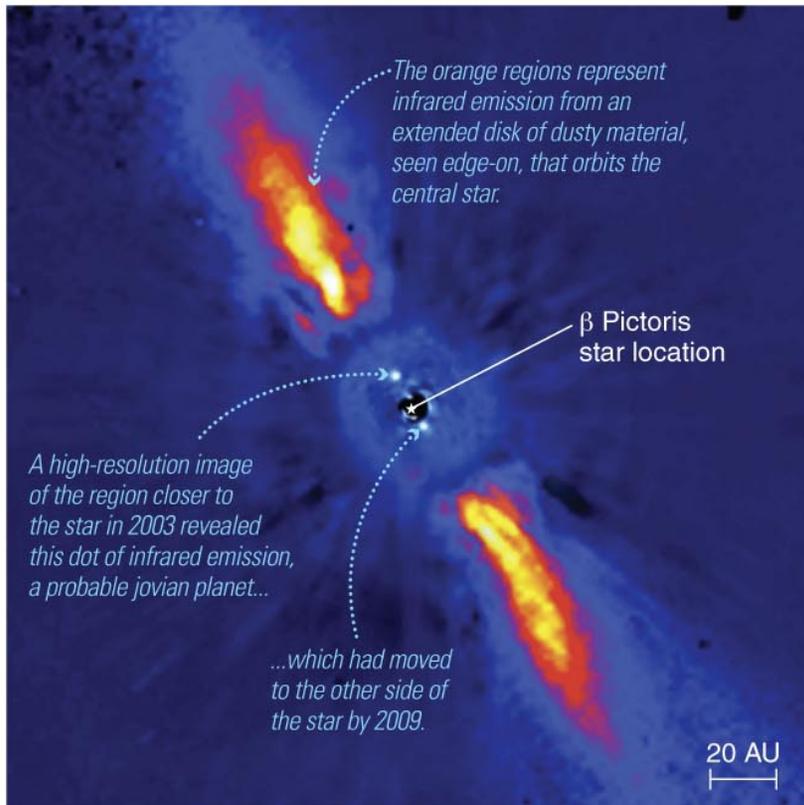
## Now let's be quantitative...

- Is there enough angular momentum to cause flattening when the nebula contracts to the size of the Solar System?
- Magnitude of angular momentum:  $mr v_{\text{perp}}$   
But  $v_{\text{perp}} = r\Omega$ , where  $\Omega$  is angular velocity  
Thus the magnitude of angular momentum is  $mr^2\Omega$   
So at constant  $m$ , and constant ang. mom.,  $\Omega \sim 1/r^2$
- Initial size and angular velocity? Maybe 1 pc (about 200,000 AU) and  $10^{-8}$  radians/year (rotation of MW)
- If the cloud contracts to 5 AU (orbital radius of Jupiter), how fast would it rotate?

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- If the cloud contracts to 5 AU (orbital radius of Jupiter), how fast would it rotate?  
 $10^{-8} \text{ rad/yr} \times (2 \times 10^5 / 5)^2 = 16 \text{ rad/yr}$
- Jupiter:  $\sim 2\pi / 12$  years, or  $\sim 30$  x less
- Quantitatively, contraction leads to orbits in same direction!

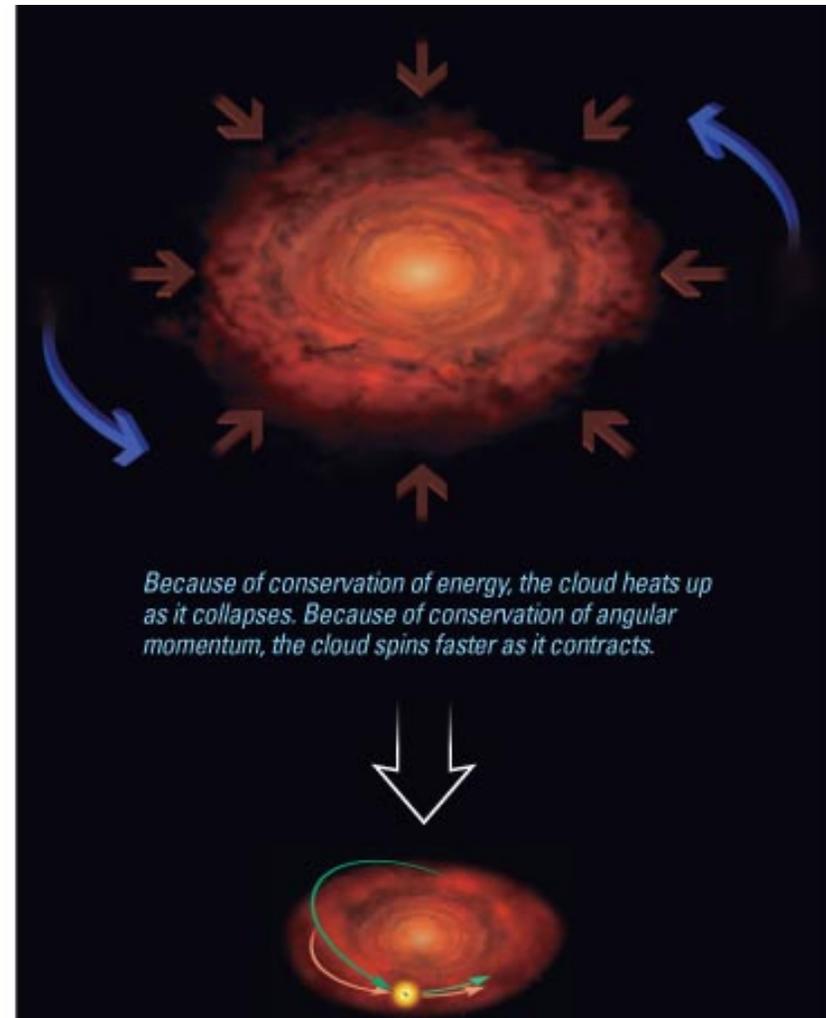
# Disks Around Other Stars



- Observations of disks around other stars support the nebular hypothesis.

# Collisions

- Collisions between particles in the cloud caused it to flatten into a disk:
  - Collisions *reduce random motions* by converting orbital energy into heat: particles tend toward *orderly circulation*.
  - Collisions also *reduce up-and-down motions*.
  - The result is the spinning cloud flattens as it shrinks.



# Are Circular Orbits Special?

- Yes! At constant angular momentum, circular orbit has the least energy. Let's see why.
- Energy of orbit of semimajor axis  $a$  is  $E = -GMm/(2a)$
- If orbit has eccentricity  $e$ , apocenter distance is  $a(1+e)$   
 Thus  $E = (1/2)mv^2 - GMm/[a(1+e)]$   
 But still must equal  $-GMm/(2a)$ . We can cancel "m"
- $(1/2)v^2 - GM/[a(1+e)] = -GM/(2a)$   
 $v^2 = (GM/a)[2/(1+e) - 1] = (GM/a)[(2-1-e)/(1+e)]$   
 $v^2 = (GM/a)[(1-e)/(1+e)]$ , so  $v = (GM/a)^{1/2}[(1-e)/(1+e)]^{1/2}$
- Angular momentum is  $mr_v$  at apocenter (why?)  
 $L = ma(1+e)(GM/a)^{1/2}[(1-e)/(1+e)]^{1/2}$   
 $L = m[GMa(1-e^2)]^{1/2}$
- So why does circ. orbit have least energy at constant  $L$ ?  
 And what does this imply if stuff runs into itself in disk?

## Same principle for other motion

- Example: if gas, or many particles, move in orbits with different orbital planes, collisions will release energy and the orbits will settle into the same plane

**That's why rings can occupy a very thin plane**

- When objects orbit around each other, the lowest-energy configuration with a fixed total angular momentum is (1) circular orbit, (2) synchronous orbit (rotation periods equal to orbital periods), and (3) rotation and orbit in the same plane

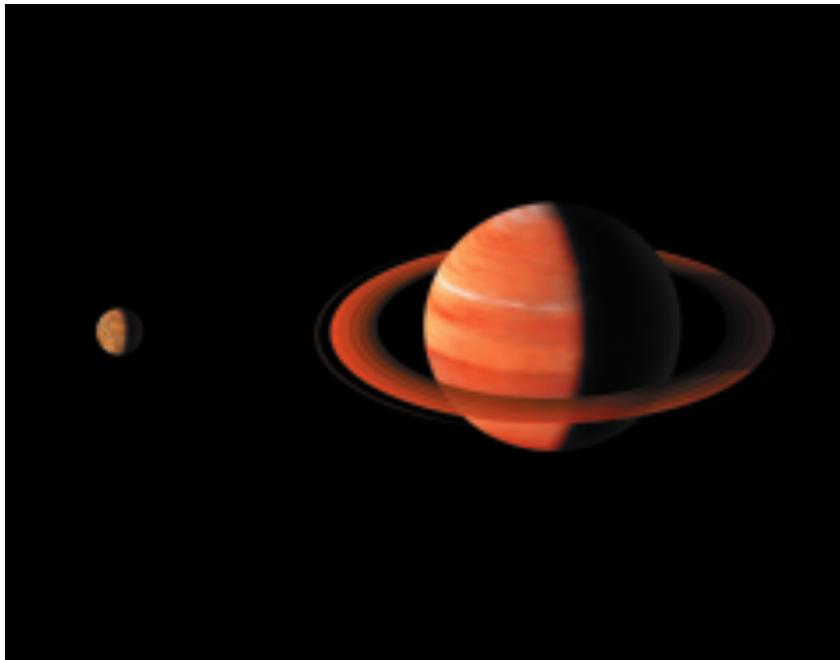
**Moon presents one face to Earth; most moons are “tidally locked” to planet; Pluto and Charon mutually synchronous**

- Can you think of why this might not always be the case?

# Why Conserve L, not E?

- Maybe you're starting to wonder:  
**Energy and angular momentum are both conserved**  
**But here we're talking about only L; why not also E?**
- Conservation is only for a closed system
- Photons can escape easily from the system  
**For photons,  $E=pc$ , so  $p=E/c$**   
**For particle,  $E=(1/2)mv^2=(v/2)mv=(v/2)p$ , so  $p=(2/v)E$**
- Particle speeds are  $v \ll c$ . Thus the energy taken away by photons takes away a tiny amount of linear momentum  
 **$L=rxp$ , so also a tiny amount of *angular* momentum**
- Another factor: photons go in all directions; every photon carries energy, but angular momentum nearly cancels  
**Questions or discussion about these points?**

# Why are there two major types of planets?



	<i>Examples</i>	<i>Typical Condensation Temperature</i>	<i>Relative Abundance (by mass)</i>
Hydrogen and Helium Gas	hydrogen, helium 	do not condense in nebula	 98%
Hydrogen Compounds	water (H <sub>2</sub> O) methane (CH <sub>4</sub> ) ammonia (NH <sub>3</sub> ) 	<150 K	 1.4%
Rock	various minerals 	500–1,300 K	 0.4%
Metals	iron, nickel, aluminum 	1,000–1,600 K	 0.2%

# Temperature Variation Over Disk

- As gravity causes the cloud to contract, it heats up.
- Inner parts of disk are hotter than outer parts.
- Why? Think about the orbital speed; for a circle,  
 $v_{\text{orb}} = (GM/r)^{1/2}$
- Faster motion=>hotter
- Rock can be solid at much higher temperatures than ice. **Why does that matter?**

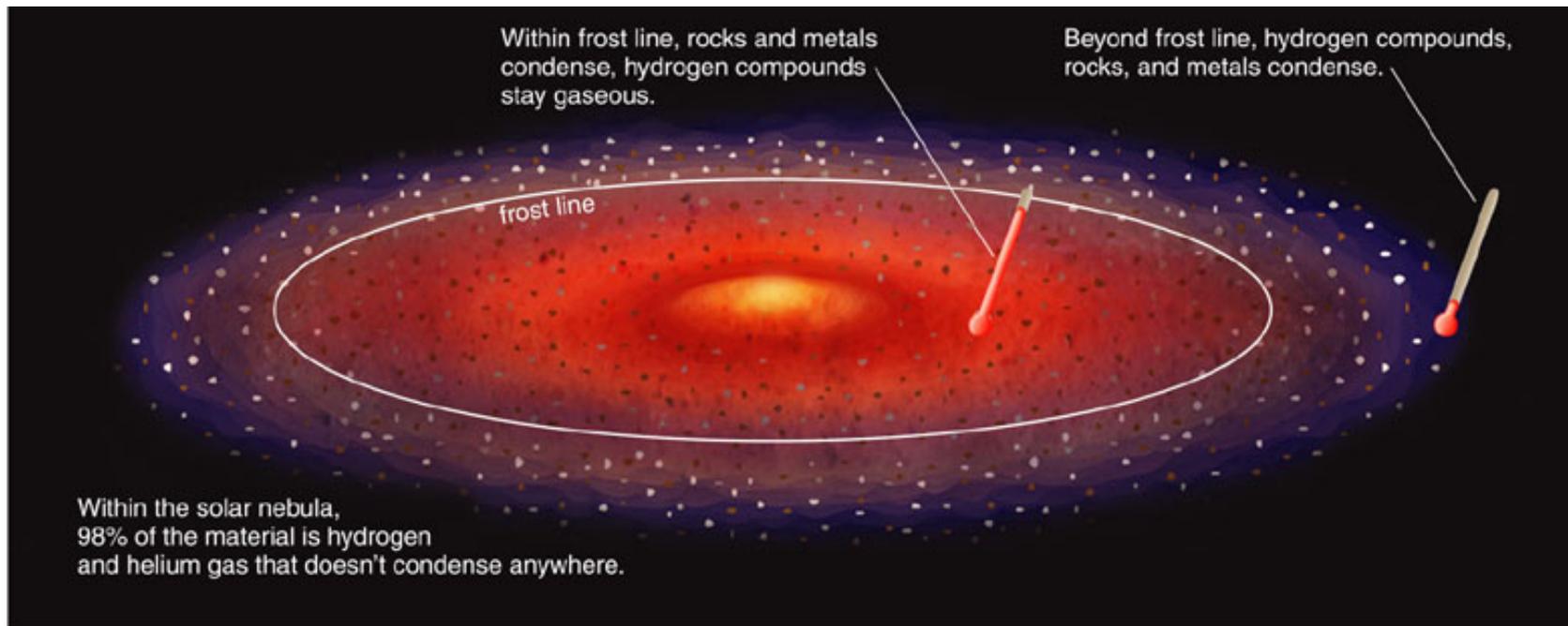


# What if There is a Star at the Center?

- First, how does the intensity (flux) of light  $F$  vary with distance  $d$  from a star?
  - Answer:  $F \propto 1/d^2$  (inverse-square law, like gravity).
- If you put a sphere at that distance, how does the amount of light (power) it intercepts depend on  $F$  and its radius  $R$ ?
  - Answer: Power absorbed  $\propto R^2 F$  (cross-section times flux).
- How does the power emitted by the sphere depend on  $R$  and its temperature  $T$ ?
  - Answer: Power emitted  $\propto R^2 T^4$  (Stefan-Boltzmann law).
- At equilibrium, power absorbed = power emitted, so

$$\frac{R^2}{d^2} \propto R^2 T^4 \quad \text{or} \quad T \propto \frac{1}{\sqrt{d}}.$$

# The Frost Line



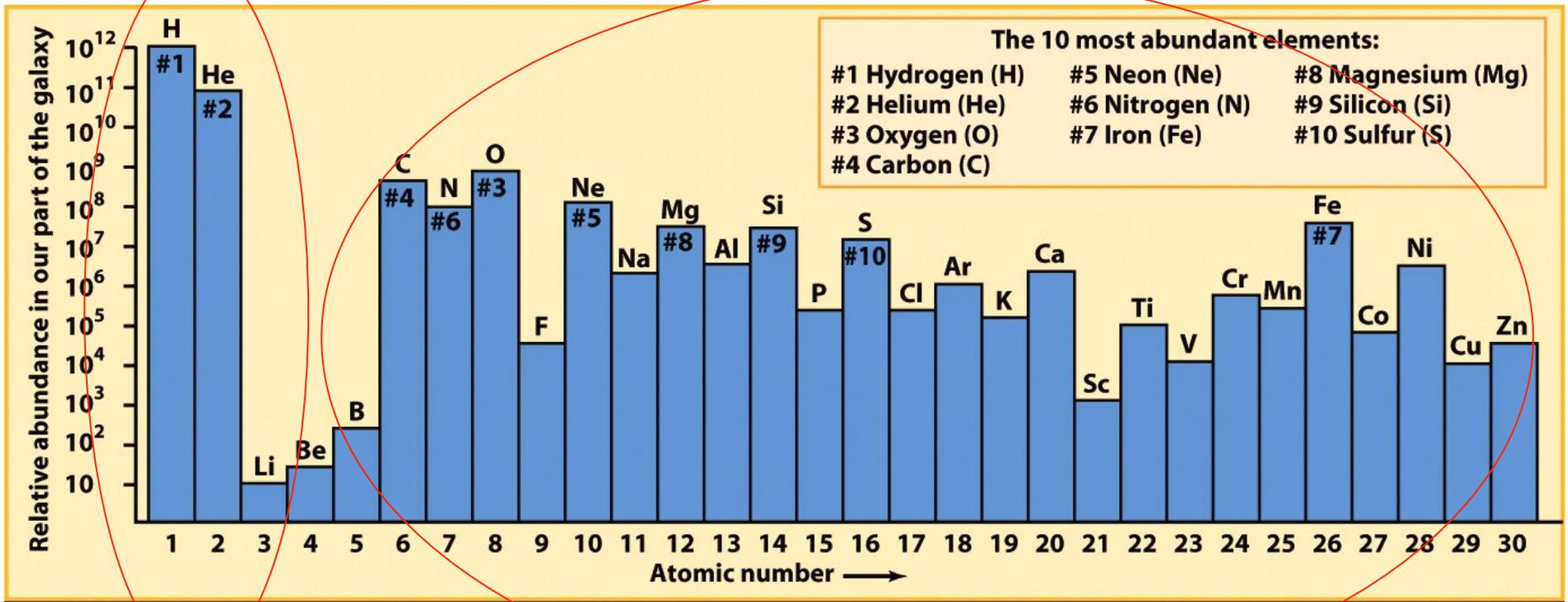
- Inside the ***frost line***: too hot for hydrogen compounds to form ices. **Most mass in the nebula is in hydrogen**
- Outside the ***frost line***: cold enough for ices to form. **Ices can adhere to each other, grow, attract gas**

Mount Fuji (Japan)

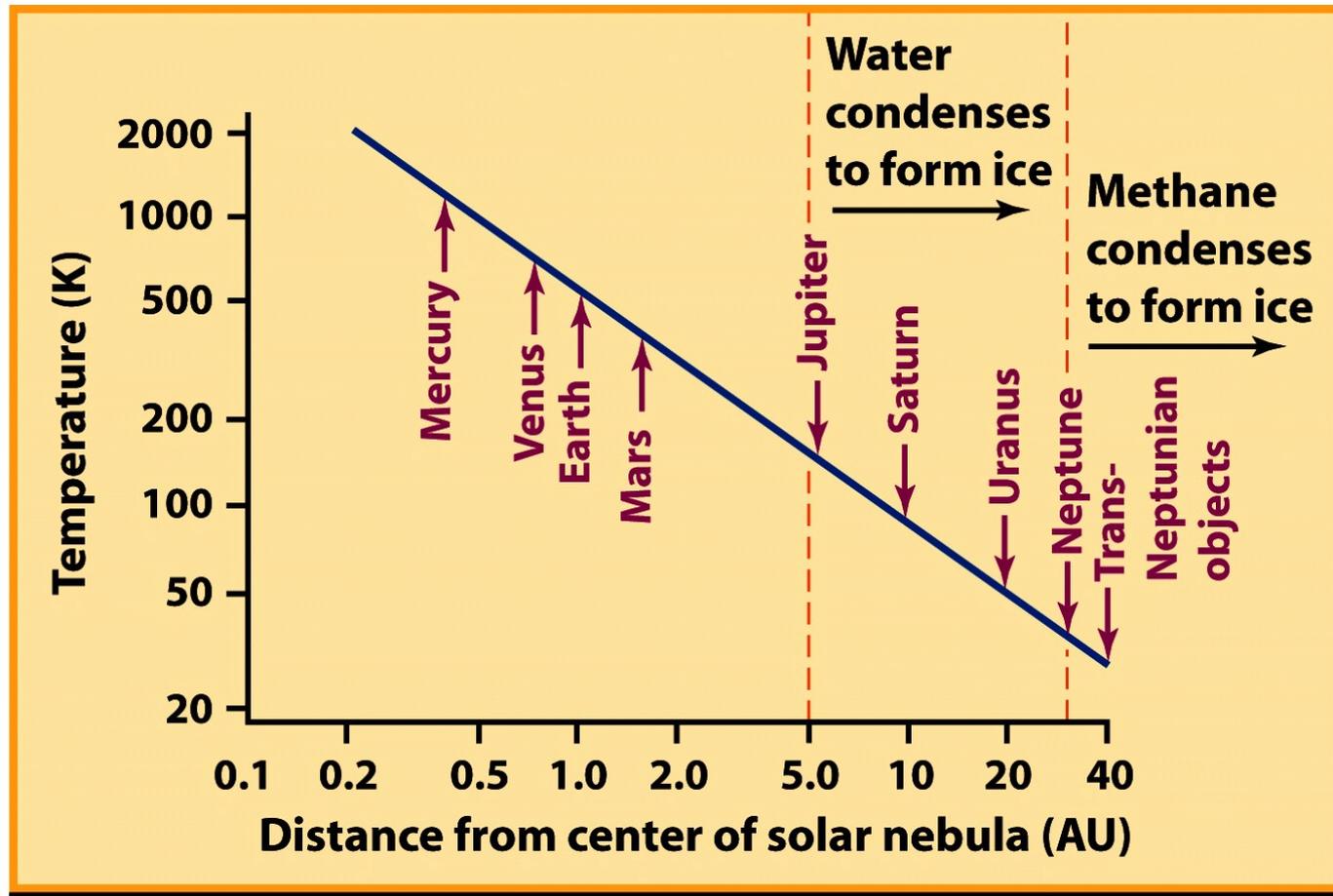


Big Bang

Made in Stars



Composition of interstellar gas in our part of the galaxy (log scale!).



- Likely temperature distribution in the solar nebula.
- Much warmer then than now—why? And why doesn't ice form closer to the Sun (at temperatures near 273 K)?

# Summary: Planet Formation 1

- The raw material comes from a very large, very tenuous nebula of gas
- That matter contracts (because it can radiate energy), and flattens (angular momentum conservation).  
**Leads to orbits in the same direction**
- Disk is hotter nearer center  
**Naturally faster motions**  
**If star has already formed, hotter because closer to star**
- When hot, only rock grains can condense  
**Outside the “frost line”, ices can condense**  
**Much more mass in ices (which contain hydrogen) than in rock (which does not)...**