ASTR 120 Problem Set 4: Due Tuesday, September 26, 2017

General reminders: You must show all your work to get full credit. Also, if any website was useful, you need to give the URL in your answer. Note that any website is fair game; you just have to cite it. If any book including our textbook was useful, you need to indicate where in the textbook you used a particular fact. This will be true in all homeworks.

1. [10 points] Lord Voldemort has decided to perform an astronomical experiment. Waving his wand, he magically transports the asteroid Vesta into an orbit that allows it to collide with Venus. He makes the orbit of Vesta exactly the same as that of Venus, but in the opposite direction, so that the collision is head-on. When Vesta hits Venus, the two stick together; thus the new Venus+Vesta planet has the sum of their two original angular momenta (note that Vesta's angular momentum was negative!). We also assume that all of the energy of the collision goes into heating Venus uniformly. With those assumptions:

a. [3 points] Calculate the kinetic energy of the impact, in Joules.

b. [3 points] Suppose that the average atom that makes up Venus has a mass of 2.68×10^{-26} kg. The thermal energy per atom is $\frac{3}{2}k_BT$, where $k_B = 1.38 \times 10^{-23}$ J K^{-1} is Boltzmann's constant. Use these numbers to calculate the temperature rise of Venus due to the collision, assuming that every atom gets the same amount of energy. Note that late planetary formation has collisions like this (although not usually involving Lord Voldemort).

c. [4 points] Assuming that prior to the collision Venus moved in an exactly circular orbit, calculate the *ratio* between the speed of the new Venus+Vesta planet just after the collision, and the speed of Venus just before the collision.

For these problems, please give the URL of the websites you used to look up values such as the masses of Venus and Vesta, et cetera.

2. [5 points] Stars form by the contraction of interstellar clouds. In this and the next problem, you will do some sample calculations to get a sense of the influence of energy and angular momentum conservation on the process of contraction.

For this problem, let us assume that we have a 1 kg bit of gas 0.1 parsecs away from a one solar mass object. The gas temporarily moves laterally (i.e., perpendicular to the direction to the central object) with a speed of 3 meters per second. Note that at this speed the gas is *not* in a circular orbit.

Calculate the radius, in AU, of a *circular* orbit that has the same angular momentum as the initial orbit. **Hint:** you can save yourself a lot of effort by noting that the Earth's nearly circular orbit, which is of course 1 AU away from a one solar mass object (the Sun!), moves at very close to 30 km s^{-1}. Also note that the angular momentum of a small mass m (such as the Earth) in a circular orbit of radius r around a much larger mass $M \gg m$ (such as the Sun) is $m\sqrt{GMr}$.

3. [5 points] For exactly the same setup as in problem 2, calculate the total (kinetic plus gravitational potential) energy of the bit of gas initially (when it moved at 3 meters per second 0.1 parsecs away) and in its final state (when it is in a circular orbit with the same angular momentum as its initial motion), in Joules. Recall that the kinetic energy of an object of mass m and speed v is $E_{\rm kin} = \frac{1}{2}mv^2$, and the gravitational potential energy of an object of mass m a distance r away from an object of mass M is $E_{\rm pot} = -GMm/r$.

You should find that the energies are different, and indeed something like this really does happen as interstellar clouds contract. But energy is conserved. Explain, qualitatively, where the extra energy goes.

4. [5 points] The angular momentum of an object with moment of inertia I and angular velocity ω is $I\omega$. Note that "angular velocity" is the rate at which the object turns a radian; for example, the angular velocity of Earth in its orbit around the Sun is 2π yr^{-1}, because we travel 2π radians (one full circle) in a year. Again, the angular momentum of a small mass m (such as Jupiter) in a circular orbit of radius r around a much larger mass $M \gg m$ (such as the Sun) is $m\sqrt{GMr}$. Using these formulae:

Calculate the rotational angular momentum of the Sun (using its angular velocity at its equator), and compare that with the orbital angular momentum of Jupiter. Recall that by "compare" we mean "find the ratio". Comment on what this means for the formation of the Solar System; most of the mass went into the Sun, but where did most of the angular momentum go?

Bonus Question [2 points]

We have discussed three things that are conserved in isolated systems (linear momentum, angular momentum, and energy). Do a Web search to find one other quantity that is absolutely (not approximately!) conserved for isolated systems, and give the URL of the page that gives you this information.