ASTR 120 Problem Set 5: Due Tuesday, October 3, 2017

General reminders: You must show all your work to get full credit. Also, if any website was useful, you need to give the URL in your answer. Note that any website is fair game; you just have to cite it. If any book including our textbook was useful, you need to indicate where in the textbook you used a particular fact. This will be true in all homeworks.

1. We mentioned in the notes that light can exert a force. In this problem, we will do a computation related to that force. The outward force of light on an object is

$$\text{Force} = \frac{\sigma}{c} F \ , \tag{1}$$

where F is the flux (radiation energy per area per time), c is the speed of light, and σ is the effective area of the particle, which is called the "cross section". We will assume that the light is emitted by a source so small that we can consider it to be a point, and that the light is emitted equally in all directions; given those assumptions, the flux a distance r from the source is related the luminosity L (radiation energy per time, summed over all directions) of the source by

$$F = \frac{L}{4\pi r^2} \ . \tag{2}$$

Use these expressions, combined with the standard expression for the force of gravity, to derive the formula for the critical luminosity $L_{\rm crit}$ such that the outward force of light exactly balances the inward force of gravity, around a massive object of mass M. Assume that the particle has a mass m, which is much smaller than M.

2. An important concept regarding how radiation moves through matter is the "random walk". In a random walk (sometimes called a "drunkard's walk"!), a photon (in this case) moves a certain distance, then scatters and moves a similar distance in a new, random, direction. This is a fairly good description of how photons bounce around inside the Sun, for example.

Recall that the optical depth τ is $\tau = R/\ell$, where R is the straight-line distance through the medium and ℓ is the mean free path. You can show that if $\tau \gg 1$, then the average number of steps of length ℓ that are required to escape the medium is $N_{\rm steps} \approx \tau^2$.

Suppose that the mean free path for a photon in the Sun is 10^{-5} meters. Calculate how long it would take the typical photon to escape from the center of the Sun. Please assume that scatterings are instantaneous and that between scatterings the photons travel at the speed of light.

3. In class we gave the blackbody spectrum

$$B_\nu(T) = \frac{8\pi\nu^2}{c^3} \frac{h\nu}{e^{h\nu/kT} - 1} \tag{3}$$

and we noted that the peak frequency ν_{peak}, which maximizes $B_\nu(T)$, must be proportional to kT, where T is the temperature. Determine, to at least two significant figures, the proportionality constant; that is, $h\nu_{\text{peak}} = xkT$, and you are to determine x to at least two significant figures. Please note that you need to show your work; a simple quote of a webpage is insufficient.

Hint 1: when getting the peak, does the constant factor $8\pi h/c^3$ matter?

Hint 2: there are ways to get this answer *without* calculus, including **intelligent** guessing!

4. Prior to the full development of quantum mechanics, Niels Bohr introduced a phenomenological description of the hydrogen atom (here "phenomenological" means a model that is descriptive rather than being based on fundamental physical principles). In his model, the angular momentum of the electron's orbit around the nucleus could only be multiples of $\hbar = h/2\pi$, where h is Planck's constant. That is, the angular momentum could only be $L = m_e v r = n\hbar$, where $n = 1, 2, 3, \ldots$ and m_e is the mass of the electron.

Let's follow Bohr's logic. Suppose that we treat the "orbit" of an electron around a nucleus like the orbit of a planet around the star. Electrical forces, rather than gravitational forces, are at play, so for a circular "orbit" we have

$$E = -e^2/(2r) \tag{4}$$

for the energy of an orbit of radius r, and the speed of that orbit is

$$v = e/\sqrt{m_e r}\ . \tag{5}$$

Here e is the charge of the electron, in a particular set of units.

Using these formulae, derive the energy of the orbit that has $L = n\hbar$. The quantization you find (meaning that the energy can take only a specific set of values) matched perfectly with the experiments available at that time, so that even though Bohr's model is not actually correct, it helped usher in the era of quantum mechanics.

5. Suppose that you radiate like a blackbody, with a temperature of $T = 310$ K, and that we can approximate you as a sphere of radius $R \approx 40$ cm. Calculate your luminosity, in Watts. Compare your luminosity divided by an average human mass of 70 kg, with the Sun's luminosity divided by its mass. Are you surprised?

Bonus Question [2 points]

What *experimental* method did people use to learn the blackbody spectrum in the late 1800s? That is, we know that Max Planck derived the spectrum, but he did so to account for the experimentally observed spectrum; how did people get that? As always, give the URL (or other reference) that you used.