

Cosmology, the study of the universe as a whole, has intrigued people since they first started looking at the skies. It deals with some of the deepest questions ever asked: where did it all come from? How will it all end? Is the universe finite or infinite?

Although it is clear that *final* answers cannot be given for such questions (if I give such an answer, you can always ask “why” to unlimited levels of depth), it is fortunately the case that in the past several decades remarkable progress has been made on these questions. As is often the case, the answers have also lead to unexpected new questions: for example, what is the dominant component of the universe? For much of the history of cosmology we have been starved for data, but this is changing and it is expected that the next many years will yield a rich array of cosmological observations.

In this course we will explore many aspects of modern cosmology, with a theoretical focus but bringing many observations to bear. By the end of it I hope you will have a sense of what ideas are firmly established as well as the uncertainties on the frontier.

By the way, a quick word about units: fields of astronomy that focus on phenomena outside the solar system tend to use the cgs system of units (cm, g, s). However, in physics classes the SI system is more common (m, kg, s). In addition, many special units are needed at times (AU, parsecs, solar masses). For this reason, and to conform to our textbook, we will use SI units in this class. It is essential to be able to convert between them, though, and on occasion I might slip into cgs by force of habit, so be alert!

Throughout the semester we will apply many techniques to understand the concepts. Some, of course, will involve derivation of equations. In such cases it is all too easy to derive away and then blithely accept the final answer without checking. In this class, however, I will place strong emphasis on standing back from your equations to see if they make sense. This is an important step in developing astrophysical reasoning skills, so let’s talk about that.

### **Developing Astrophysical Reasoning Skills**

As discussed in detail in the “Hints about doing research in astrophysics” file on the class web page, there’s quite a transition between classwork and research. In this course I will encourage development of research-oriented skills. One of these is the ability to size up a problem and determine how best to approach it, given the goal of the research and the needed accuracy. Some things are best solved analytically and some with a computer; some require great accuracy and some are best done with order-of-magnitude estimates; and so on. In all cases, though, you’ve got to be able to sit back and ask yourself “Does this make sense?” so that a programming bug doesn’t convince you that energy isn’t conserved!

One aspect of “does this make sense” is that you need to be able to look at a result and determine if it satisfies several “common-sense” criteria, from simple to complex. Does

it have the right units? Is it correct in limits that I can check easily? Does it possess the appropriate symmetries? Does it depend on what it should depend on, and no more? Ideally, you should do this before you embark on a calculation, and also afterwards, to check your result. You'd be surprised at how often you can catch errors this way or sharpen your intuition. Here's an example, due to Doug Hamilton:

### Units, Limits, and Common Sense

You launch a rocket straight up from the Earth's North pole, and it rises up to a maximum height  $H$ , then falls back to Earth. The maximum height above the Earth is given by one of the expressions below. Here  $R_E$  is the Earth's radius,  $X = v^2 R_E / GM_E$ ,  $G$  is the gravitational constant,  $M_E$  is the Earth's mass and  $v$  is the launch velocity. Without solving the problem, rule out as many of the incorrect equations as possible using simple physical arguments.

- A)  $H = R_E X / (1 + \sqrt{X})$
- B)  $H = R_E X / (1 - X)$
- C)  $H = R_E X / (2 - X)$
- D)  $H = R_E (1 - X) / (2 - X)$
- E)  $H = v X^2 / (2 - X)$
- F)  $H = R_E X / 2$
- G)  $H = R_E X^2 / (2 - X)$
- H)  $H = R_E X |1 - X| / (2 - X)$

You can rule out all but one of these possible answers using relatively simple arguments. This means a huge savings in time, and if you get in the habit of thinking about answers in this way your intuition will improve dramatically.

### Simplification of equations

A separate skill that is often useful is the ability to examine a problem and determine what complications can be dropped and yet retain the essence of the physics or at least the desired accuracy. Here's one example. Suppose you have a photon with energy  $\hbar\omega$  scattering off an electron at rest. The total cross section, with  $x \equiv \hbar\omega / m_e c^2$ , is

$$\sigma = \frac{3}{4} \sigma_T \left\{ \frac{1+x}{x^3} \left[ \frac{2x(1+x)}{1+2x} - \ln(1+2x) \right] + \frac{1}{2x} \ln(1+2x) - \frac{1+3x}{(1+2x)^2} \right\}. \quad (1)$$

Looking at such an equation, I don't feel any surges of intuition! Moreover, in many circumstances where this result might apply, there are other uncertainties in the problem that make unnecessary accuracy superfluous. In such cases, you could approximate by assuming that for low-energy photons,  $x < 1$ , the cross section is the low-energy limit of this expression, which is just  $\sigma = \sigma_T = 6.65 \times 10^{-25} \text{ cm}^2$ , the Thomson cross section, and that for  $x > 1$  the

cross section is the high-energy limit,  $\sigma \approx \frac{3}{8}\sigma_T x^{-1}(\ln 2x + 0.5)$ .

Here's another example, from the 2000 ASTR 688R midterm:

Suppose that low-mass stars ( $M < M_\odot$ ) have a uniform density equal to that of the Sun ( $\rho=1.4 \text{ g cm}^{-3}$ ) and a central temperature

$$T_c = 2 \times 10^7 \left( \frac{M}{M_\odot} \right) \left( \frac{R}{R_\odot} \right)^{-1} \text{ K} . \quad (2)$$

Suppose also that the stars are pure hydrogen ( $X = 1$ ), and that the main reaction burning hydrogen to helium has an energy generation rate of

$$\epsilon_{\text{eff}} \approx \frac{2.4 \times 10^4 \rho X^2}{T_9^{2/3}} e^{-3.38/T_9^{1/3}} \text{ erg g}^{-1} \text{ s}^{-1} , \quad (3)$$

where  $T_9 = T_c/10^9$  K. Stars stay on the main sequence until they have exhausted most of the hydrogen in their cores, which we will assume have a uniform temperature equal to  $T_c$ . To within a factor of 2, calculate the mass of the lowest-mass stars, which have main sequence lifetimes of  $\sim 10^{13}$  yr.

*Answer:* Now, this looks like a killer equation. If you tried to solve it exactly you'd need a computer. However, if you have the insight that the burning rate is extremely low and that this implies that the exponential must be very small, you have a dramatic simplification open to you. In particular, to the required accuracy you can drop the power-law prefactor (!) and treat it as simply an exponential equation, which is trivial to solve. To get the numerical answer you also need to know the energy released by hydrogen burning, to get the energy generation rate. In any case, a nasty problem is solved simply and the insight is retained, by just making an easy simplification.

### Approaches to Creativity in Astrophysics

Creativity can be said to have two steps: (1) coming up with a list of possibilities, (2) going through those and throwing out what doesn't work, to focus effort on the more promising explanations.

Let's say you want to explain a phenomenon. One approach is to simply make a big list of anything you can think of that might explain it (without culling them at this stage), then later go through the list and see if observations or other constraints absolutely rule out some of the proposals. Doing it in a two-step way like this gives you a chance to come up with something really original (by not cutting it down first), but also serves as a check against errors.

To do this successfully, you need to have a wide range of knowledge of physics and astrophysics, both to generate ideas and to test them. In this class we will try to generate a

set of tools to approach problems in cosmology, so that we can come up with ideas and cull them for the most promising.