

Distances in Cosmology

One of the most basic measurements that can be performed is that of distance. How tall am I? How about that building? How far is it to my school or travel destination? In fact, throughout the history of astronomy, distance measurements have played a key and often surprising role. In this lecture, we will first discuss direct observations, then some of the different measures of distance. See Advanced Topic 2 in Liddle for additional information.

Observations and History

Eratosthenes got the ball rolling by using clever arguments about shadows to get a remarkably good estimate of the size of the Earth. At the base of this was *direct measurement* of distance; he had soldiers pace off the distance between Alexandria and Syene in Egypt, then used geometry.

Since one can't pace off distances in space, another approach is needed. The simplest approach is one of *parallax*. If an object is viewed from two different locations, then its direction appears to change, by an amount that is inversely proportional to its distance from the viewing points. This is one of the ways we obtain depth perception; looking out with first one eye, then the other, shows that close things appear to move more than distant things. If you know the distance between your viewing points and the angle of difference, you know the distance to an object. This principle was used to determine the distance to the Sun (by measuring the apparent times at which Venus crossed the Sun's limb). Then, knowing this distance, parallax has been used to determine the distance to many stars.

Parallax, however, has its limits. The problem is that something sufficiently distant has such a tiny angular shift that it can't be measured. For example, consider a parsec. This is the distance an object has to be so that with a baseline shift of one astronomical unit (about 150 million kilometers, the average distance of the Earth from the Sun), the object appears to shift by one second of arc ($1/3600$ of a degree). This is about 3.26 light years, not quite as far as the distance to the nearest star from the Sun. The problem is that a second of arc is already really small; it is the apparent size of a basketball as seen from 60 kilometers, about the distance from Baltimore to Washington, DC. For comparison, the center of our Milky Way galaxy is about 8,000 parsecs away, so the parallax shift would be $1/8,000$ arc seconds, less than a basketball at the distance of the Moon! This is already an impractical measurement with current technology (although future spacecraft such as GAIA might be able to manage, barely), and galaxies are much farther away still, so some other method is needed.

The key turns out to be that you can also measure distances if the thing you're looking at has a known luminosity or size. For example, if you saw a friend of known height walking

down the street, then by seeing the angle your friend subtended you could estimate his/her distance. If your friend had some strange penchant for carrying lit 100 watt light bulbs, then measurement of the apparent brightness of the bulb would also tell you the distance. Incidentally (and this will be important later), in normal Euclidean geometry those two measures would give you the same distance, but in a curved universe they won't if the distance is large. We will therefore call them, respectively, the "angular diameter distance" and the "luminosity distance".

With this in mind, if we can find a "standard candle", whose luminosity we know, then we can measure distances. By analogy, we could do the same with a "standard ruler" whose size we know, but this plays less of a role and we won't focus on it. The ideal standard candle would be one that (1) is easy to identify due to other properties, and (2) can be seen to a large distance, so that observations of it probe a significant volume of the universe.

One might think that the best standard candles are rock-steady in their luminosity, but in practice the most useful ones are variable. The reason is partially that they are thus easier to pick out in other galaxies (which have billions of steady stars), but also partially happenstance.

The first of these, and arguably still the workhorse of extragalactic distance determinations, is the Cepheid variables. Named after delta Cephei (the fourth brightest star in the constellation of Cepheus), these stars are typically hundreds to thousands of times brighter than the Sun. Their brightness changes periodically, and the longer it takes to change the brighter the star is. These are well-understood theoretically and observationally, and played an essential role in Edwin Hubble's demonstration that the distance to galaxies is proportional to the apparent speed with which they recede from us (more directly, to their redshift, which is the amount by which the wavelength of light is changed from emission to our observation).

However, bright though Cepheid variables are, they can't be seen to great enough distances. Beyond roughly a hundred million light years it becomes very difficult to identify Cepheids, but to really learn about cosmology requires several billion light years. That's because over small distances, the relation between, say, angular diameter distance and redshift, or luminosity distance and redshift, does not depend on whether there is or is not a cosmological constant, or the total value of Ω . These differences only become apparent over much larger distances. It is therefore necessary to find a standard candle that is much brighter still.

Since there aren't stars much brighter than Cepheids, we have to look elsewhere. Supernovae are a good candidate; they are extremely bright (they can briefly outshine their host galaxy), and unlike the integrated emission from a galaxy are highly variable, so they draw attention to themselves. It turns out that there are in fact two fundamental types of

supernovae. In one case, they result from the collapse of a core of a massive star, which then forms a neutron star or a black hole. In the other, a white dwarf close to its maximum mass (the “Chandrasekhar mass”, which is a little less than 1.4 times the mass of the Sun) accretes mass from a companion, starts collapsing, then fusion of the carbon and oxygen in the white dwarf runs away and blows the white dwarf to bits, leaving no remnant. Through an unfortunate historical accident, the white dwarf type is called a Type Ia supernova, and all the rest (including Types Ib, Ic, Ip, and II) are the core collapse kind.

Since, according to prevailing theory, all Type Ia supernova involve white dwarfs of about the same mass, we might hope that they all behave similarly and thus give about the same actual peak brightness. It isn’t quite that simple, but in the 1990s it was realized that most of the variation in peak brightness was related to how long the light takes to die away; the longer it takes, the brighter it is. Correcting for this gives a very nice standard candle (or “standard bomb”, since this is an explosion). These are bright enough to be seen to when the universe was 1/4 of its present age, hence they are ideally suited for probing cosmology.

Different Distance Measures

An excellent summary of various distance measures is given by David Hogg

(see <http://www.arxiv.org/abs/astro-ph/9905116>). He discusses many different measures:

- The *proper distance* D_P that we measure to an object is the distance we would get if we were to take a snapshot of the universe and directly measure (e.g., pace off) the distance between where we are and where the object is, at some fixed time.
- The *luminosity distance* D_L is how far an object of known luminosity L (measured in energy per time) would have to be in Euclidean space so that we measure a total flux F (measured in energy per area per time), that is, $D_L = \sqrt{L/(4\pi F)}$.
- The *angular diameter distance* D_A is the distance an object of known size l (say, one meter) would have to be in Euclidean space so that it appeared to be its measured angular size θ ; that is, $D_A = l/\theta$.

See Figure 1 as well.

Let us now try to understand some of the trends in these distances. First, in his figures, Hogg shows that, as a function of redshift z , all these measures are very similar for low z (i.e., close by things), but that at higher z they are different. Specifically, as z becomes

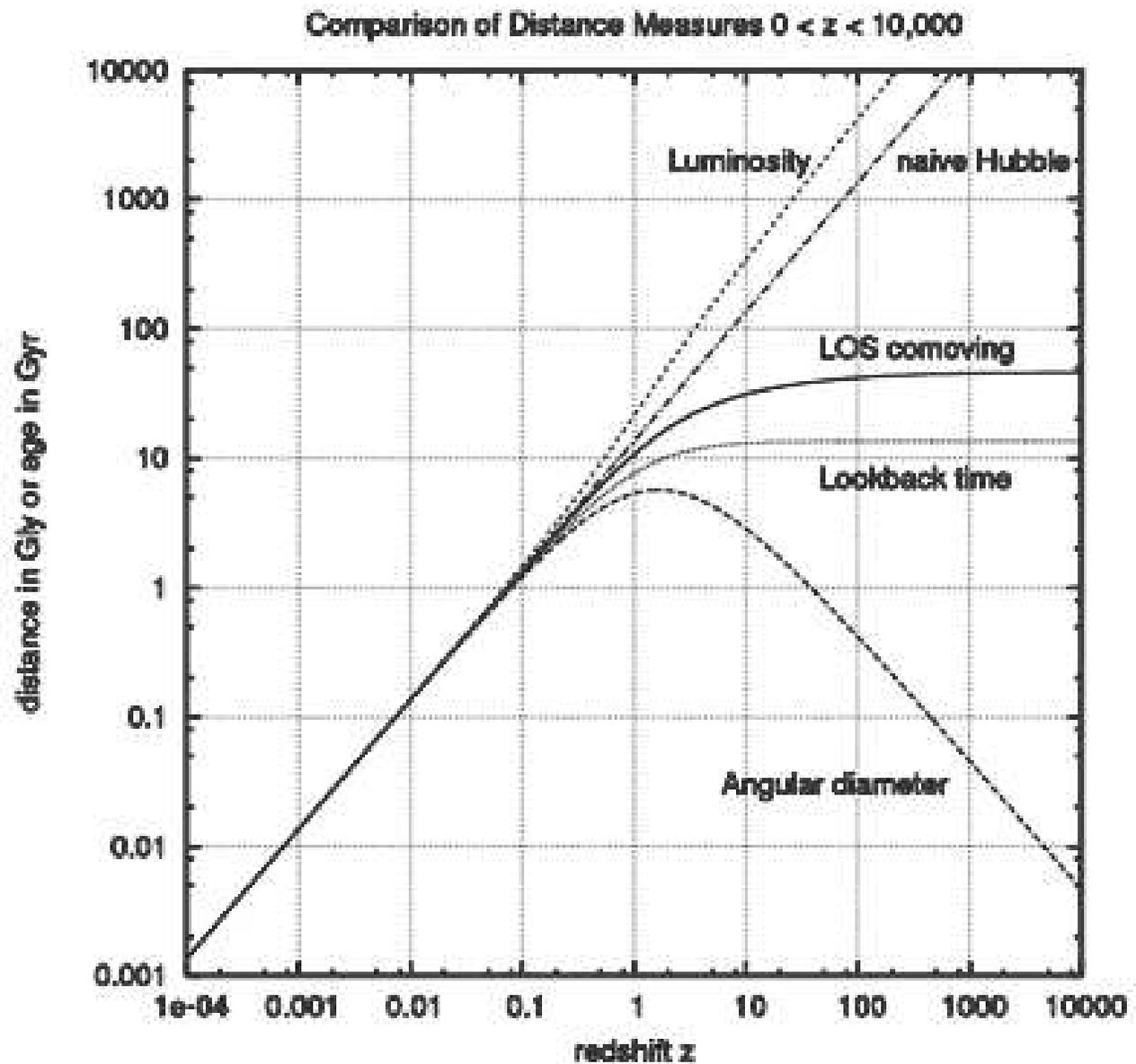


Fig. 1.— Different distance measures as a function of redshift. Note, and try to understand, why luminosity distance increases indefinitely, the comoving distance saturates, and the angular diameter distance reaches a peak and returns to zero. From <http://content.answers.com>

larger the proper distance keeps increasing, but tends to a maximum; the luminosity distance increases indefinitely; and the angular diameter distance reaches a maximum and then actually decreases at higher z . **Ask class:** qualitatively, explain these behaviors as well as their agreement at low z .

Answer:

We do expect all these distances to be the same close by. This is because even if spacetime is warped, in a small patch it looks flat and Euclidean. At larger distances it's different. The proper distance has to reach a maximum because the size of the observable universe is finite. The luminosity distance, though, is a measure of flux, and flux is also affected by redshift. In fact, there are two factors of $1+z$ that enter (one because the energy of each photon decreases by that factor, and the other because the time between photon arrivals also increases by that factor). Since the redshift can be arbitrarily large, so can the luminosity distance. The angular diameter distance is another story. Clearly, if you start a meter stick close to us and move away, it looks progressively smaller. However, at some point it starts looking bigger. The reason is that if the light from the meter stick started off when the universe was much smaller than it is today, the meter stick occupied a big fraction of the size of the universe! Since the emission from any redshift fills the whole sky (a full 4π steradians), the meter stick looks bigger, hence its angular diameter distance is *smaller*. To make this more dramatic, consider the angular diameter distance if the light was emitted when the universe was just two meters in diameter. The meter stick would then appear to span 180° !

Of course, even cosmologists have to encounter reality at some point. For example, when we measure the luminosity distance, we really want the *total* flux F , measured over all wavelengths, and the *total* luminosity L . However, as we go to higher redshifts, the wavelengths are themselves redshifted, so that for example the light we measure in the visible band might have started in the ultraviolet. **Ask class:** what are some complications that this might introduce in the measurements?

Answer: The problem is that we can only measure some of the radiation emitted by any given object. For example, hard ultraviolet radiation is tough to measure because it is easily absorbed by intervening dust and gas. If we use visible light to observe a galaxy at redshift $z = 4$, the light was very much in the ultraviolet when it was emitted (the wavelengths were a factor of $1+z = 5$ shorter at the time).

The result is that corrections have to be made for this. One way is to look at longer wavelengths; thus, for example, the rest-frame optical radiation emitted by galaxies at high redshift will be received in the infrared by us, so infrared telescopes are needed. This, however, works best if you already know the redshift range that you want to observe. When looking at, say, Type Ia supernovae, you don't know this in advance. However, if you know

the spectrum very well (as you do in that case), you can use the redshift to determine the rest-frame wavelength range you see with your telescope and then figure out what the total luminosity is by extrapolating to the rest of the spectrum.

Intuition Builder

We have focused on standard candles. Can you think of any objects of known lateral size (i.e., “standard rulers”) that might be used with the angular diameter distance to constrain cosmology?