Observational Parameters

Classical cosmology reduces the universe to a few basic parameters. Modern cosmology adds a few more, but the fundamental idea is still the same: the fate and geometry of the universe can be characterized fairly simply. It is the evolution of the components that is complicated and increasingly the focus of research. In this lecture we will follow Chapter 6 in Liddle on the classical parameters of cosmology.

The Hubble Parameter

The first, and in principle simplest, of the parameters is the Hubble "constant" that we've encountered several times. The general idea is straightforward. Since the apparent recession speed is proportional to distance, all we have to do is measure the apparent speed and distance for a whole bunch of objects, then figure out the proportionality constant. Simplicity itself! But how does it work in practice?

The apparent speed really is easy, because it just amounts to measuring a redshift. In practice, what one does is take a spectrum of an object, identify spectral lines, then figure out the factor by which they are shifted compared to those same lines at rest. If the factor is $1+z$, then your redshift is z and (for close objects with $z \ll 1$), your apparent speed is cz. Voila.

Hold on, though. **Ask class:** how do we know that a particular observed line can be identified with a given line at rest? If we can't make such an identification, we're hosed.

The answer is that typically a *single* line or edge doesn't give you enough information. There are occasions when a lousy spectrum of a very distant galaxy yields only one feature, and you can guess that this is hydrogen because that's the most abundant element in the universe. Most of the time, though, you use multiple lines to make your redshift identification. Some lines come in pairs whose relative strengths are fixed by atomic physics, so that's enough. If your spectrum is good, though, you'll see lots of lines. You then shift the whole thing by a factor that lines up all those features with known lines, and you're set. It is true that every now and then something weird happens. For example, the spectrum of the first quasar (3C 273) looked bizarre until Maarten Schmidt realized that it was just several hydrogen lines redshifted by a then-unprecedented amount.

In practice, then, the major stumbling point is distance instead of apparent speed. We can get an idea of how tough this is in the following way. Bright Cepheids are about 10,000 times brighter than the Sun. The absolute visual magnitude of the Sun is about +5, and since 5 magnitudes is a factor of 100, the absolute visual magnitude of bright Cepheids is about -5. Absolute magnitudes are calibrated for a distance of 10 pc from us. At 10 Mpc (slightly closer than the Virgo Cluster), the extra factor of a million means a factor of 10^{12} less flux. This is 30 magnitudes, meaning that if you want to detect Cepheids directly you need to get to at least 25th magnitude. That takes seriously big glass on Earth (at least 5 meter mirrors), or space. In addition, to also get the redshift you need spectra. That makes it tough, but obviously not impossible.

For example, it took a Hubble Space Telescope Key Project to get the total uncertainties under 10% at the one standard deviation level. Ironically, the final best value of 72 km s^{-1} Mpc^{-1} is right in between the values of about 50 (advocated by Allan Sandage) and about 100 (advocated by Gerard deVaucouleurs). About the only point of agreement between Sandage and collaborators, and deVaucouleurs and his collaborators, was that H_0 couldn't be, say, 75! Many broken careers later, though, that's where it stands.

We should also point out that there are important *systematic* errors for which one must correct. After all, although the *average* motion of a galaxy is with the universal expansion, any given galaxy has some motion relative to that, called "peculiar motion". For example, Andromeda will run into us in a few billion years. Nearby galaxies are particularly problematic because their peculiar motions are comparable to the expected apparent recession speed. In fact, a few years ago in Russia I had a physicist tell me he had disproved Hubble because several galaxies are coming towards us. His remedy was something called the ψ-ether. He hadn't realized that when you go far enough away (in practice, at least 20 Mpc), every galaxy recedes, and the fractional errors introduced by peculiar velocity go down just as the statistical errors of measurement go up.

Finally, note that the systematic uncertainty in H_0 will be decreased further in the future, because of the nature of the distance ladder. Recall that we can measure distances in the Solar System directly, hence with negligible error. The next step out, parallax, is good for close things but not for distant things. That means that Cepheids, which are rare, are not easy to measure with parallax. Galactic distances depend on Cepheids (or depend on things that depend on Cepheids), hence when next-generation astrometric missions such as GAIA are operational, an important low rung in the ladder will be stabilized.

The Density Parameter

The next classical parameter is the density parameter Ω. To understand this, divide the Friedmann equation by H^2:

$$1 = \frac{8\pi G}{3H^2}\rho - \frac{k}{a^2H^2} \; . \tag{1}$$

From this we note that if $\rho = \rho_{\rm crit} = 3H^2/(8\pi G)$, then $k = 0$ and the geometry is flat (recall that we are still ignoring the possibility of dark energy). Therefore, let's define $\Omega \equiv \rho/\rho_{\rm crit}$. If $\Omega = 1$, then $k = 0$ and the universe has a flat geometry. Heck, while we're at it, we can define

$\Omega_k \equiv -k/(a^2 H^2)$, so that $\Omega + \Omega_k = 1$. We can similarly define separate density parameters for nonrelativistic matter, radiation, neutrinos, photinos, and boogley boogley particles, although those last are not thought to constitute a significant fraction of the universe.

Note that if $\Omega = 1$, then it is that way forever, because this implies $k = 0$ and k is constant. If instead $\Omega < 1$, you can show as we did in the last lecture that Ω_k becomes progressively more dominant as the scale factor increases, meaning that Ω becomes closer and closer to zero as time goes on. Similarly, if $\Omega > 1$ then it becomes larger and larger with time until finally the expansion halts and reverses. Indeed, given how much the universe has expanded, Ω had to be staggeringly close to 1 at very early times. This coincidence (the "flatness problem") was one of the puzzles that motivated inflationary theory, which we will discuss later in the course.

What is the critical density today? Putting in the parameters, we get $\rho_{\rm crit} \approx 10^{-26}$ kg m^{-3}. This is a ridiculously small number. It means, more or less, that at this density a typical cubic meter of the universe contains just a few hydrogen atoms. In fact, the real density of the universe is even less than this. This is a value that gives significant mass over large regions (about $1.4 \times 10^{11}\, M_\odot$ Mpc^{-3}), but it gives us an idea of roughly how far out we have to go to get a good measure of the density of the universe.

In our solar system, say out to Pluto, we have about one solar mass in a 40 AU sphere. There are about 200,000 AU in a parsec, so this radius is 2×10^{-10} Mpc and the average density is $\langle\rho\rangle \approx 3 \times 10^{28}\, M_\odot$ Mpc^{-3}. Even our Galaxy contains some $10^{11}\, M_\odot$ within a radius of 0.01 Mpc, so the density is still far larger than average. Indeed, we would need to get to distances of 100 Mpc or so for a fair sample. Again, we need difficult measurements at large distances to estimate the density. In addition, as we will discuss in far more detail when we get to Chapter 9, measuring masses is itself tricky and full of controversy. To jump to the answer, it appears that the density parameter for "normal" matter (nuclei and electrons) is a paltry 0.04, most of which is difficult to see, and the density parameter for *any* matter is another 0.23. To the surprise of most cosmologists, it doesn't add up to 1, even though Ω_k is very close to zero. See Figure 1 for a snazzy diagram of current constraints, with a sneak look at the cosmological constant.

The Deceleration Parameter

The last of the classical parameters is the deceleration parameter q_0. The point is that if we can measure the change in the Hubble parameter (or alternatively, the second time derivative of the scale factor) then we have important leverage to determine the nature and fate of the universe.

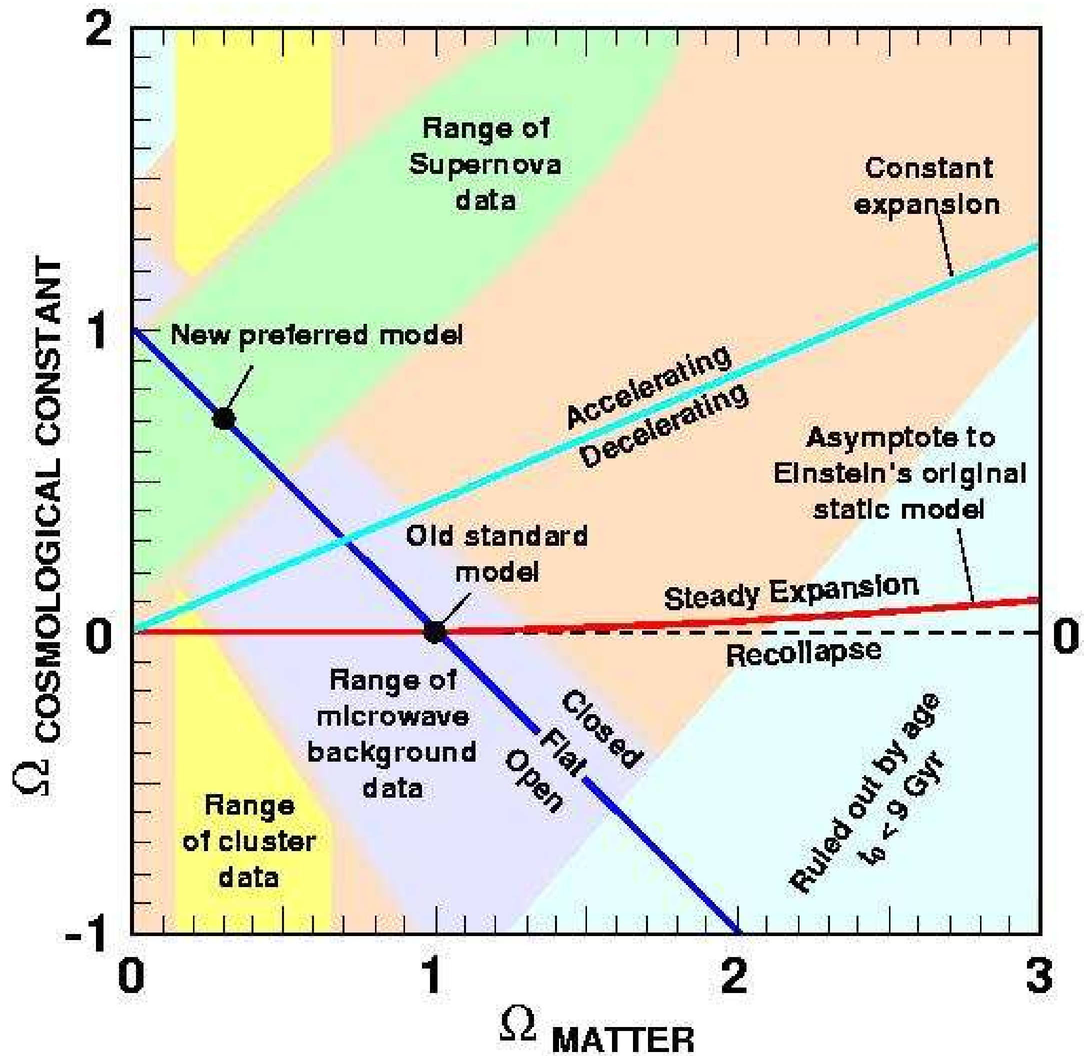

Fig. 1.— Plot showing constraints on the matter density parameter (horizontal axis) versus the effective density parameter for the cosmological constant (vertical axis), taking into account several types of data. This is the "cosmic concordance" model. From http://scipp.ucsc.edu/∼haber/ph171/cosmo.jpg

To see how this works out, let's expand the scale factor in a Taylor series:

$$a(t) = a(t_0) + \dot{a}(t_0)[t - t_0] + \frac{1}{2}\ddot{a}(t_0)[t - t_0]^2 + \cdots \; , \tag{2}$$

where t_0 is the time today. If we divide through by $a(t_0)$ and remember that $H_0 = \dot{a}(t_0)/a(t_0)$, we can write

$$\frac{a(t)}{a(t_0)} = 1 + H_0[t - t_0] - \frac{q_0}{2}H_0^2[t - t_0]^2 + \cdots \tag{3}$$

which defines the deceleration parameter q_0 as

$$q_0 = -\frac{\ddot{a}(t_0)}{a(t_0)}\frac{1}{H_0^2} = -\frac{a(t_0)\ddot{a}(t_0)}{\dot{a}^2(t_0)} \; . \tag{4}$$

To see what this gives, we go back to the acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3p/c^2\right) \tag{5}$$

which gives

$$q_0 = \frac{4\pi G}{3}\left(\rho + 3p/c^2\right)\frac{3}{8\pi G\rho_c} = \frac{\Omega_0}{2}\left[1 + 3p/(\rho c^2)\right] \; . \tag{6}$$

For example, for nonrelativistic matter with $p = 0$, $q_0 = \Omega_0/2$, and for radiation with $p = \rho c^2/3$, $q_0 = \Omega_0$.

Note, therefore, that q_0 is not independent of our other two classical parameters. In principle, though, it provides another way to measure the density parameter. That is, if we are confident that most of the mass-energy in the universe is nonrelativistic ($p = 0$), then a measurement of the deceleration parameter can tell us Ω_0 without our having to go through the difficult procedure of weighing distant things.

When I was a young, innocent graduate student back in the last millennium, it was thought that eventually it would be determined that $\Omega_0 = 1$ and $q_0 = 1/2$. Yes, it was true that those foolish astronomers persistently failed in finding Ω_0 close to 1, but it was pointed out that the value of the effective density parameter seemed to get larger and larger once one considered bigger scales, and extrapolation to $\Omega_0 = 1$ seemed possible. More compellingly, $\Omega_0 = 1$ and $\Omega_k = 0$ is a special point. If we are exactly at the critical density, we will stay at the critical density forever. If we are slightly above or below the critical density, then we will deviate from critical more and more as the universe expands. Given how much expansion has gone on, the density had to be amazingly close to critical in the very early universe. How likely would it be that the density was that close to critical, yet just at this very epoch in the universe it had significant deviations?

In retrospect, cosmologists were guilty of arguments from aesthetics, similar to the ancient Greeks. Yes, it would have been pretty that way, but the universe appears to have other ideas. Indeed, strong evidence in the last decade or so indicates that in fact $q_0 < 0$, meaning that the expansion of the universe is *accelerating*, so the term "deceleration parameter" is incorrect! This has caused a revolution in cosmology. We will discuss this and other evidence in the next lecture.

Intuition Builder

Suppose there is a component of the universe with an equation of state $p = w\rho$ such that if it is the only thing around, $q_0 < 0$. Demonstrate that this component will dominate the dynamics of the universe at sufficiently large scale factor a.